Orthogonal Drawings of Series-Parallel Graphs

by

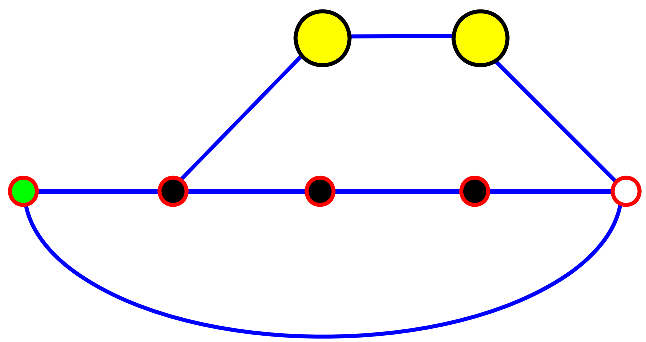
Takao Nishizeki

Joint work with Xiao Zhou

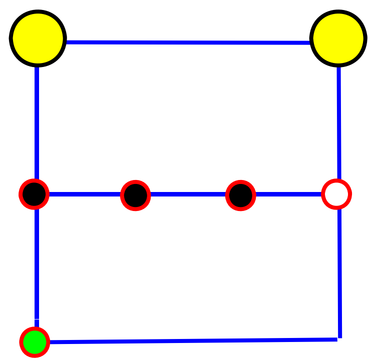
Tohoku University

Orthogonal Drawings of Series-Parallel Graphs with Minimum Bends

1. each vertex is mapped to a point
2. each edge is drawn as a sequence of alternate horizontal and vertical line segments
3. any two edges don't cross except at their common ends



planar graph



orthogonal drawings

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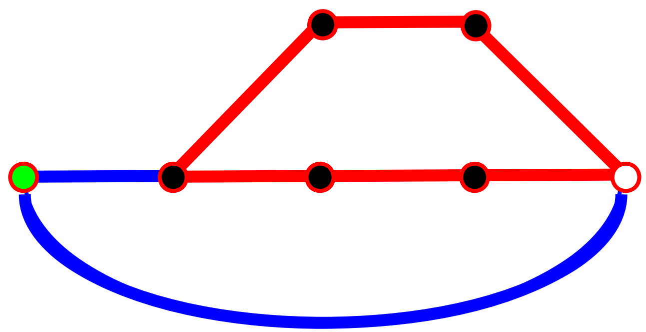
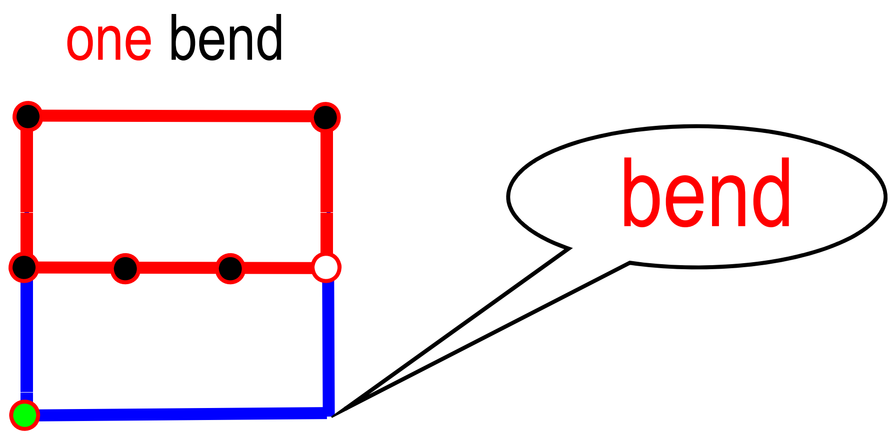


planar graph



orthogonal drawings

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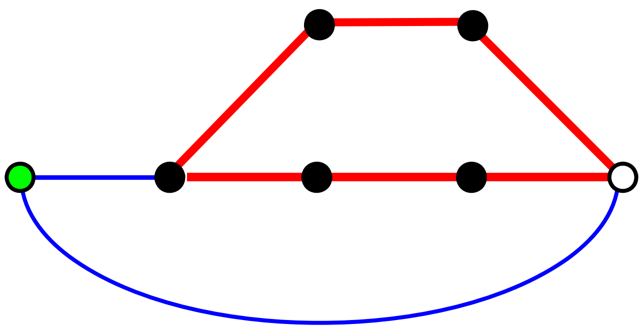
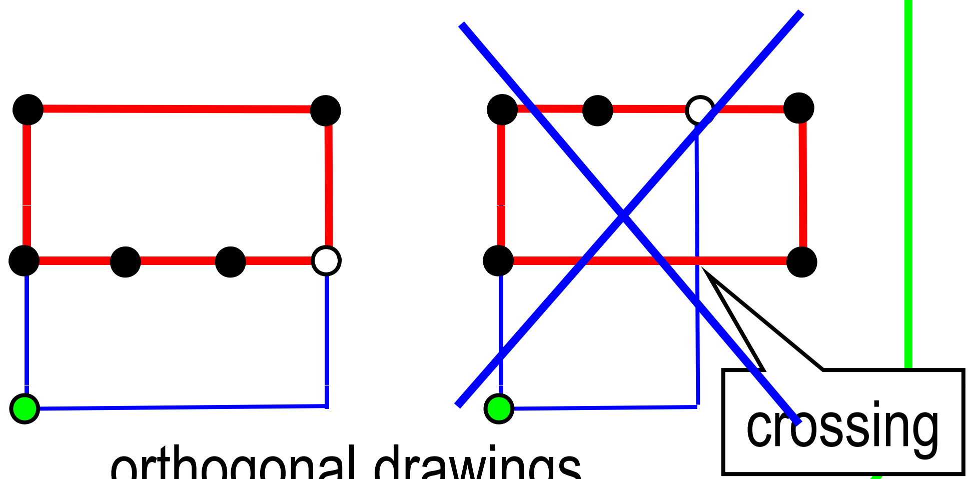
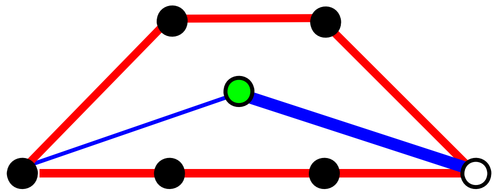

planar graph



orthogonal drawings

Orthogonal D… …arallel Graphs

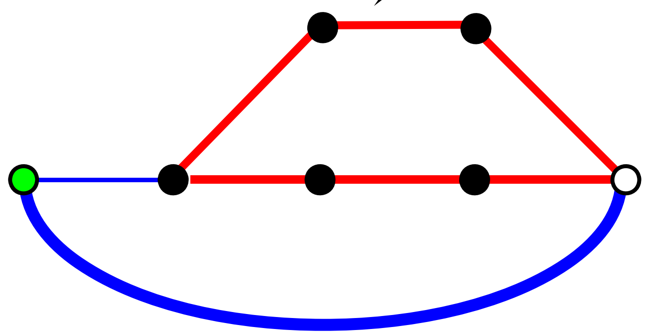
another embedding



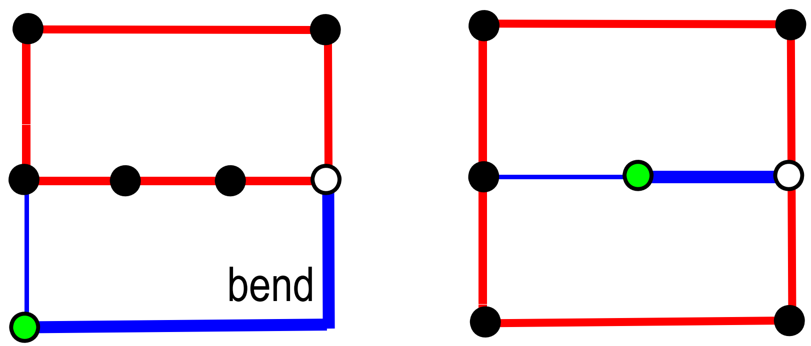
1. each vertex is …
2. each edge is dra… …quen… …nate horizontal and vertical line s… …nts
3. any two edges … …t cross except at their comm… …n ends

one bend

no bend



planar graph



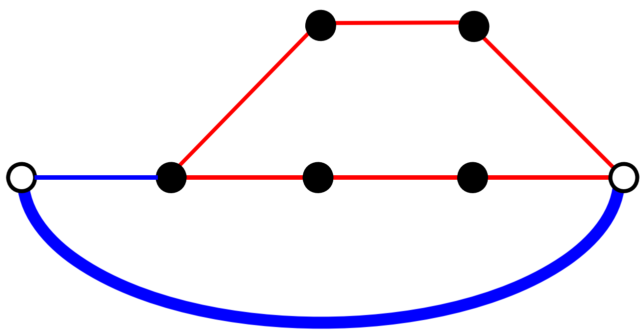
orthogonal drawings

Optimal orthogonal drawing

An orthogonal drawing of a planar graph *G* is optimal if it has the minimum # of bends among all possible orthogonal drawings of *G*.

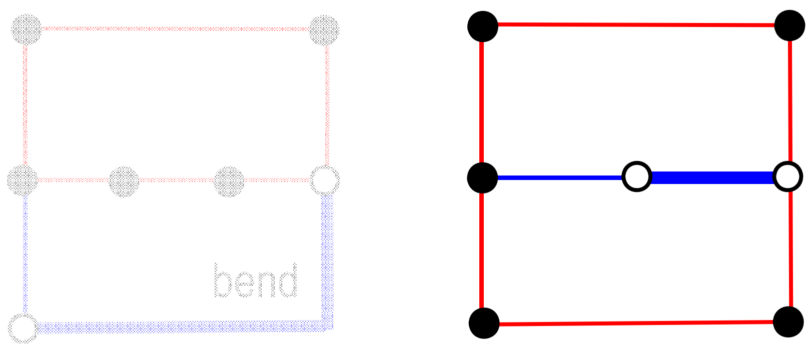
VLSI design

bend ↔ via-hole, through-hole



planar graph

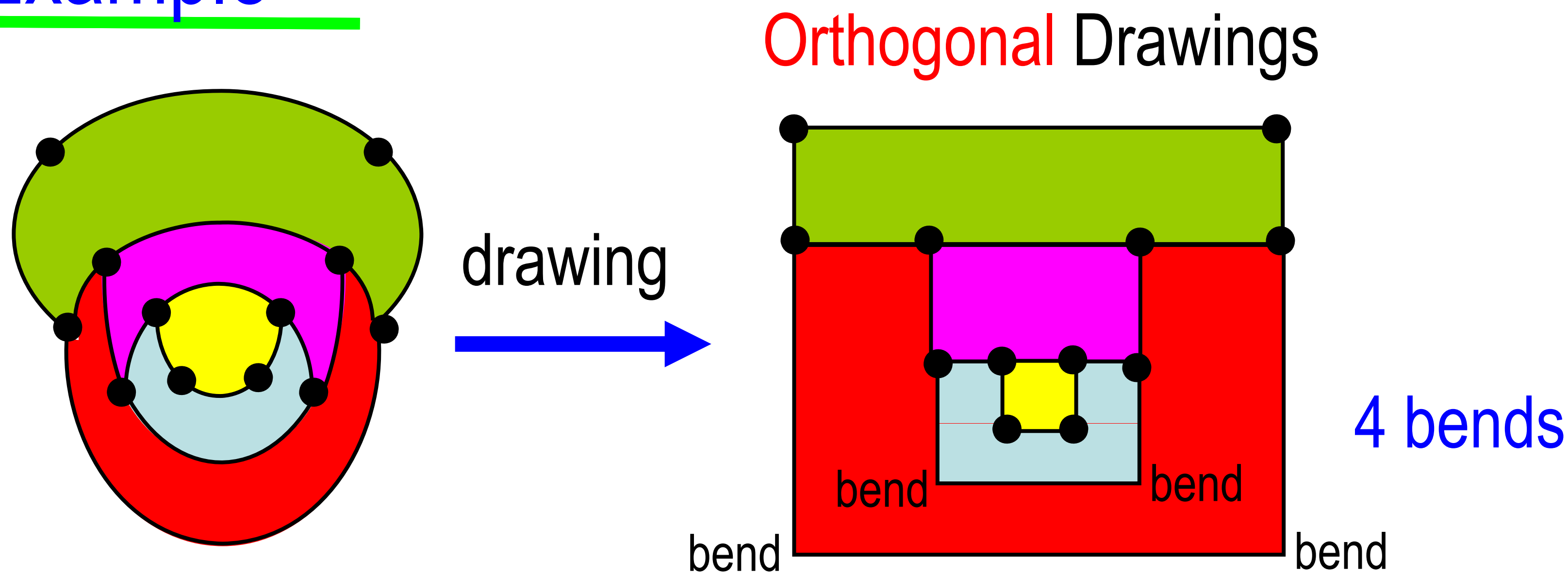
one bend no bend



orthogonal drawings

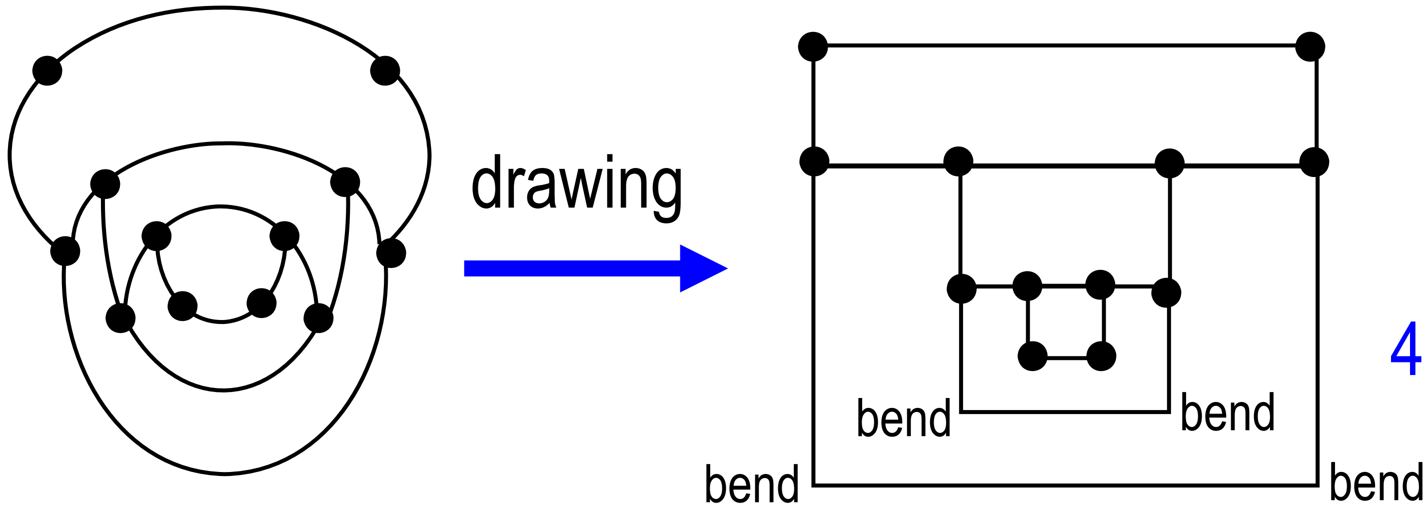
optimal

Example



Example

Orthogonal Drawings

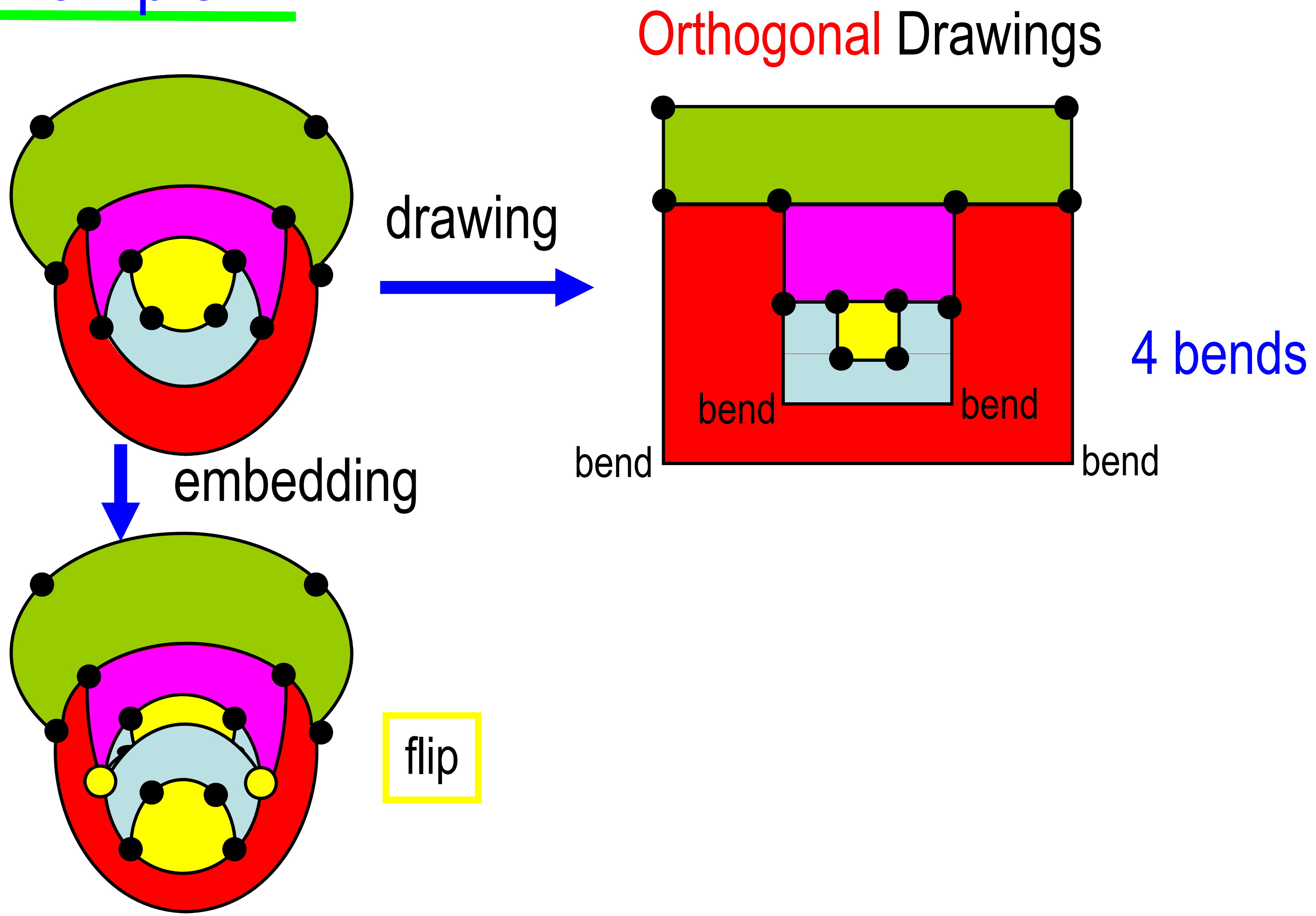


4 bends

Optimal ?

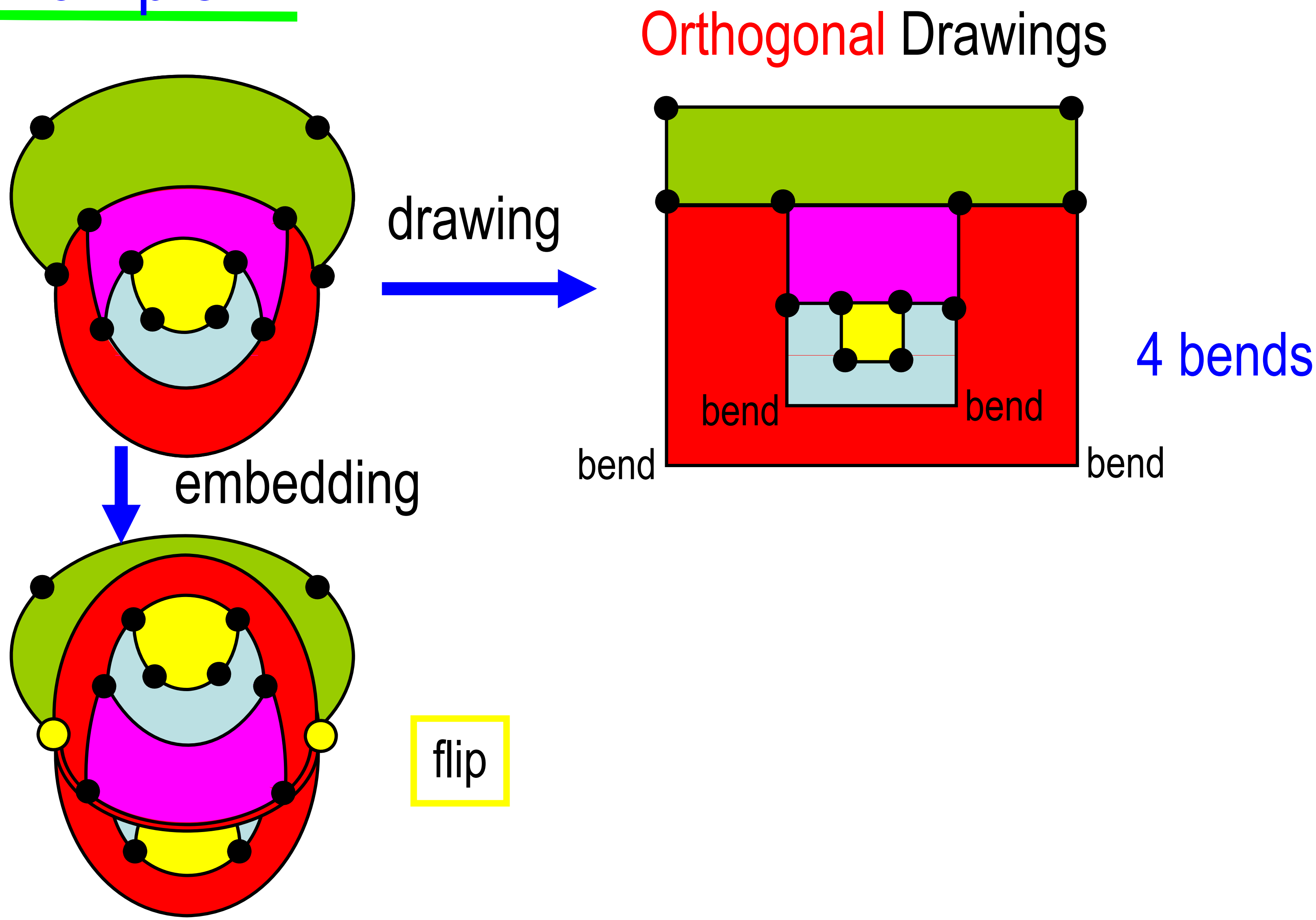
No

Example



Orthogonal Drawings
drawing
embedding
flip
bend
bend
bend
bend
4 bends

Example


Orthogonal Drawings
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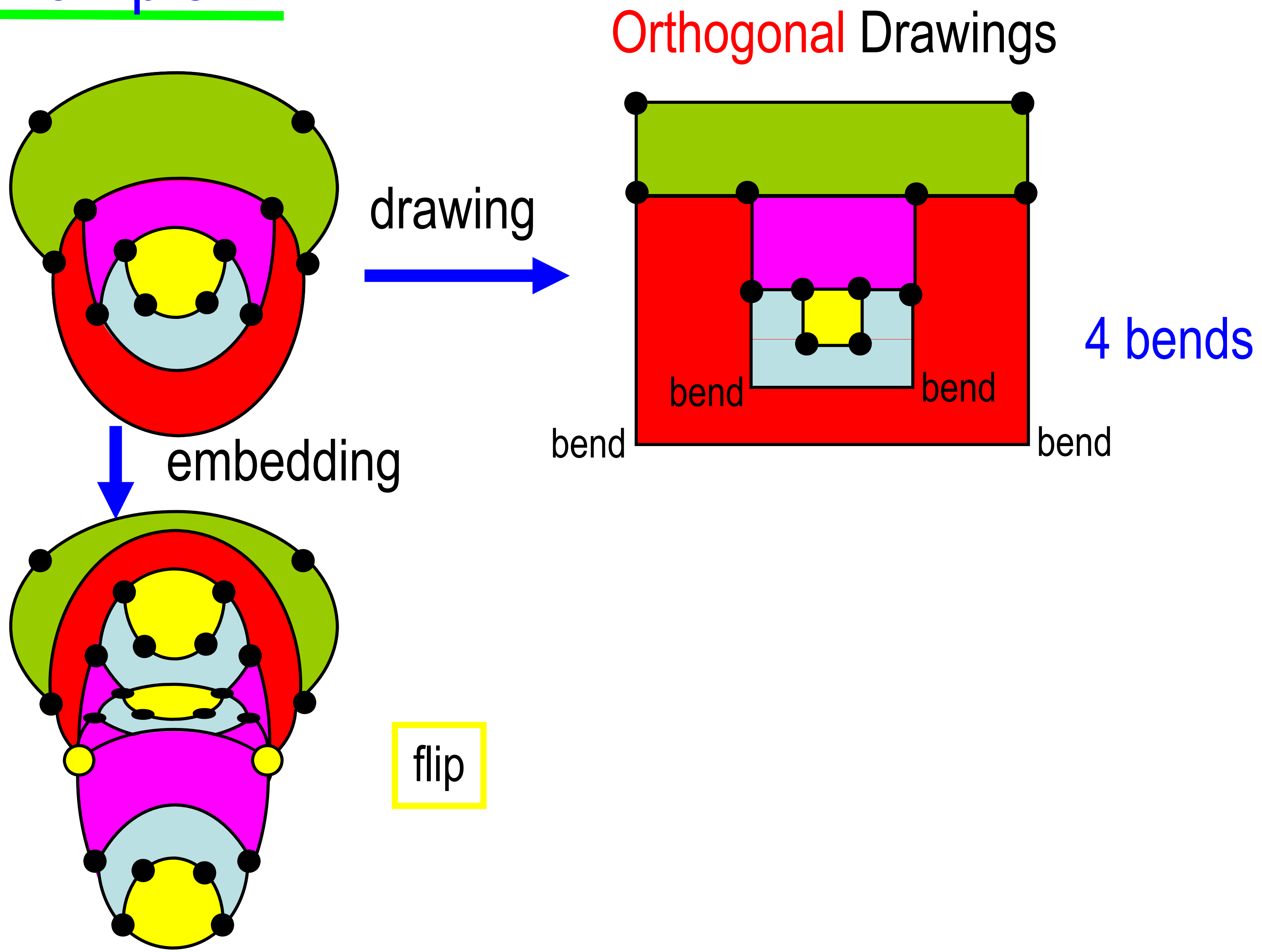
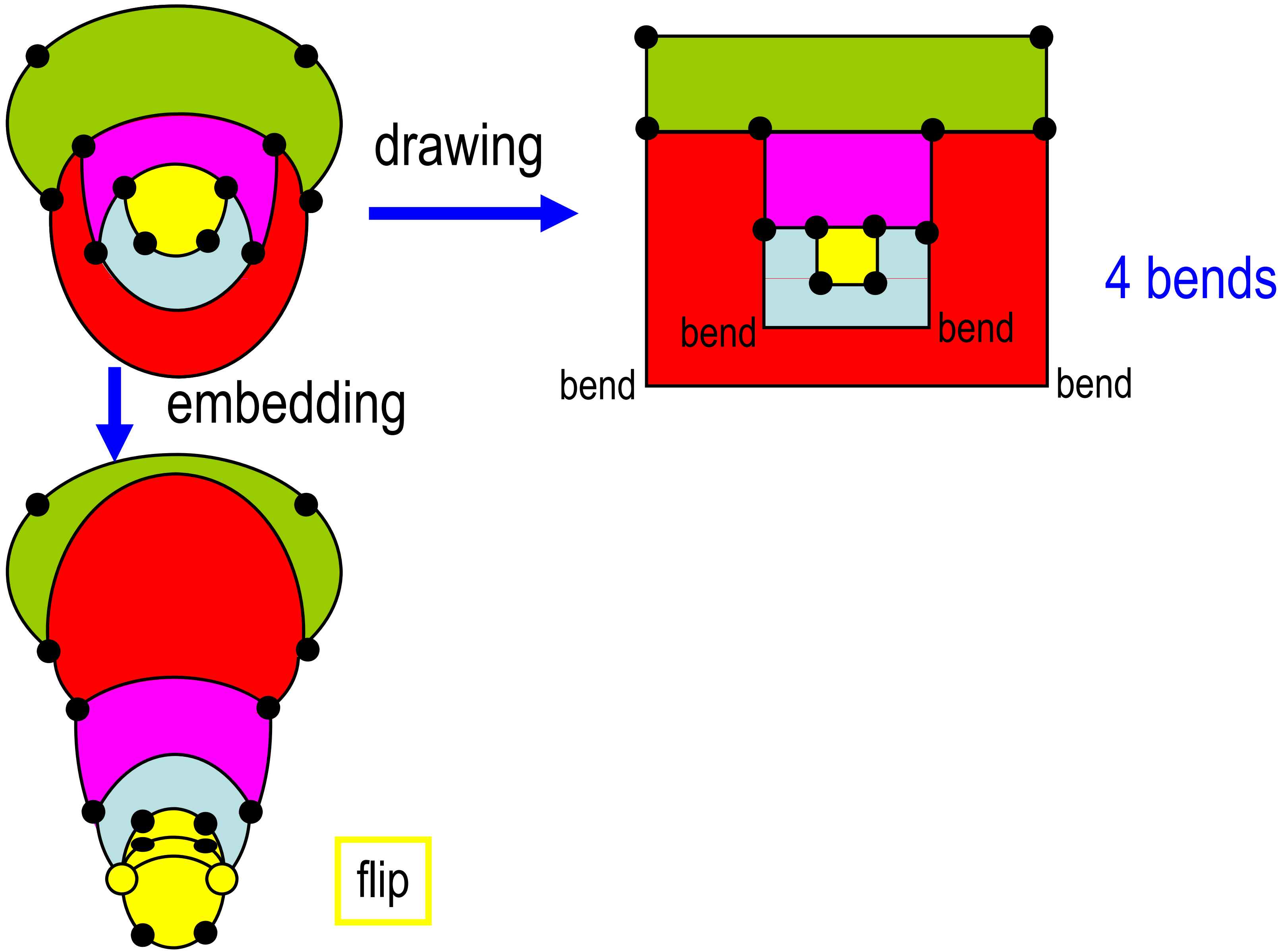


Orthogonal Drawings
drawing
4 bends
bend
bend
bend
bend
embedding
flip

Example

Orthogonal Drawings



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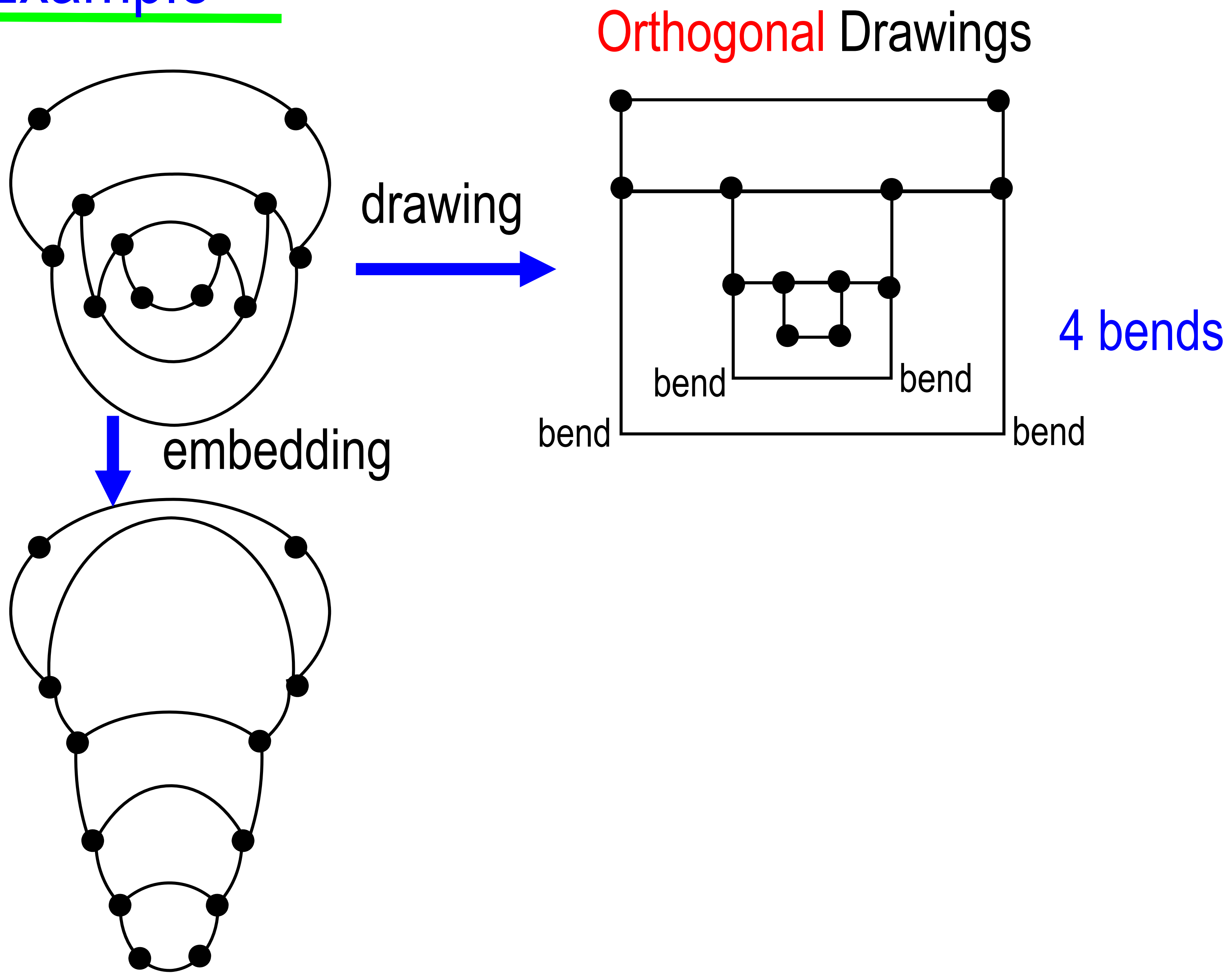
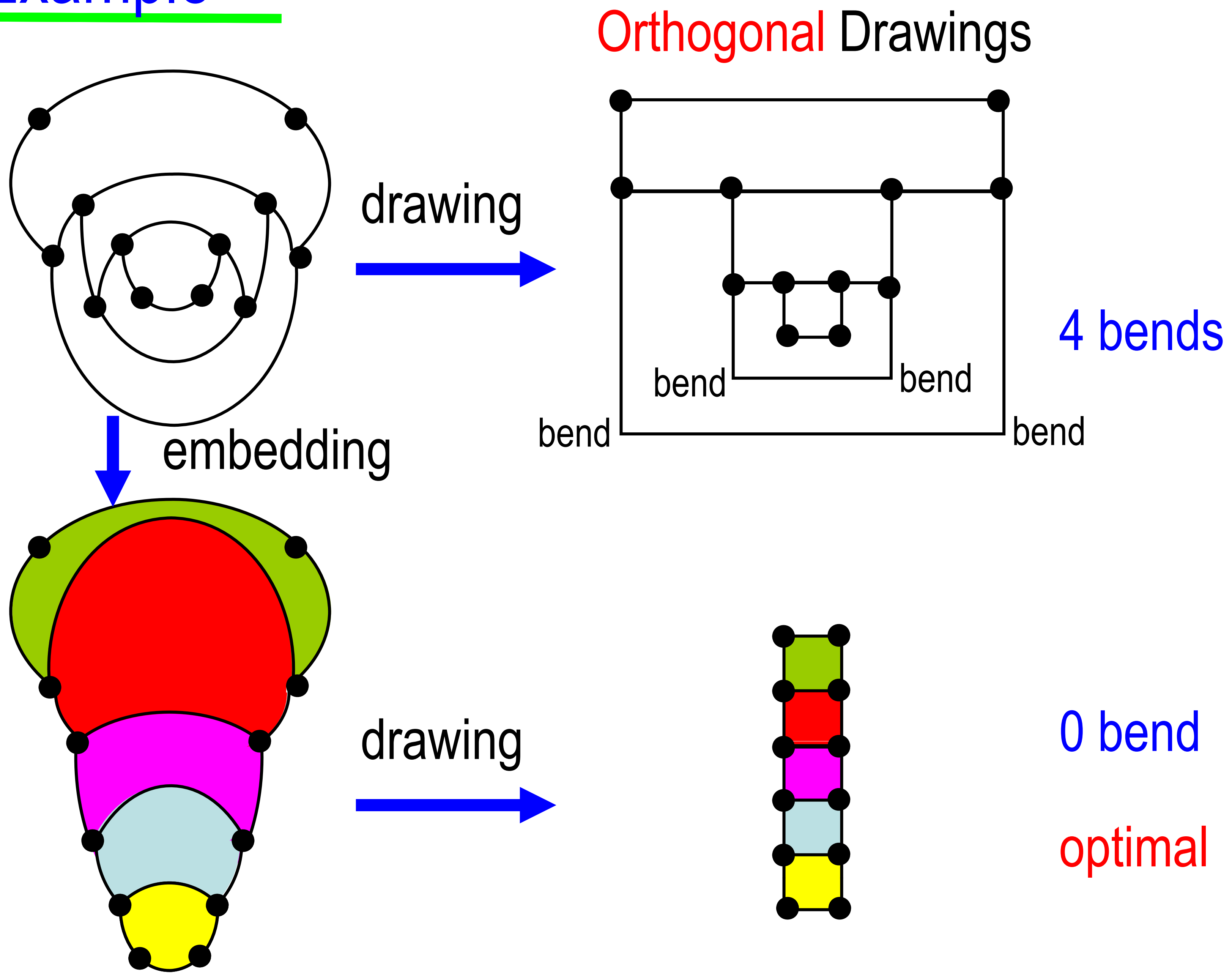
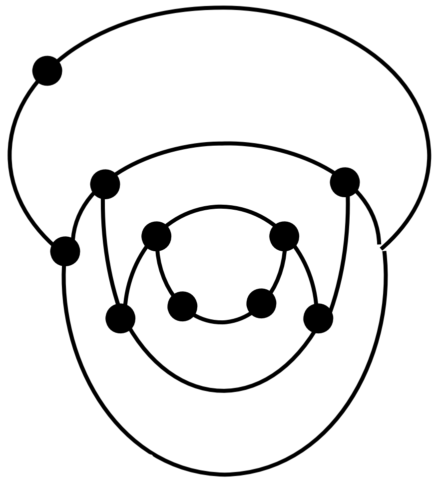

Orthogonal Drawings
drawing
embedding
bend
bend
bend
bend
4 bends

Example



Orthogonal Drawings
drawing
embedding
bend
bend
bend
bend
4 bends
drawing
0 bend
optimal

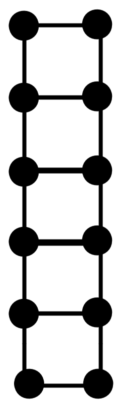
Given a planar graph



Problem

Find an optimal orthogonal drawing of a given planar graph.

Wish to find
an optimal orthogonal drawing



optimal

Optimal orthogonal drawing

An orthogonal drawing of a planar graph *G* is optimal if it has the minimum # of bends among all possible orthogonal drawings of *G*.

Problem

Find an optimal orthogonal drawing of a given planar graph.

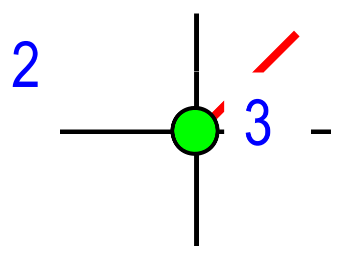
Known results

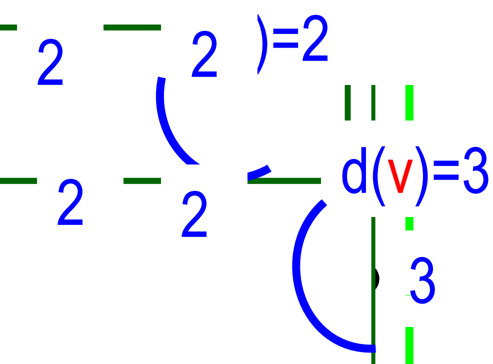
The problem:

NP-complete for planar graphs of $\Delta \leqq 4$

A. G R. Tamassia, 2001

If $\Delta \geqq 5$, then no orthogonal drawing.





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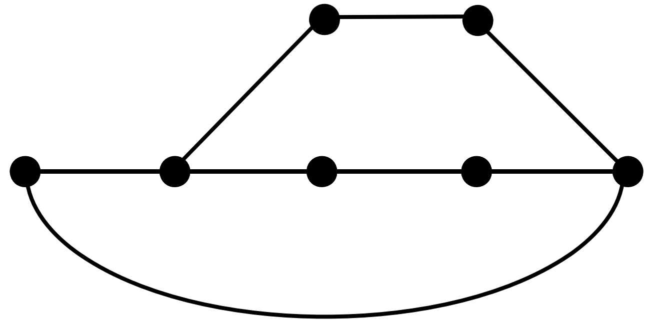
If $\Delta \leqq 3$, $O(n^5 \log n)$ time D. Battista, *et al*. 1998



Δ=3, n=7

where
n: # of vertices

Known results

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For biconnected series-parallel graphs

If $\Delta \leqq 4$, $O(n^4)$ time D. Battista, *et al.* 1998

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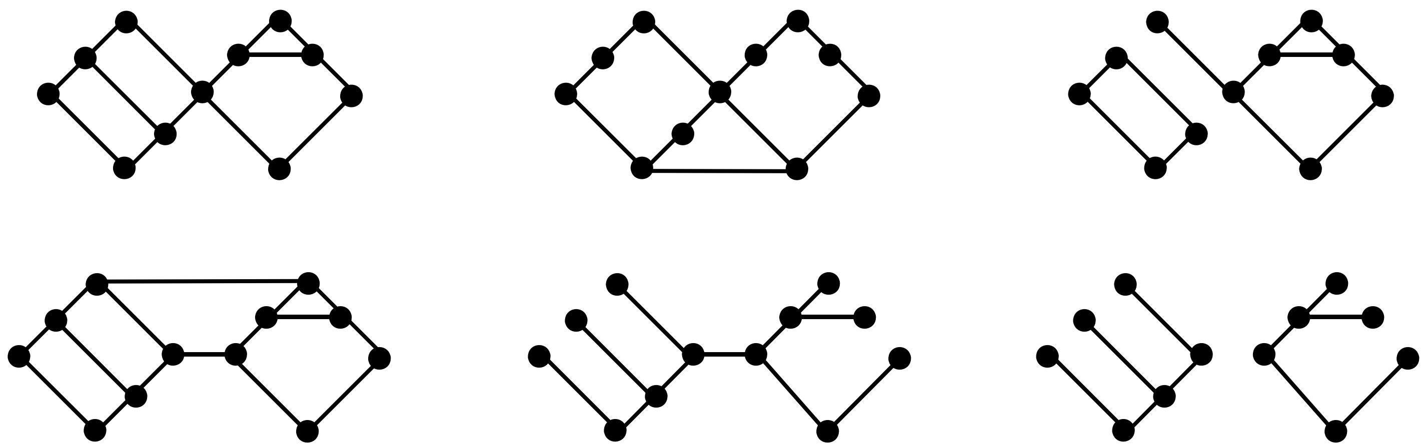
Connected graphs ?



For biconnected series-parallel graphs

If $\Delta \leqq 4$,	$O(n^4)$ time	D. Battista, *et al.* 1998
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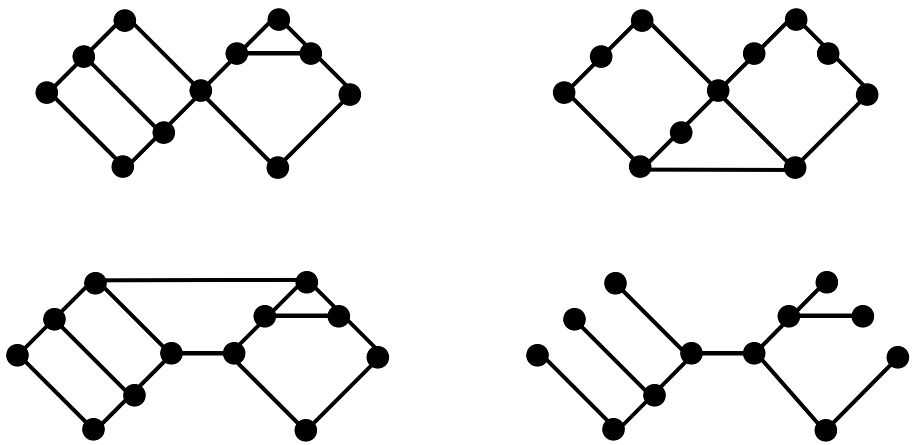
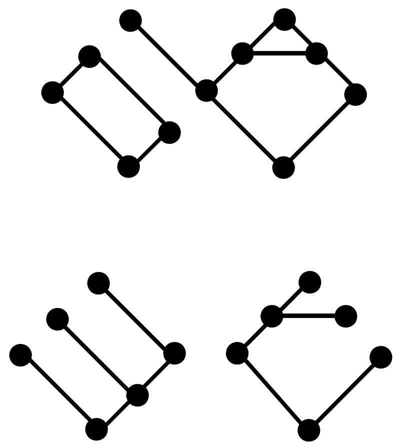


Non-connected graphs



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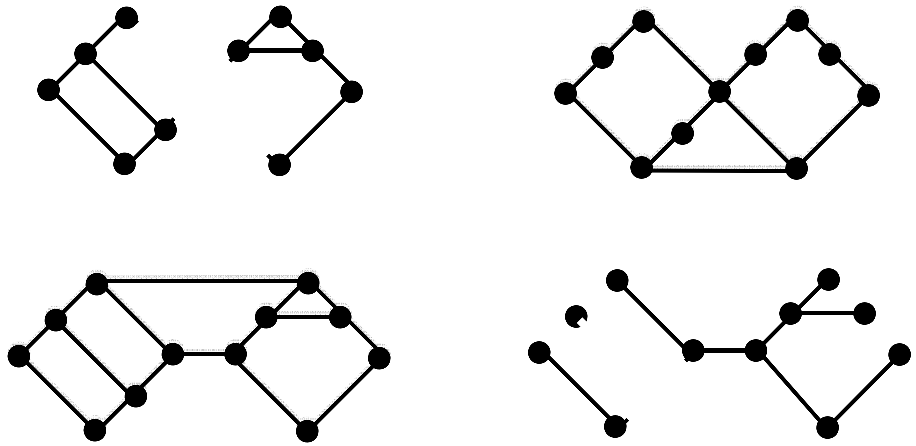

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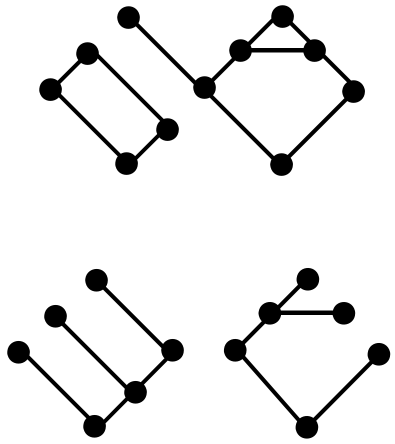

Biconnected graphs

Known results

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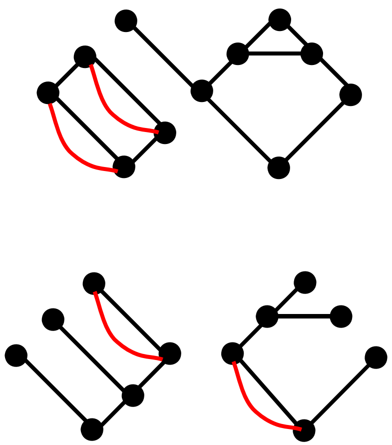
Our results

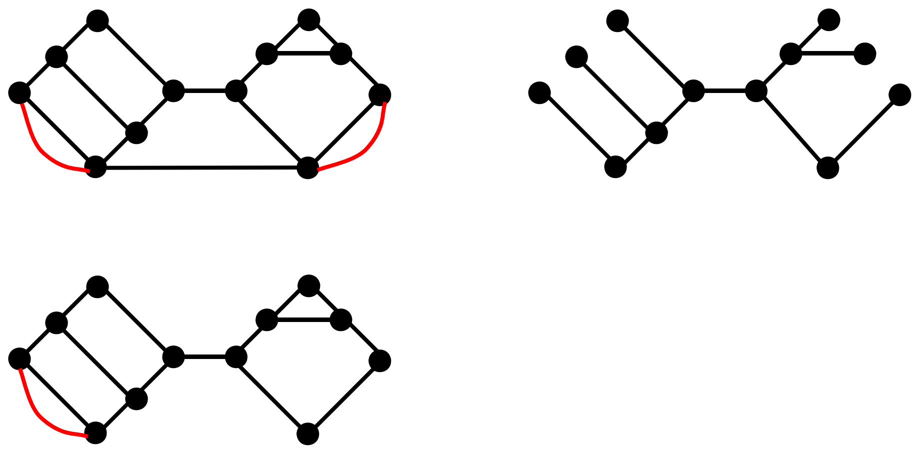
For series-parallel (multiple) graphs

If $\Delta \leqq 3$, $O(n)$ time **our result**

Non-connected



Connected



Our results

For series-parallel graphs

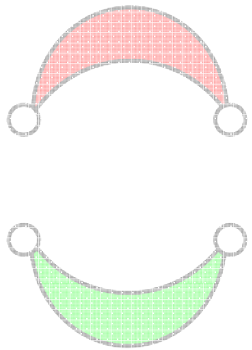
If $\Delta \leqq 3$, $O(n)$ time $\boxed{\text{our result}}$

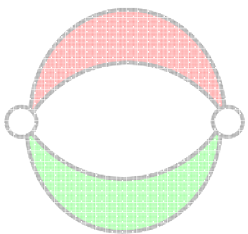
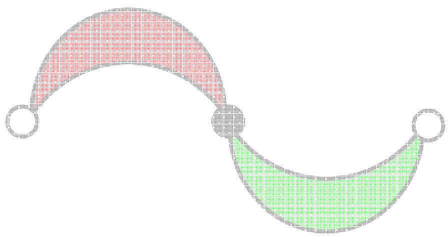
much simpler and faster than the known algorithms

Series-Parallel Graphs

A SP graph is recursively defined as follows:

(a)  a single edge is a SP graph.

(b) if  are SP graphs,

then   are SP graphs

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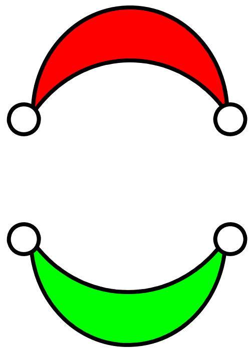
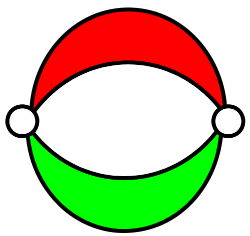
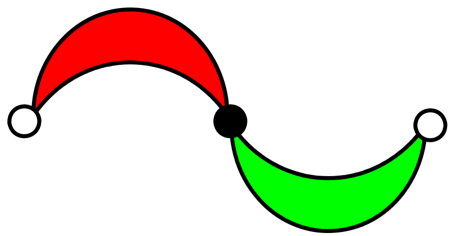
(b) if  are SP graphs,

then   are SP graphs

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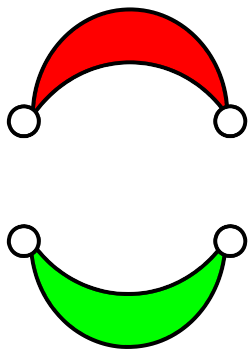
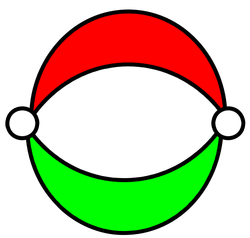
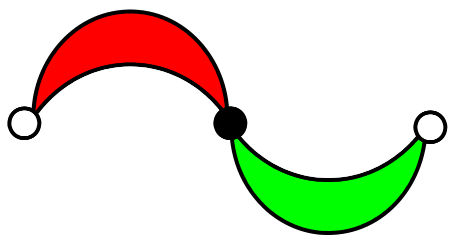
(b) if  are SP graphs,

then   are SP graphs

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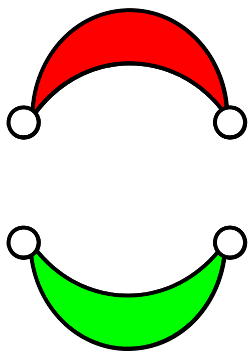
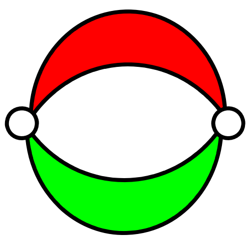
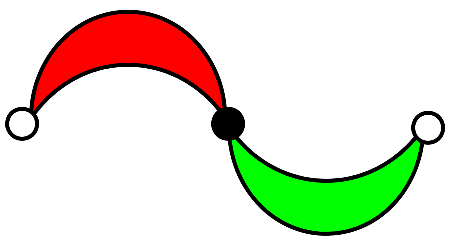
(b) if  are SP graphs,

then   are SP graphs

parallel-connection

Series-Parallel Graphs

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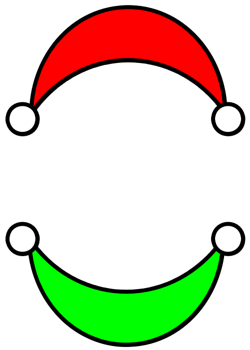
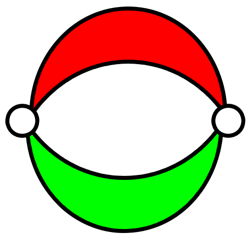
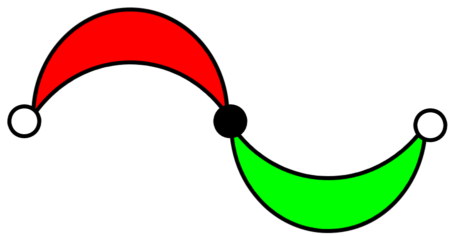
(b) if  are SP graphs,

then   are SP graphs

parallel-connection series-connection

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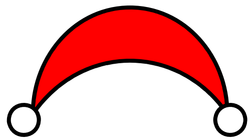
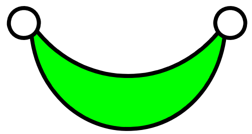
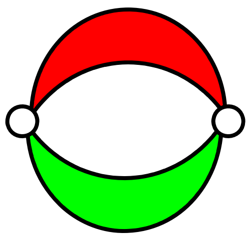
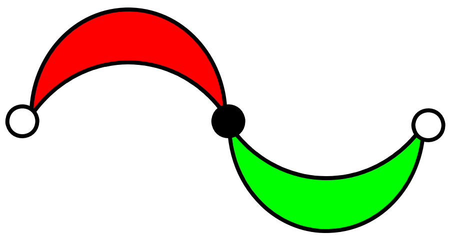
(b) if   are SP graphs,

then   are SP graphs

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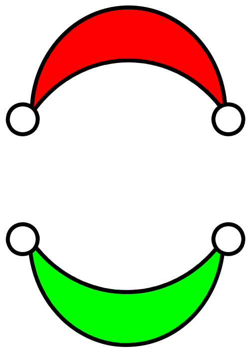
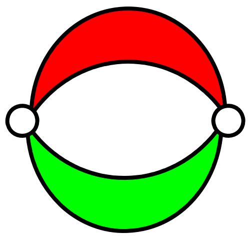
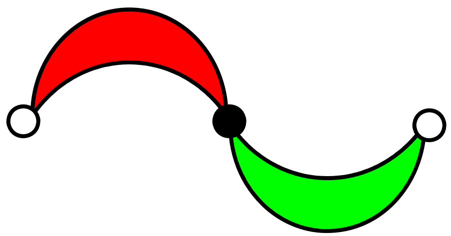
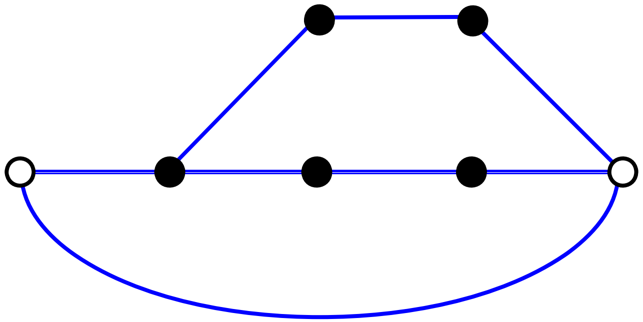
(b) if  are SP graphs,

then   are SP graphs

Series-Parallel Graphs

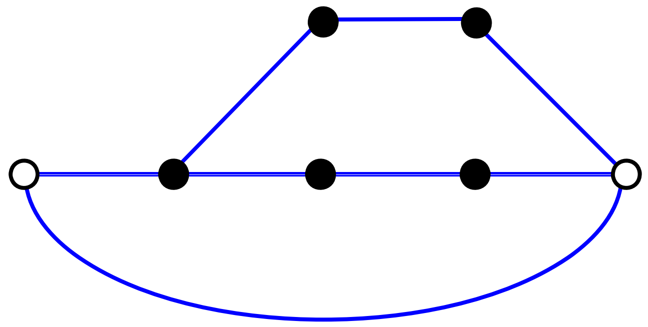
Example



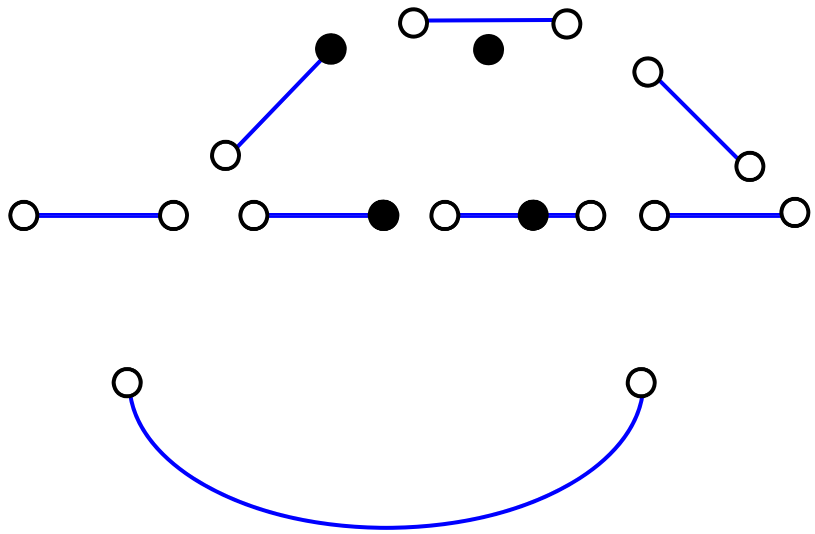
SP graph

Series-Parallel Graphs

Example



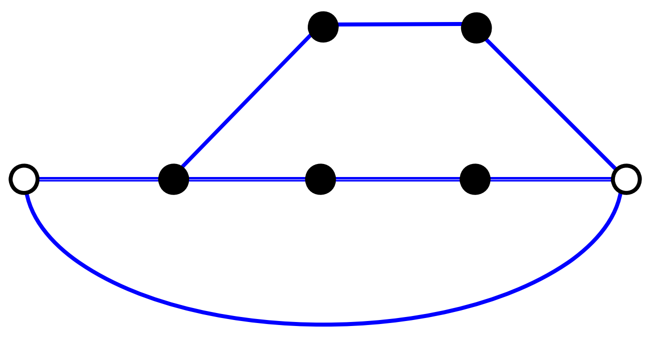
SP graph



series-connection

Series-Parallel Graphs

Example



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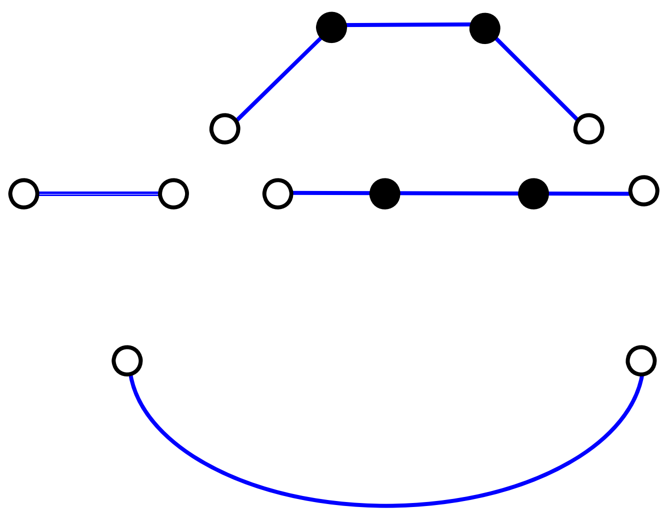
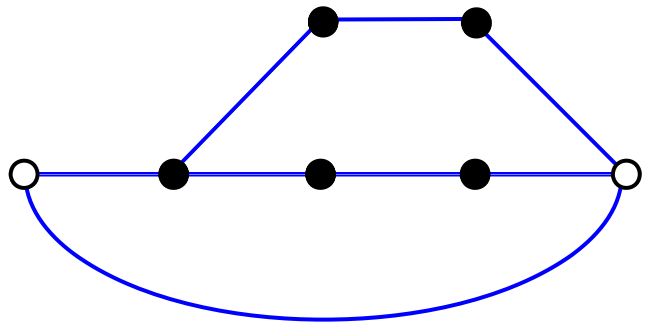


parallel-connection

Series-Parallel Graphs

Example



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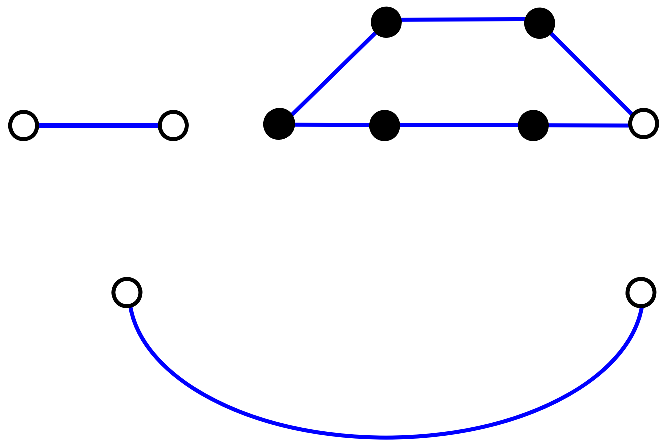
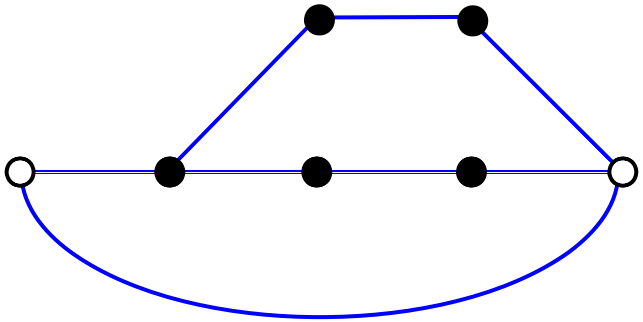


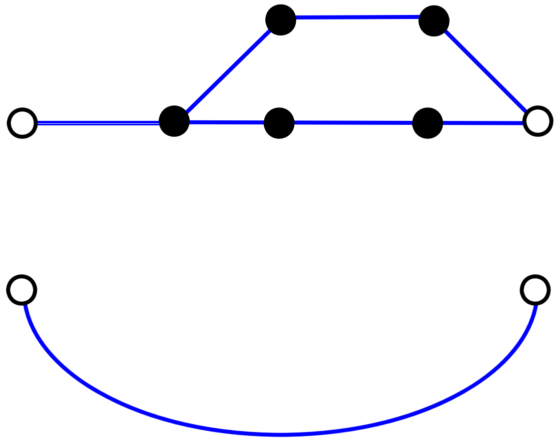
Series-connection

Series-Parallel Graphs

Example



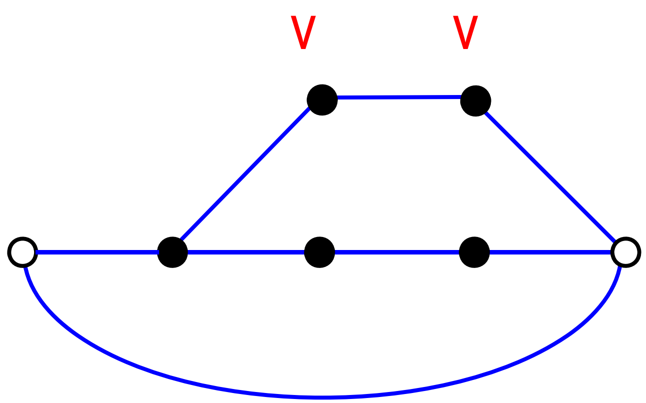
SP graph



Parallel-connection

Series-Parallel Graphs

Example



connected SP graph

Biconnected graphs G:

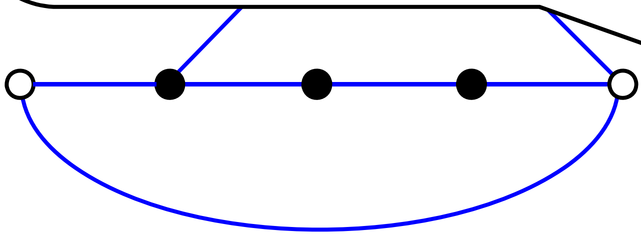
G – v is connected for each vertex v.

Biconnected SP graphs

S

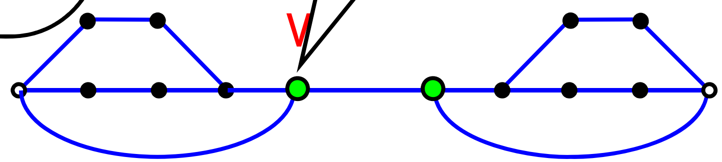
Biconnected graphs G:

$G - v$ is connected for each vertex v.



biconnected SP graph

cut vertex

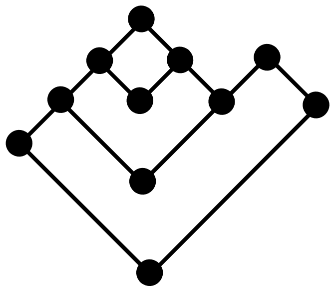


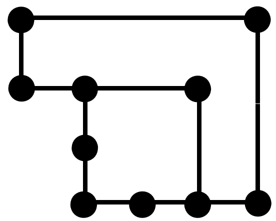
not connected

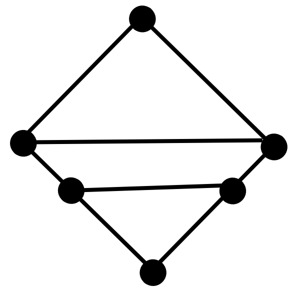
Biconnected SP graphs

Series-Parallel Graphs

Example







biconnected SP graphs

Series-Parallel Graphs

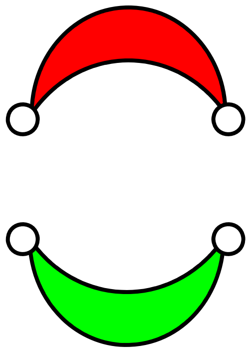
A SP graph is recursively defined as follows:

(a)

a single edge is a SP graph.

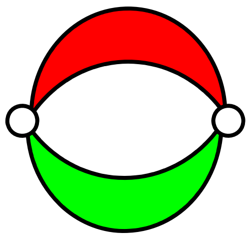
(b)

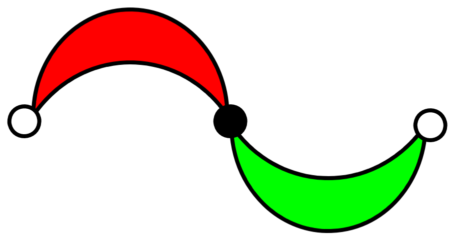
if



are SP graphs,

then





are SP graphs

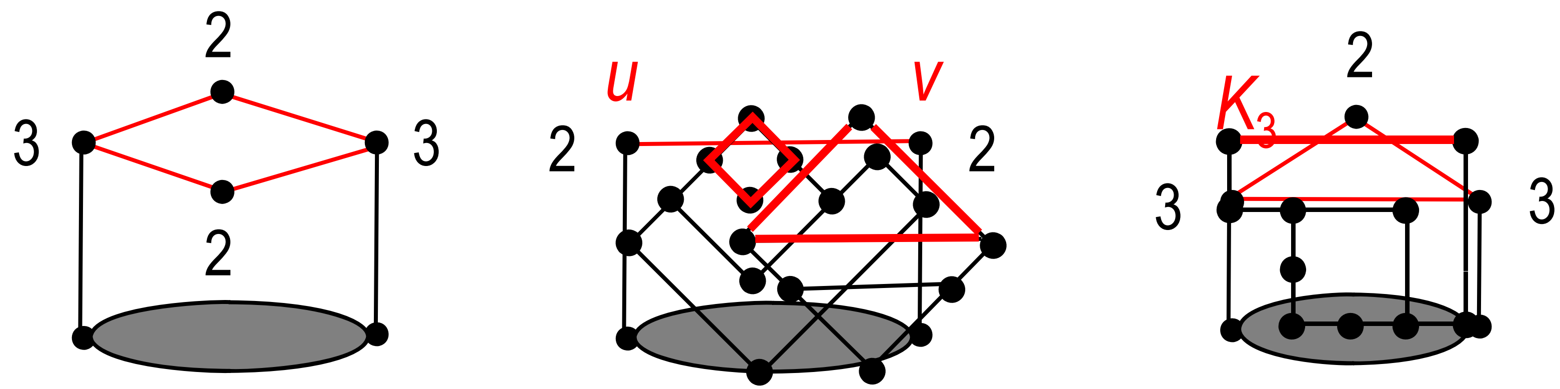
Lemma 1 (Our Main Idea)

Every biconnected SP graph G of $\Delta \leqq 3$ has one of the following three substructures:

(a) a diamond C

(b) two adjacent vertices u and v s.t. $d(u)=d(v)=2$

(c) a triangle K_3.



(a) (b) (c)

Lemma 1 (Our Main Idea)

Every biconnected SP graph G of $\Delta \leqq 3$ has one of the following three substructures:

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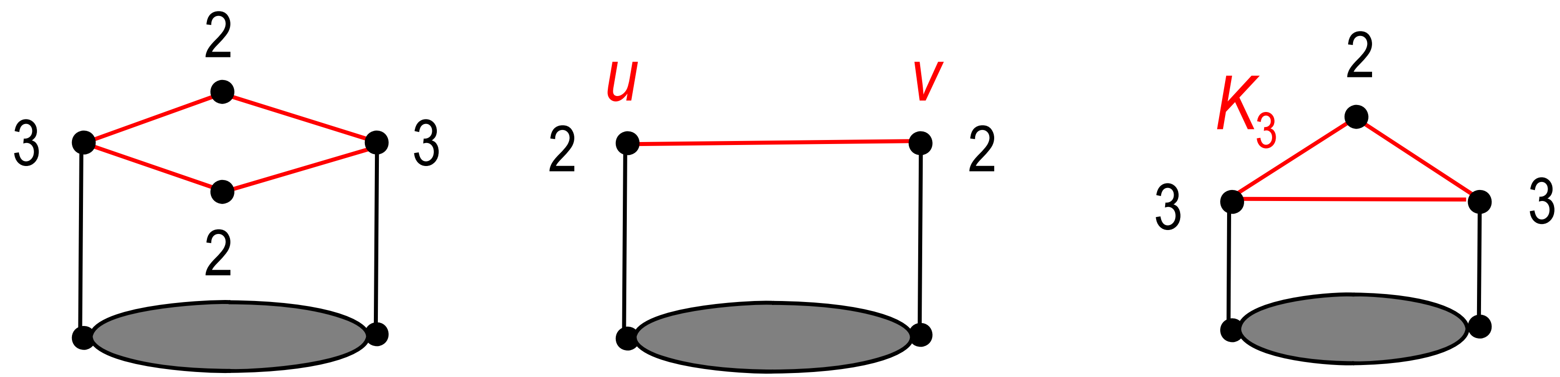
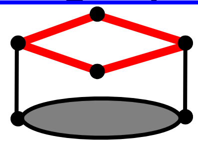


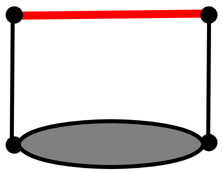
(a) (b) (c)

Algorithm(G)

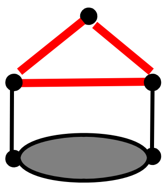
Let G be a biconnected SP graph of $\Delta \leqq 3$.

Case (a): ∃ a diamond  ,

Recursively find an optimal drawing.

Case (b): ∃ 

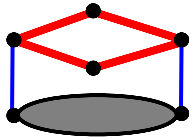
Decompose to smaller subgraphs in series or parallel, and iteratively find an optimal drawing

Case (c): ∃ 

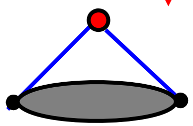
Similar as Case (b).

Algorithm(G)

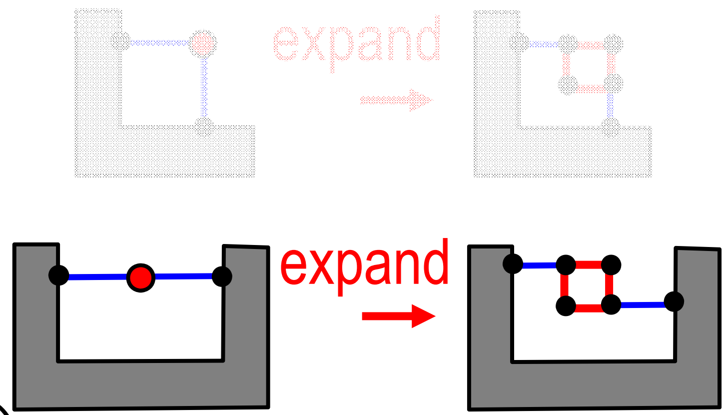
Let G be a biconnected SP graph of $\Delta \leqq 3$.

Case (a): $\exists$ a diamond  ,

contract

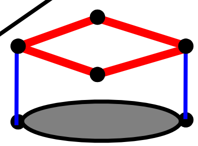
Algorithm(), find

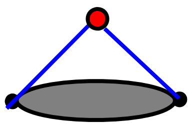
Return



optimal

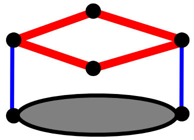
optimal

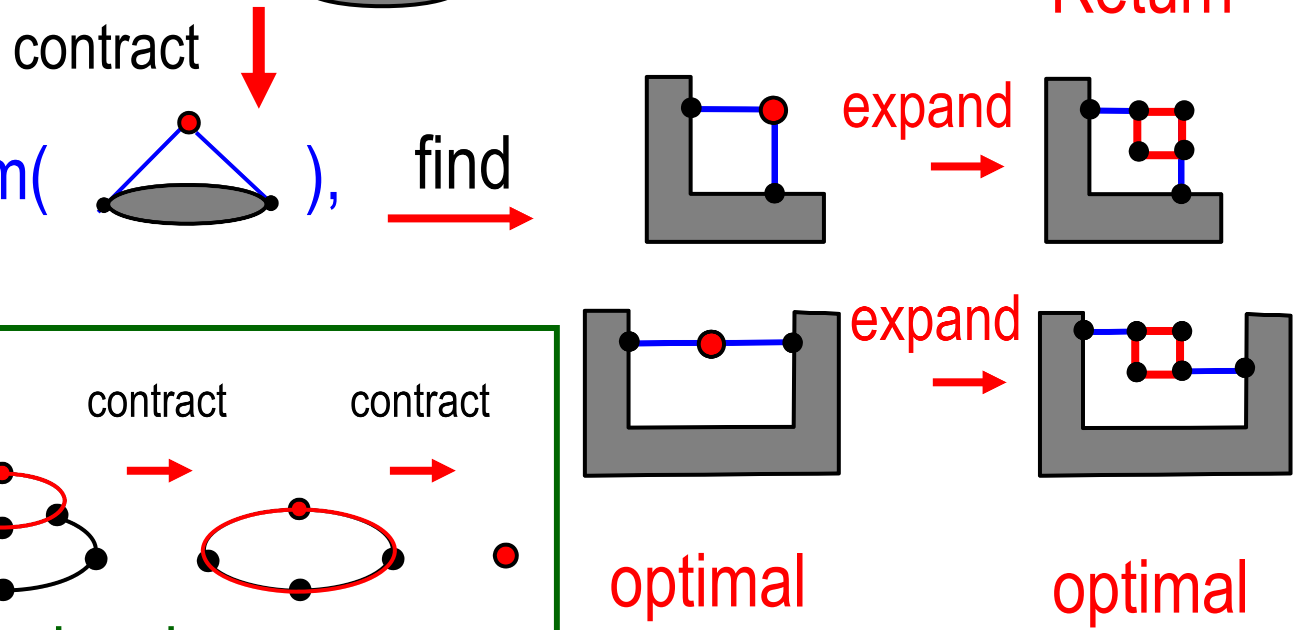


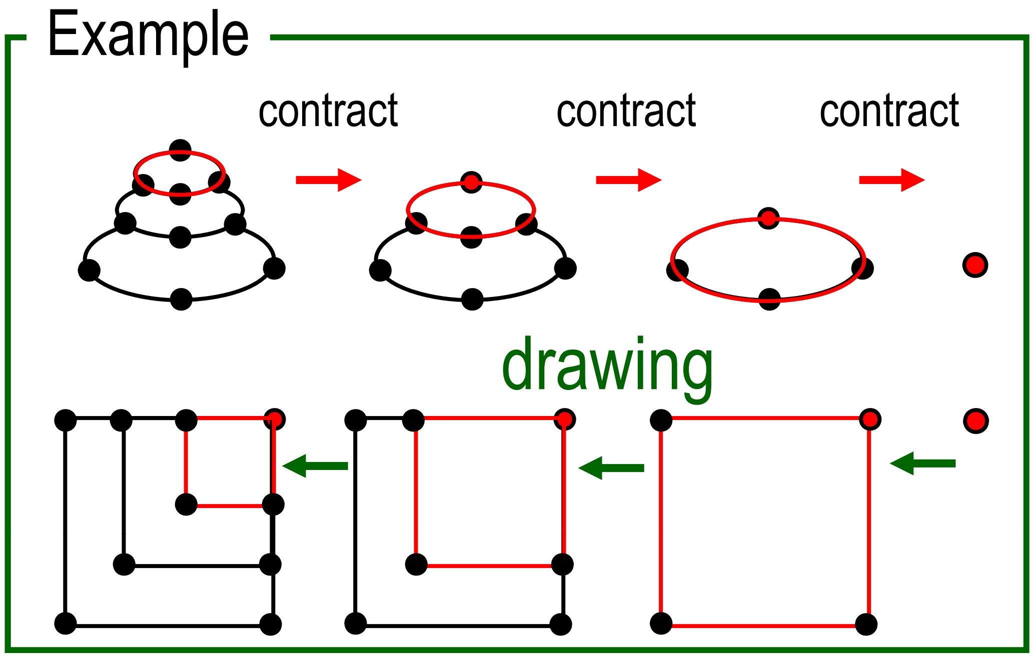
Recur G to a smaller one 

Algorithm(G)

Let G be a biconnected SP graph of $\Delta \leqq 3$.

Case (a): $\exists$ a diamond  ,

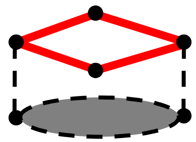




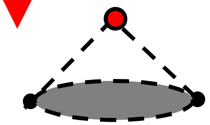
Algorithm(G)

Let G be a biconnected SP graph of $\Delta \leqq 3$.

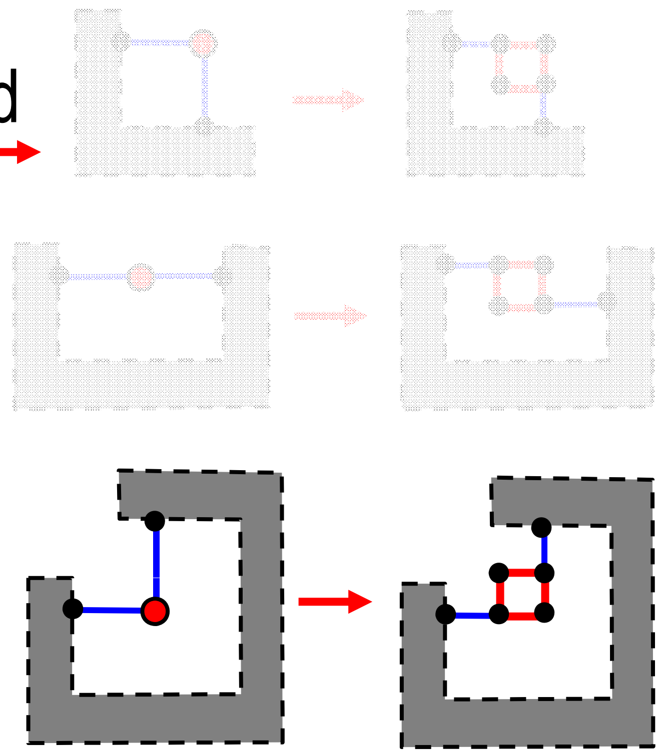
If $n(G) < 6$, Then find an optimal drawing of G

Else If $\exists$ a diamond  ,

contract

Then Algorithm(), find

Return



Recur G to a smaller one

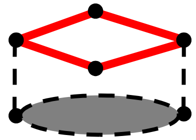
optimal

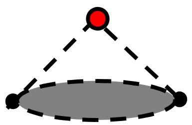
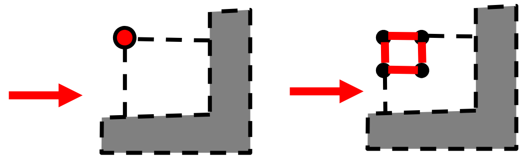
optimal

Algorithm(G)

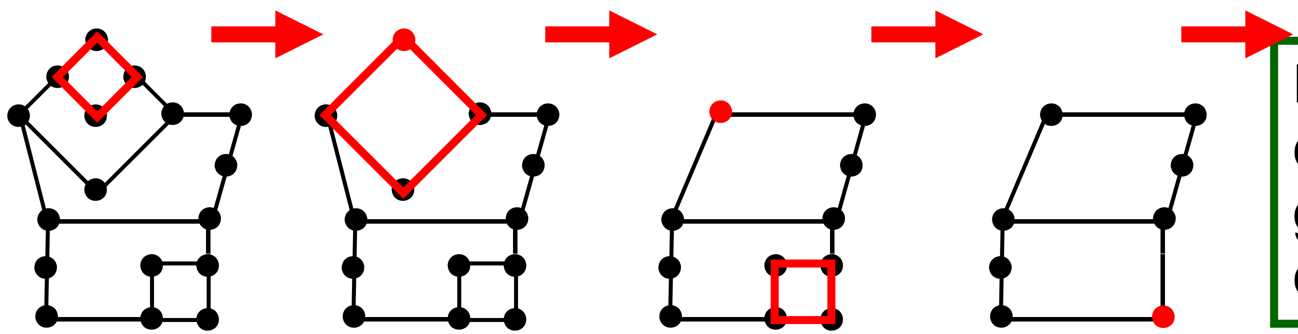
Let G be a biconnected SP graph of $\Delta \leqq 3$.

If $n(G) < 6$, Then find an optimal drawing of G

Else If $\exists$ a diamond  ,

Then Algorithm(), 

Example

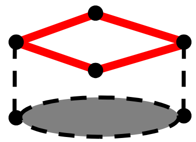


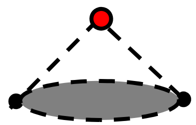
Find an optimal drawing of SP graphs without diamonds

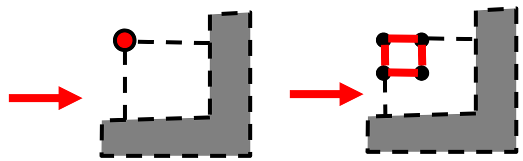
Algorithm(G)

Let G be a biconnected SP graph of $\Delta \leqq 3$.

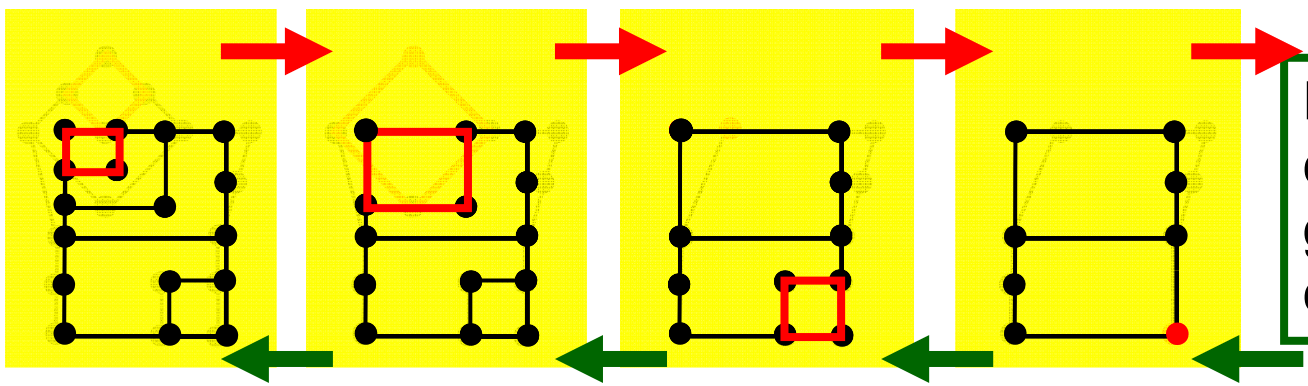
If $n(G) < 6$, **Then** find an optimal drawing of G

Else If $\exists$ a diamond  ,

Then Algorithm(),



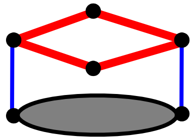
Example



Find an optimal drawing of SP graphs without diamonds

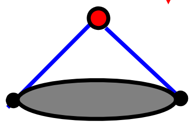
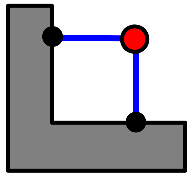
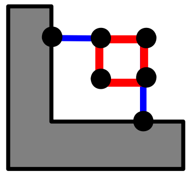
Algorithm(G)

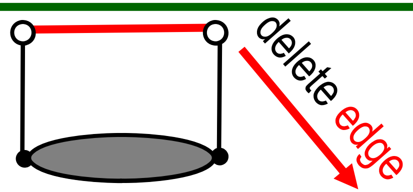
Let G be a biconnected SP graph of $\Delta \leqq 3$.

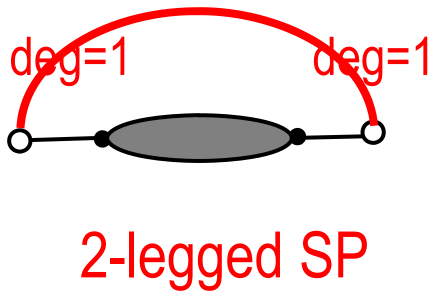
Case (a): $\exists$ a diamond  ,

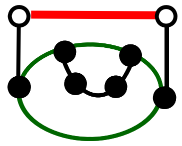
contract

Return

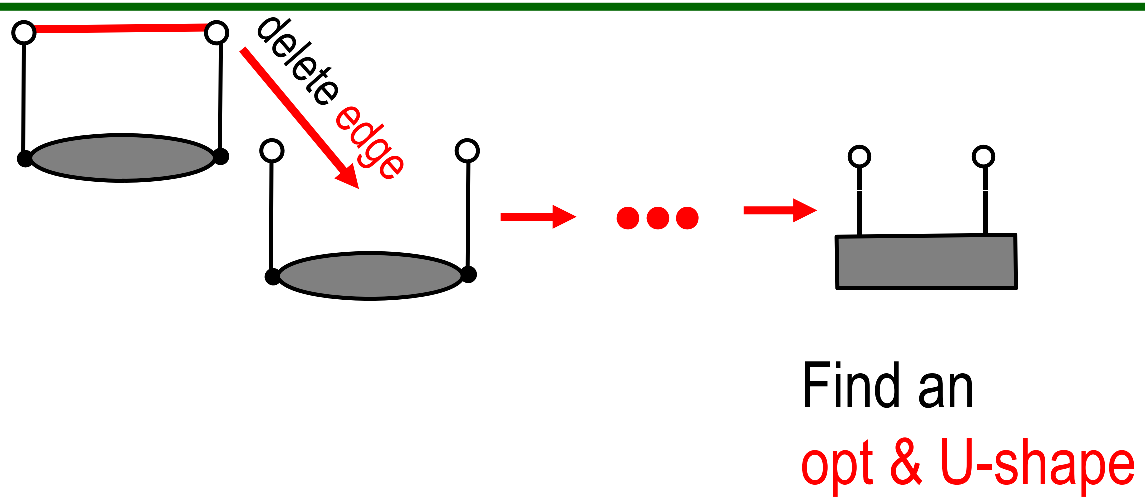
Algorithm(), find  expand 

Case (b): $\exists$ 

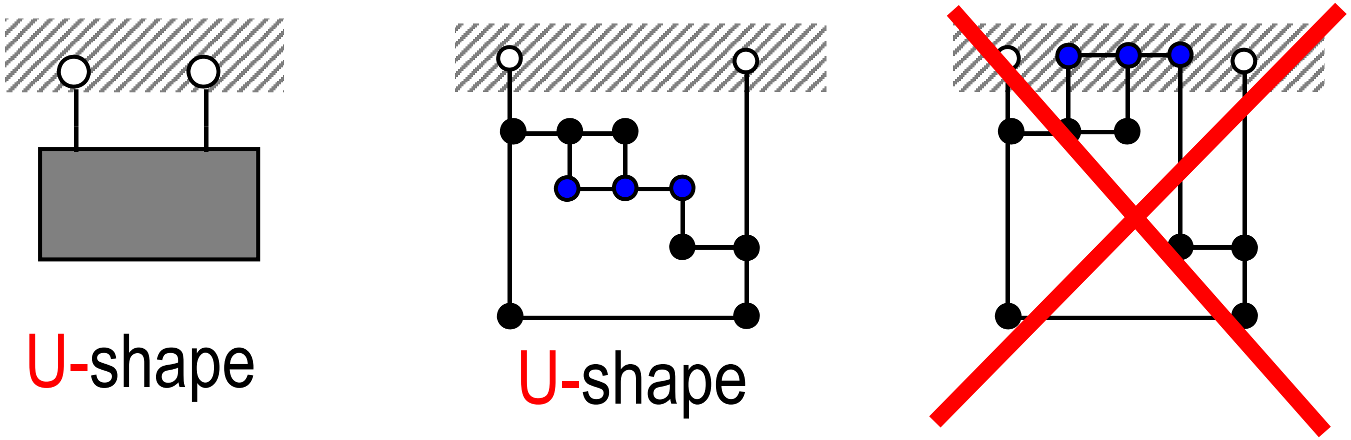




Case (b): ∃

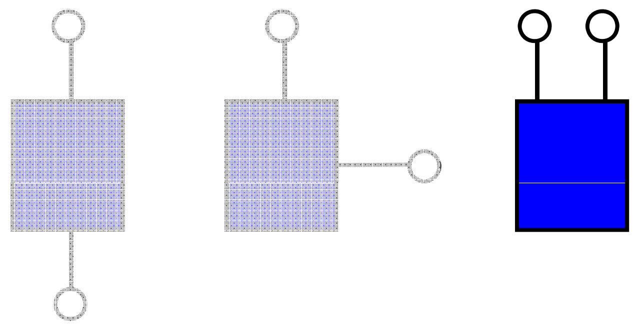


Our Main Idea



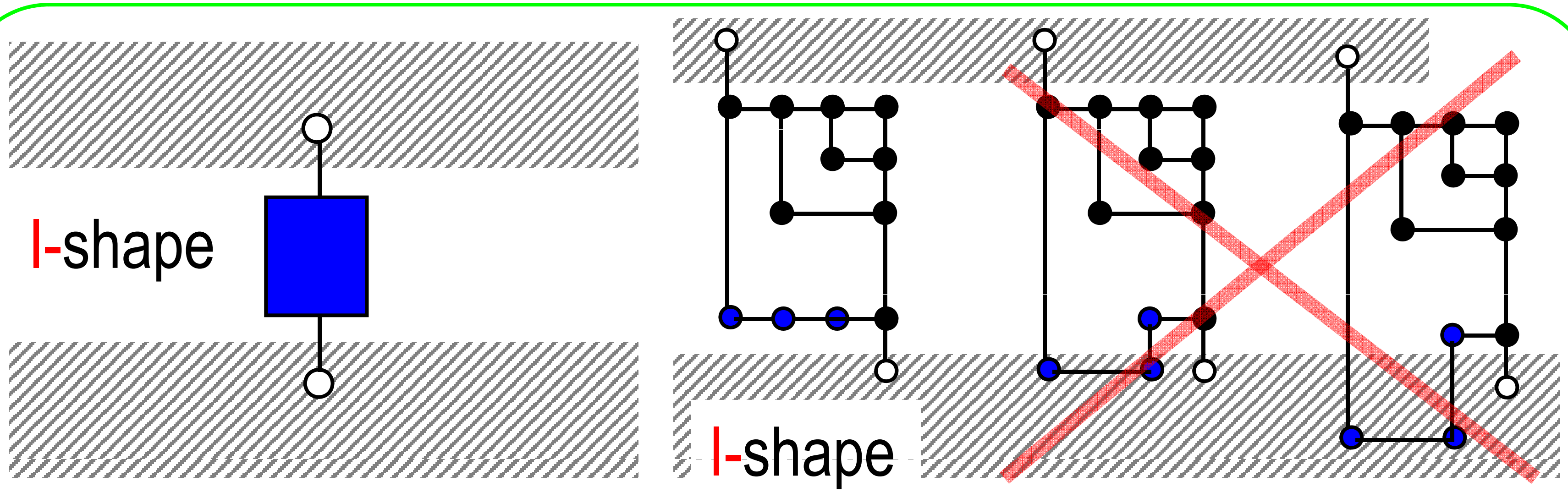
Defin , L- and U-shaped drawings

- terminals are drawn on the outer face;
- the drawing except terminals doesn't intersect the north side



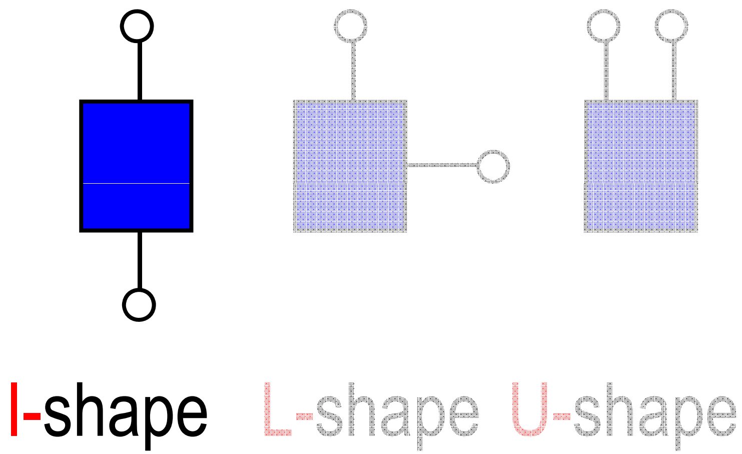
I-shape L-shape U-shape

Our Main Idea



Def… , L- and U-shaped drawings

- terminals are drawn on the outer face;
- the drawing except terminals intersects neither the north side nor the south side



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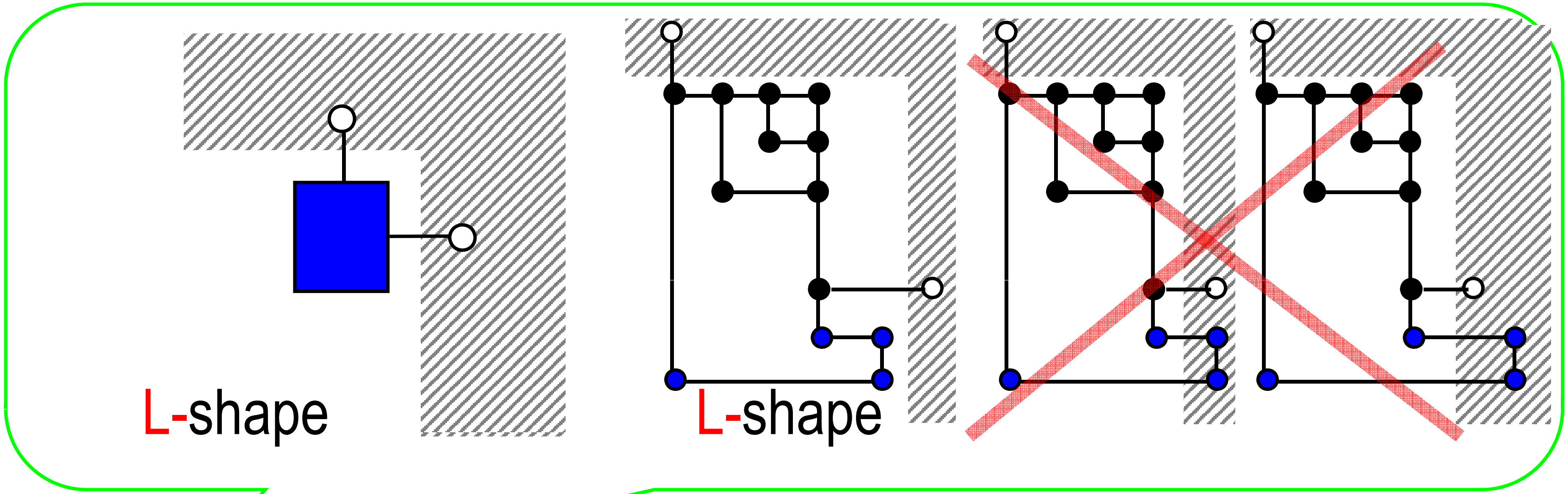
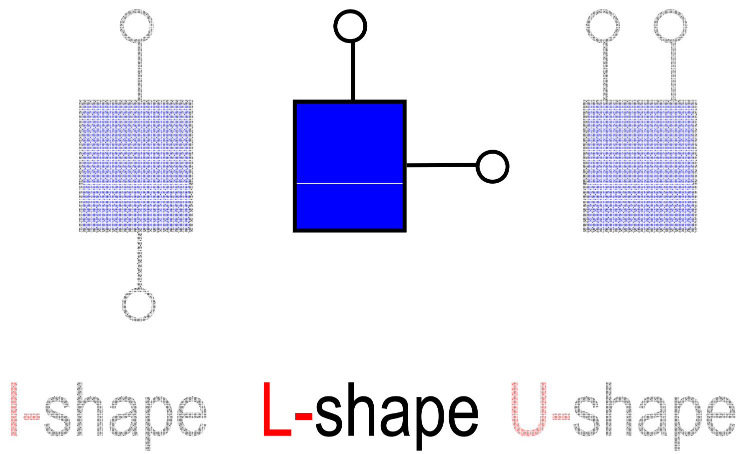


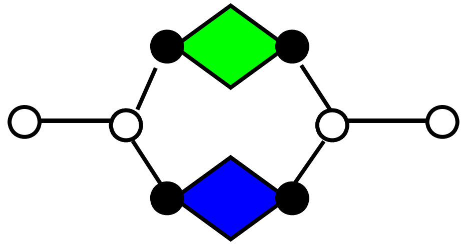
Defi , L- and U-shaped drawings

- terminals are drawn on the outer face;
- the drawing except terminals intersects neither the north side nor the east side

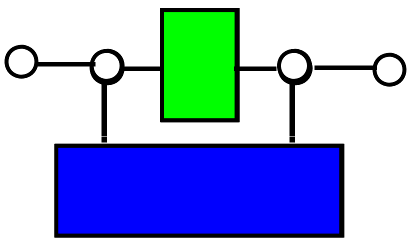


Lemma 2

Every 2-legged SP graph without diamond has optimal I-, L- and U-shaped drawings

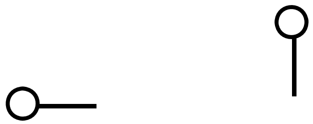


optimal



I-shape

optimal



L-shape

optimal

U-shape

Lemma 2

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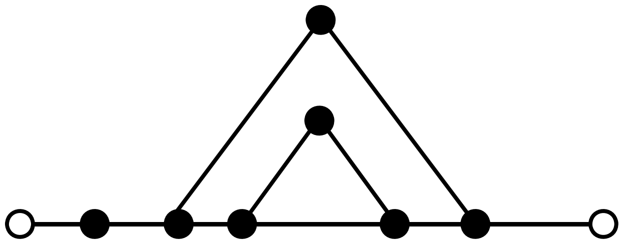
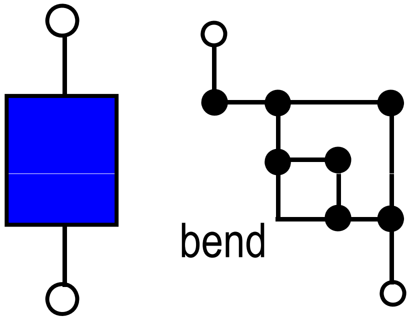
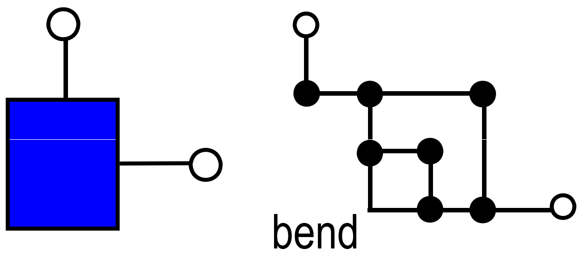


optimal



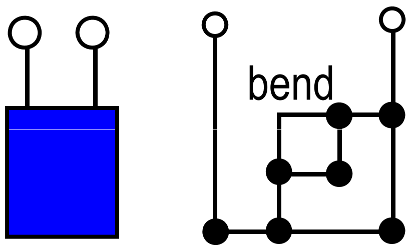
I-shape

optimal



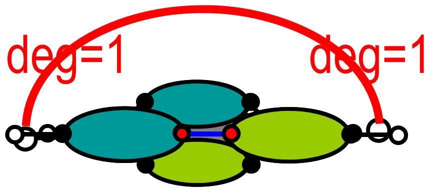
L-shape

optimal



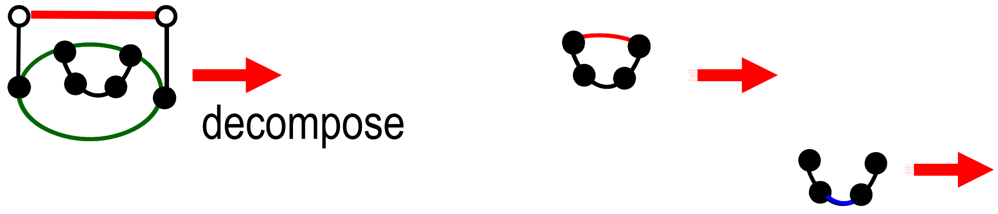
U-shape

series connection

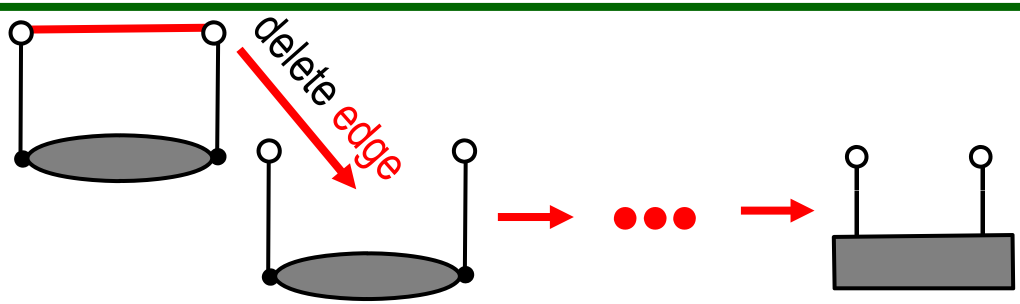


2-legged SP

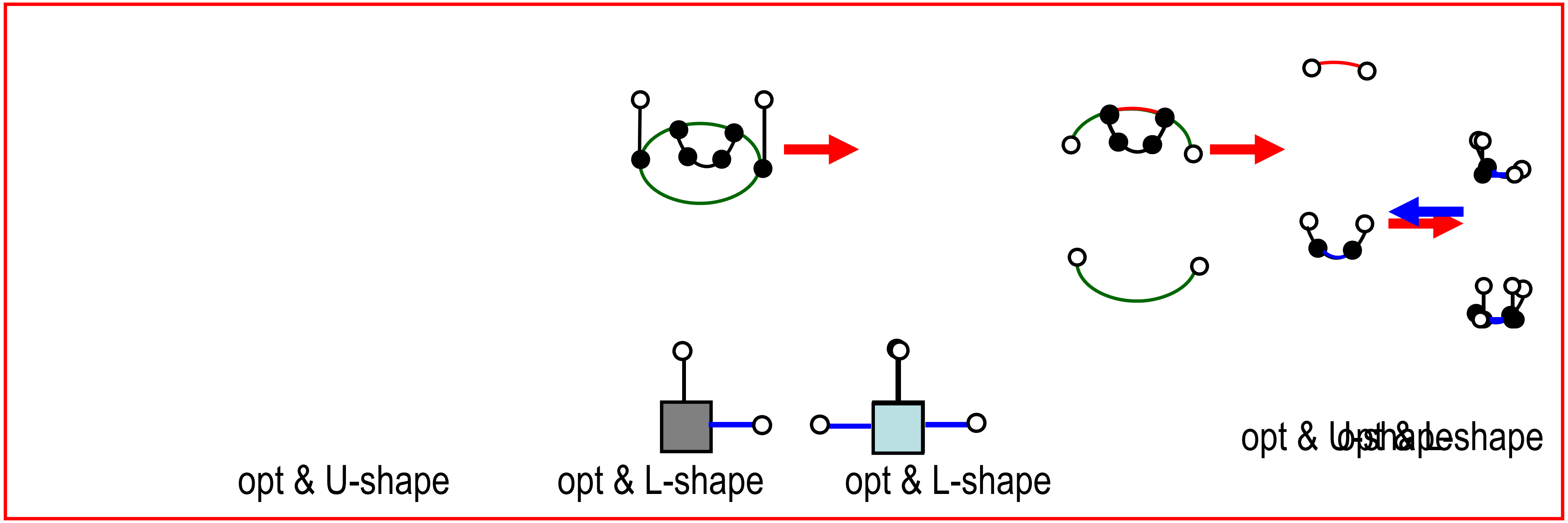
$\Delta \leqq 3$



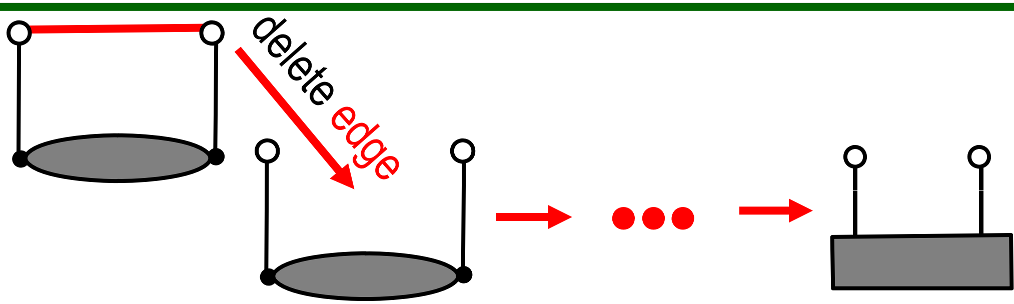
Case (b): ∃



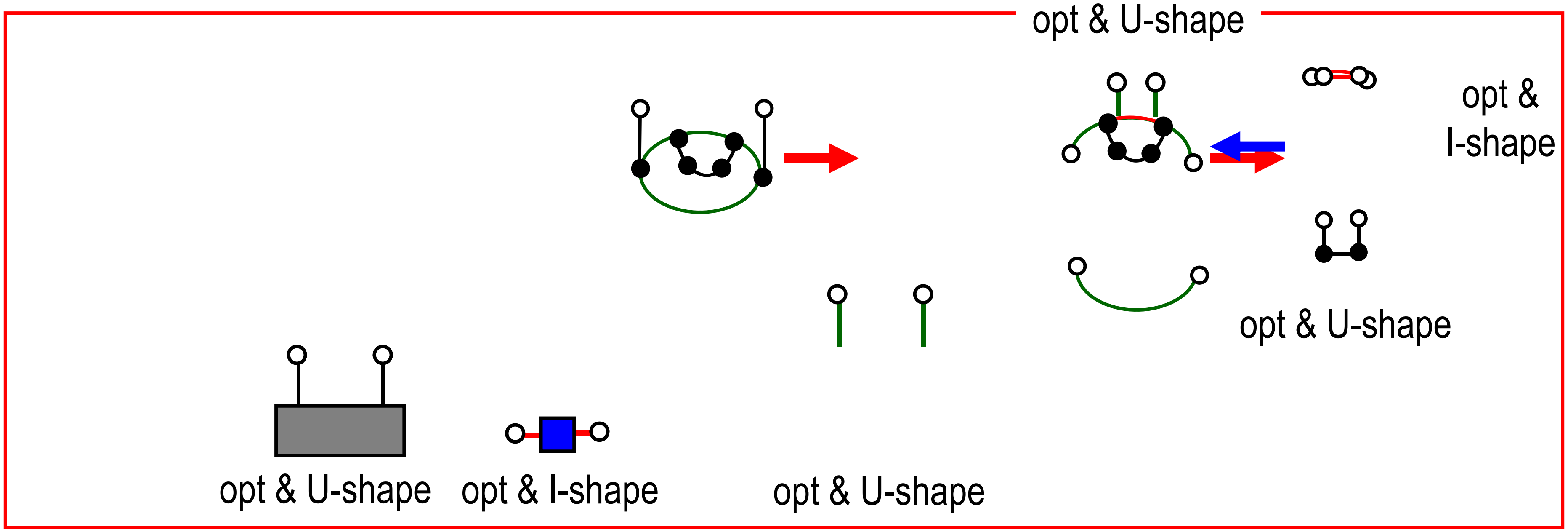
Find an
opt & U-shape

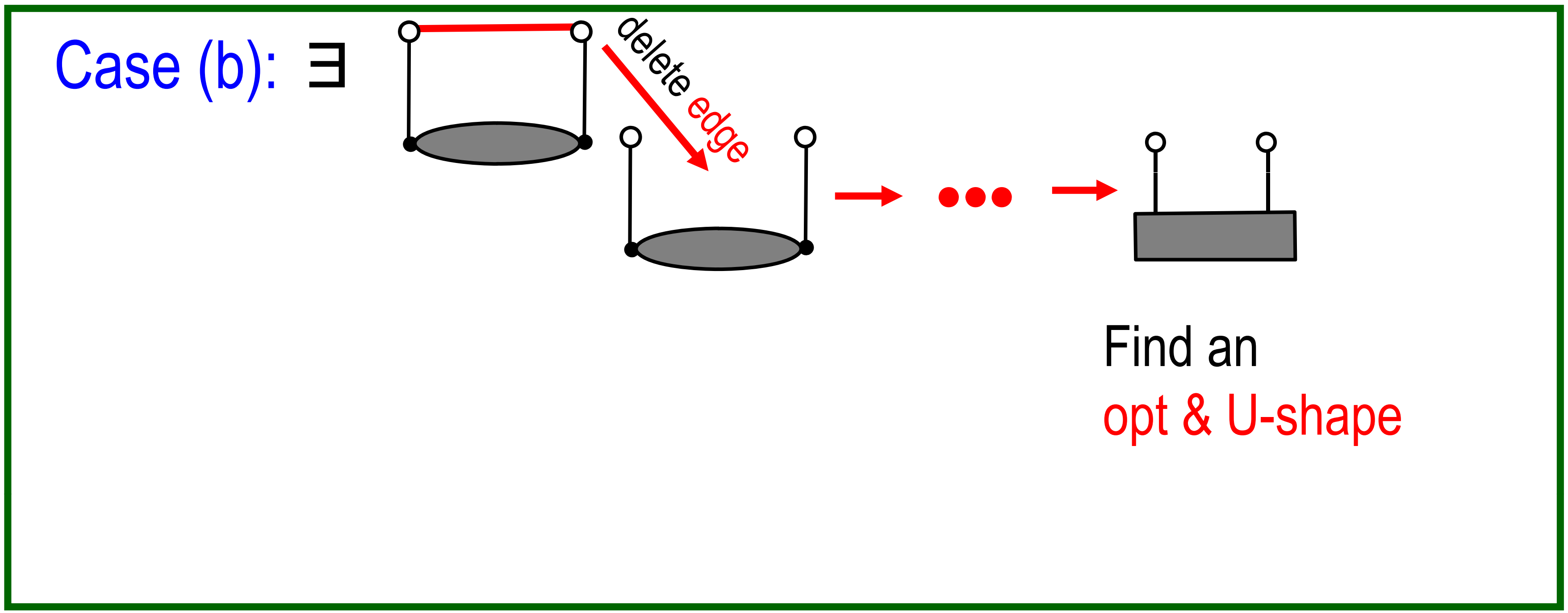


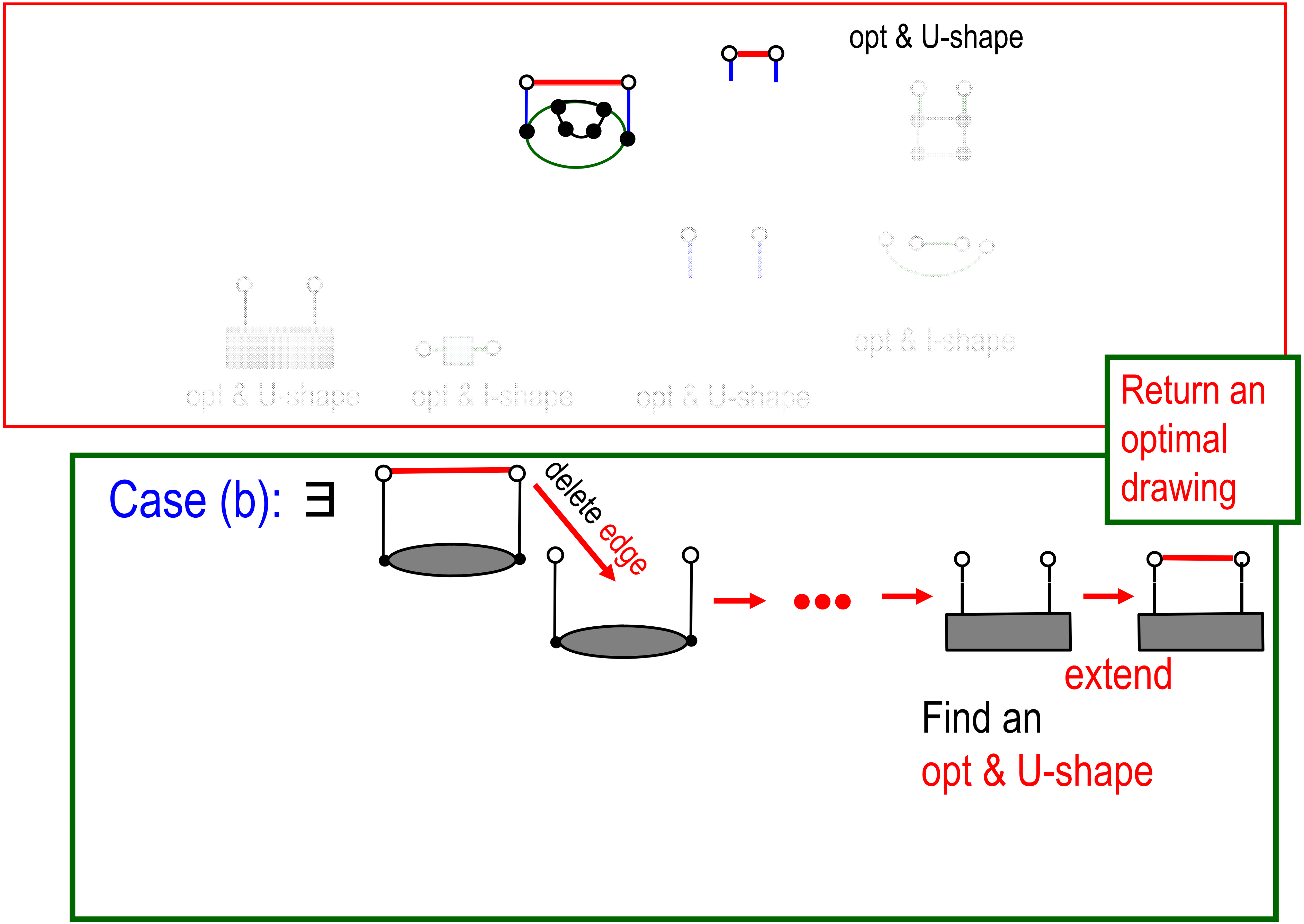
Case (b): ∃



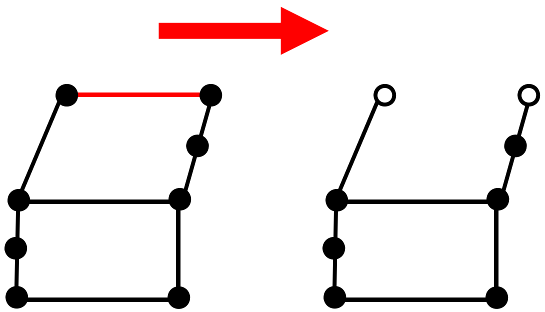
Find an
opt & U-shape


opt & U-shape
opt &
I-shape
opt & U-shape
opt & U-shape
opt & I-shape
opt & U-shape

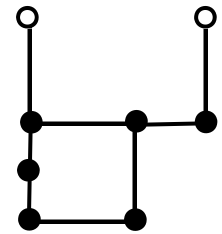

Case (b): ∃
delete edge
Find an
opt & U-shape


opt & U-shape
opt & I-shape
opt & U-shape
opt & I-shape
opt & U-shape
Return an optimal drawing
Case (b): ∃
delete edge
extend
Find an
opt & U-shape

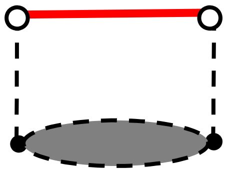
Example



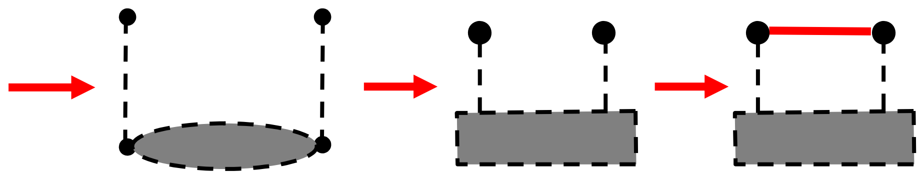
How to find an optimal U-shaped drawing ?



Else If ∃

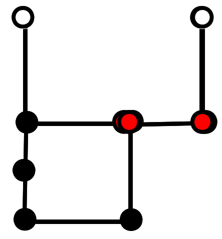


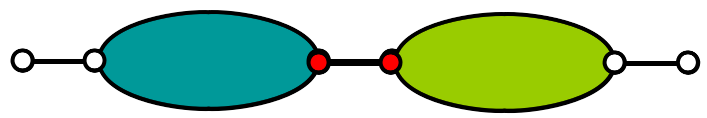
Then

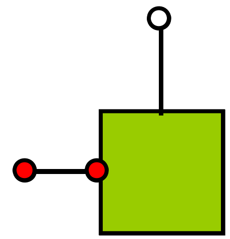


series connection

Example



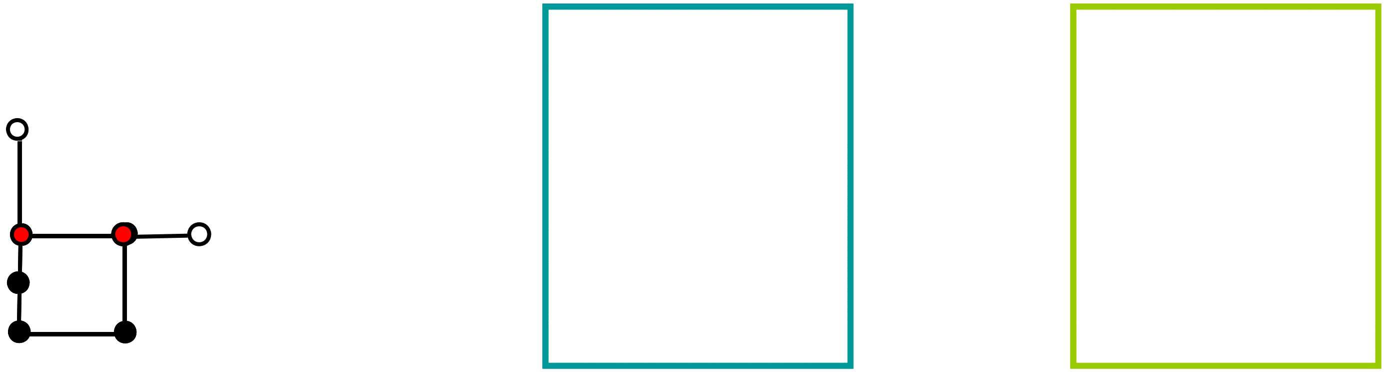


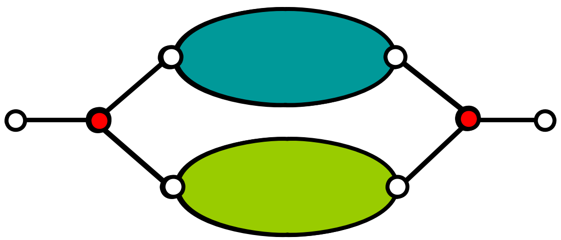


L-shaped draw

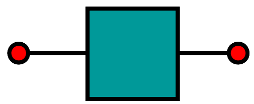
parallel connection

Example

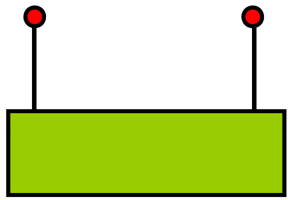




I-shape



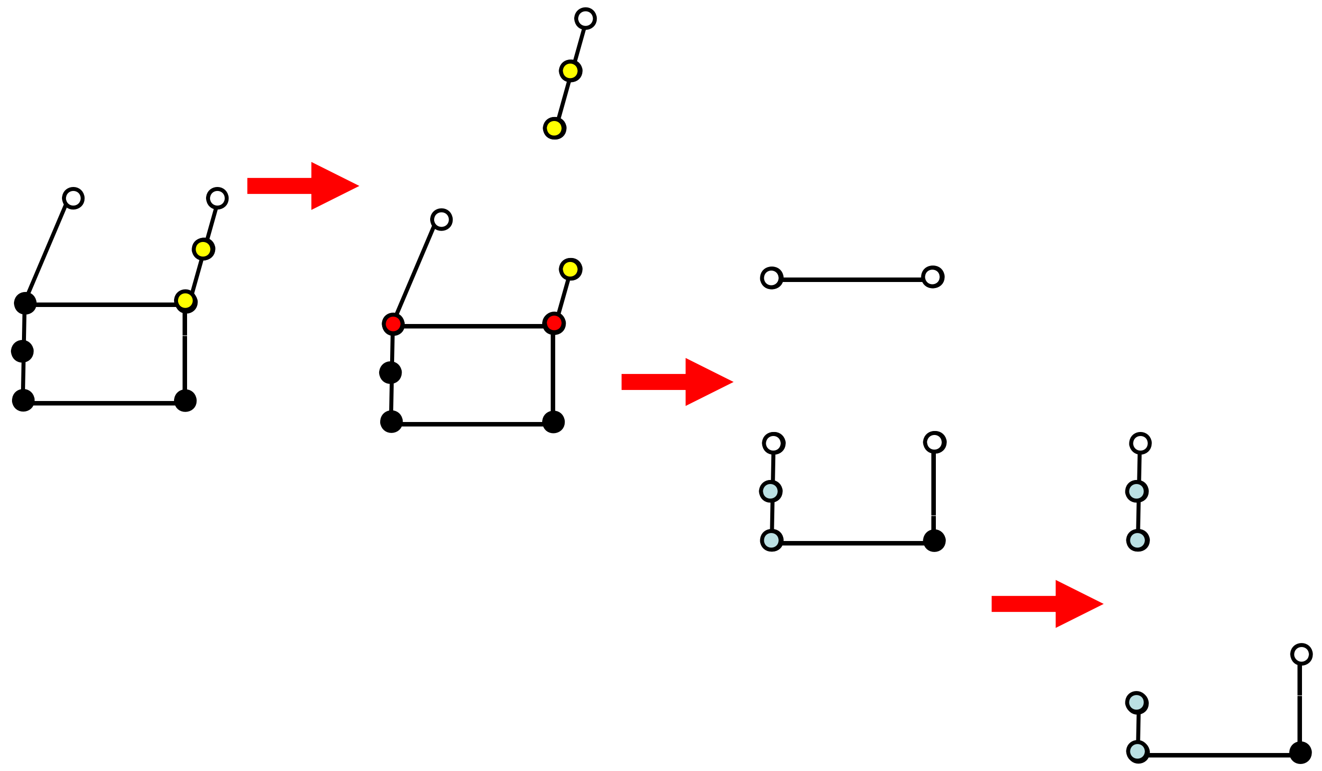
U-shape

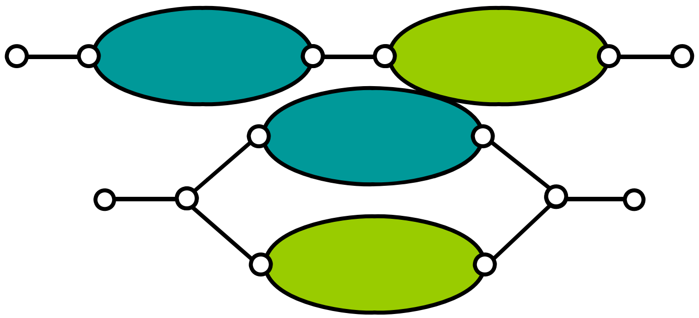


recursively

I-shape
U-shape

Example

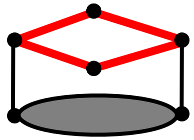




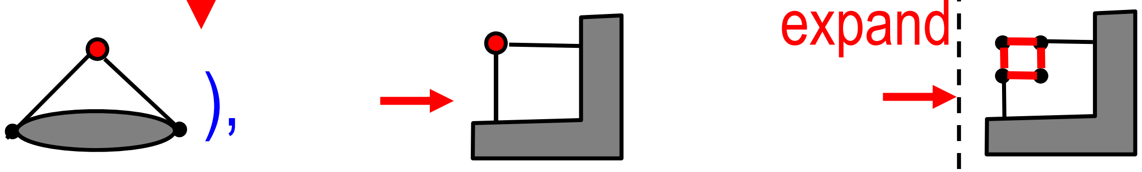
Algorithm(G)

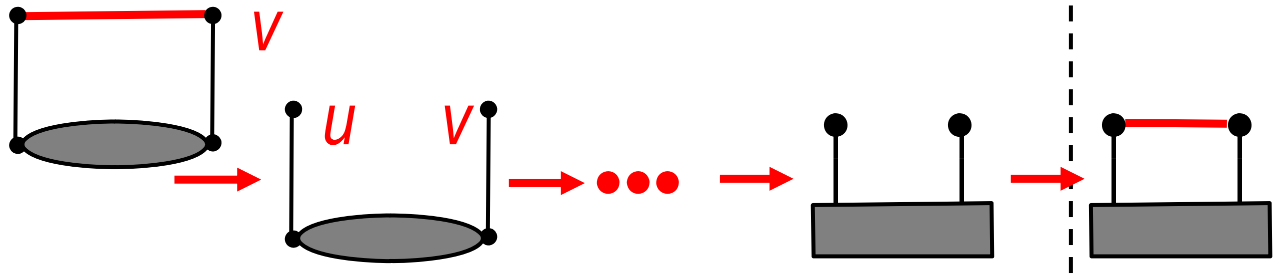
Let G be a biconnected SP graph of $\Delta \leqq 3$.

Return optimal drawing

Case (a): ∃ a diamond ,

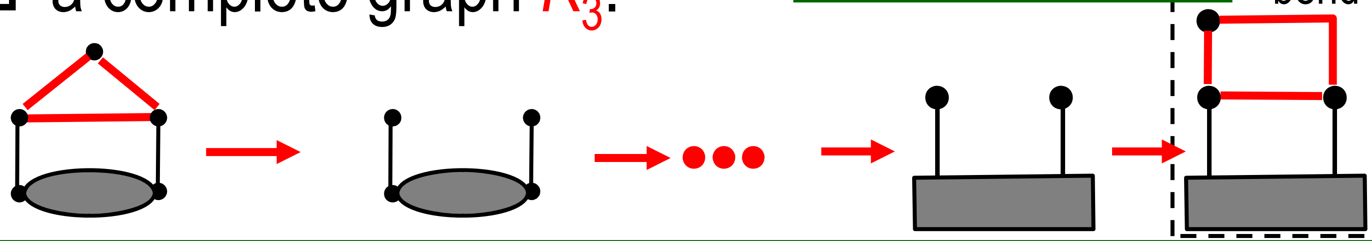
contraction

Algorithm(

Case (b): ∃ 

opt & U-shape

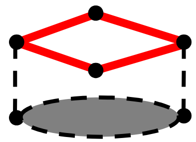
Case (c): ∃ a complete graph K_3.

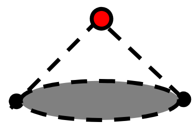
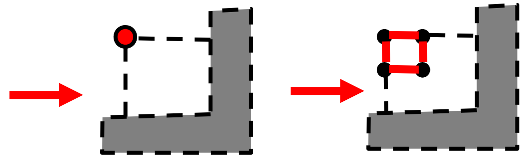


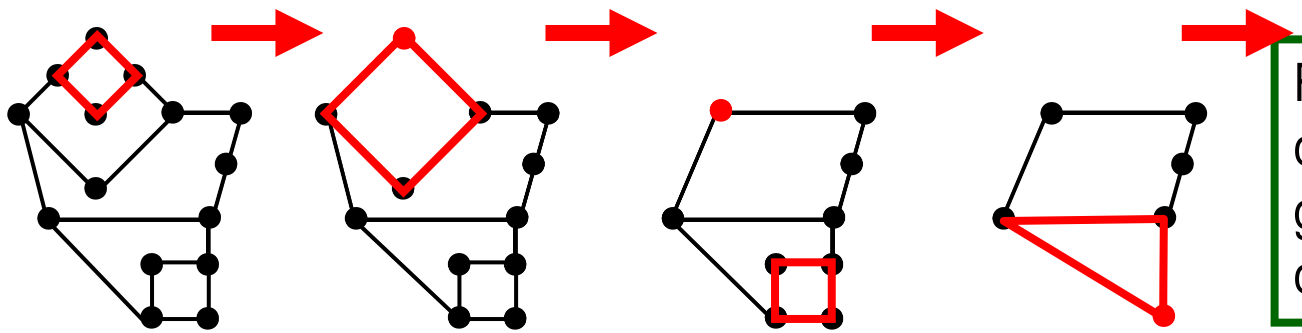
Algorithm(G)

Let G be a biconnected SP graph of $\Delta \leqq 3$.

If $n(G) < 6$, Then find an optimal drawing of G

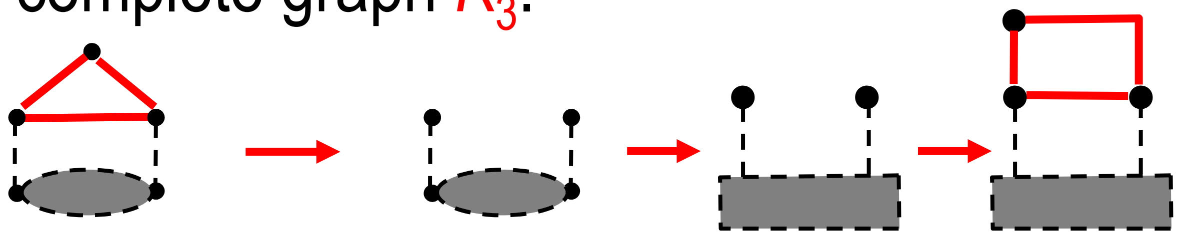
Else If $\exists$ a diamond  ,

Then Algorithm(), 



Find an optimal drawing of SP graphs without diamonds

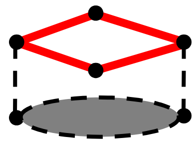
Else $\exists$ a complete graph K_3.

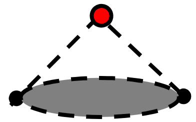
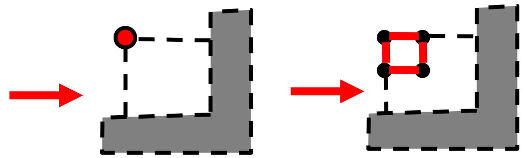


Algorithm(G)

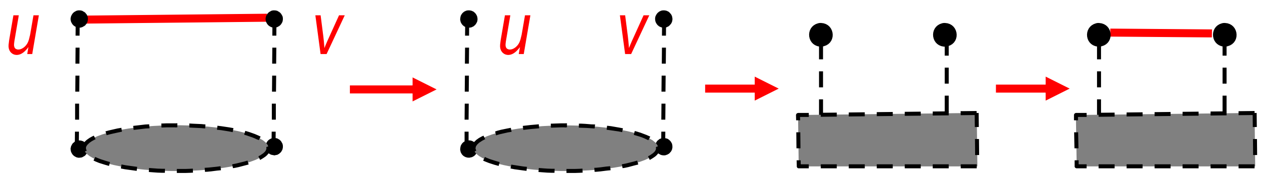
Let G be a biconnected SP graph of $\Delta \leqq 3$.

If $n(G) < 6$, Then find an optimal drawing of G

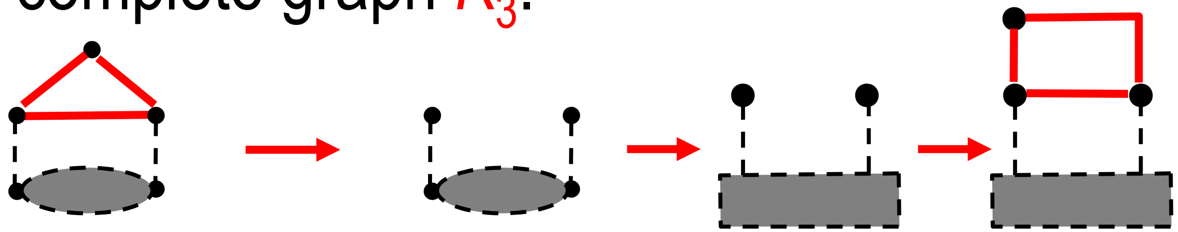
Else If $\exists$ a diamond  ,

Then Algorithm(), 

Else If $\exists$ two adjacent vertices u,v s.t. $d(u)=d(v)=2$

Then 

Else $\exists$ a complete graph K_3.

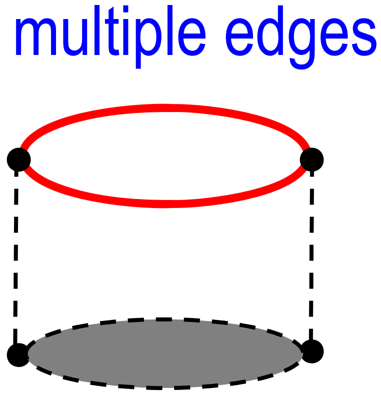


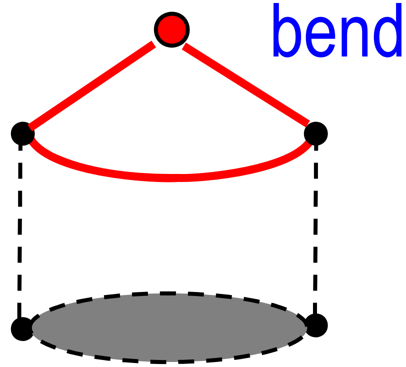
Conclusions

Theorem 1

An optimal orthogonal drawing of a biconnected SP graph G of $\Delta \leqq 3$ can be found in linear time.

Our algorithm works well even if G has multiple edges





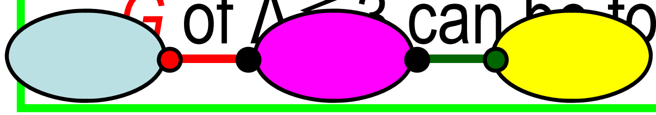
Theorem 1

An optimal orthogonal drawing of a biconnected SP graph G of $\Delta \leqq 3$ can be found in linear time.

Theorem 1

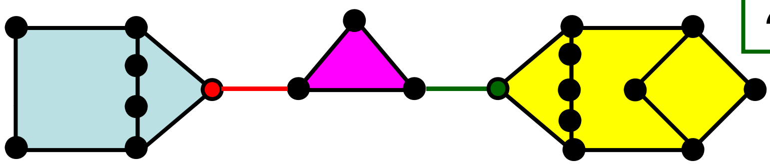
Our algorithm works well even if G is not biconnected.

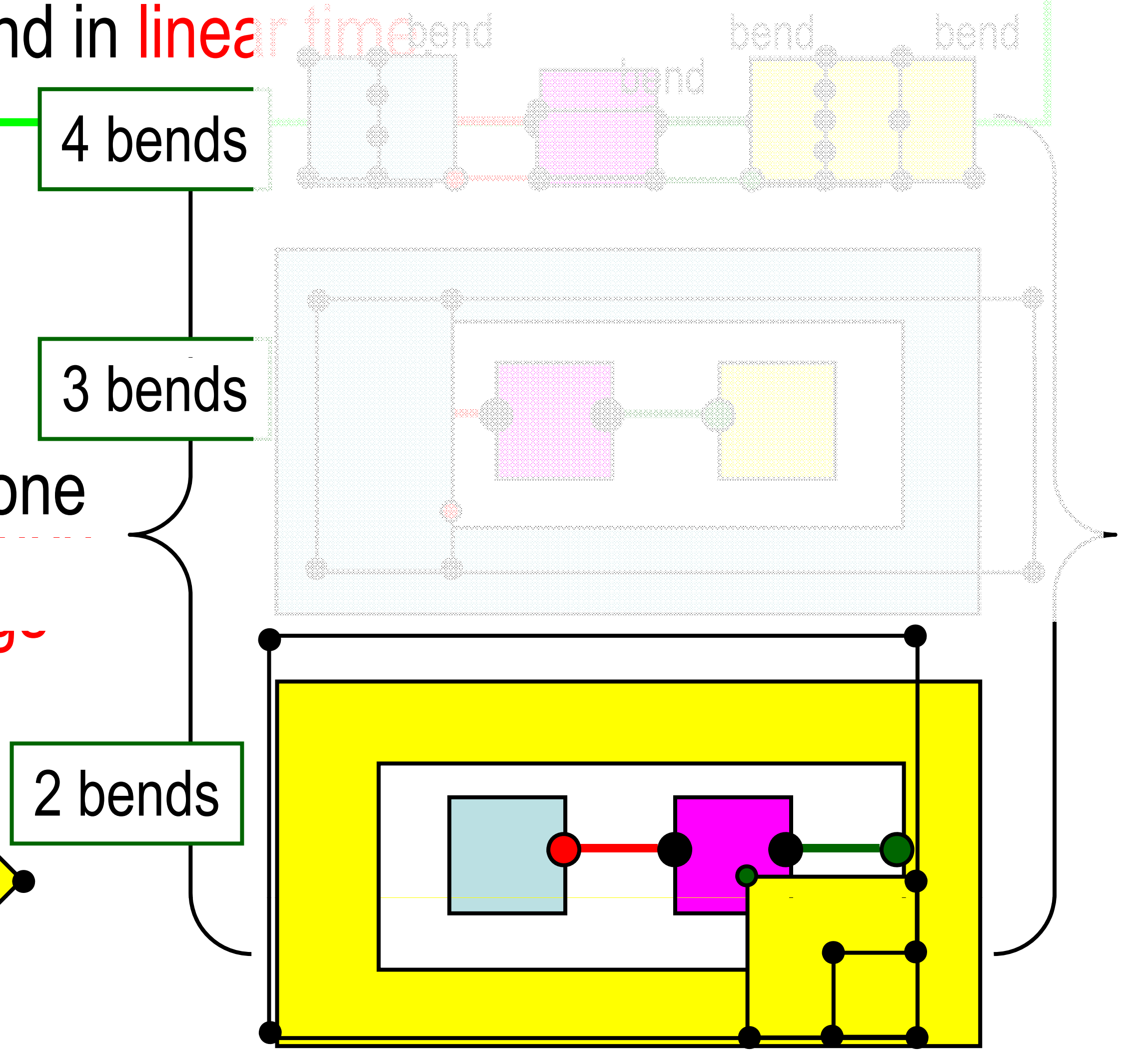
An optimal orthogonal drawing of a biconnected SP graph G of $\Delta \leq 3$ can be found in linear time.



Optimal drawing

find the best one of drawings





Conclusions

Theorem 1

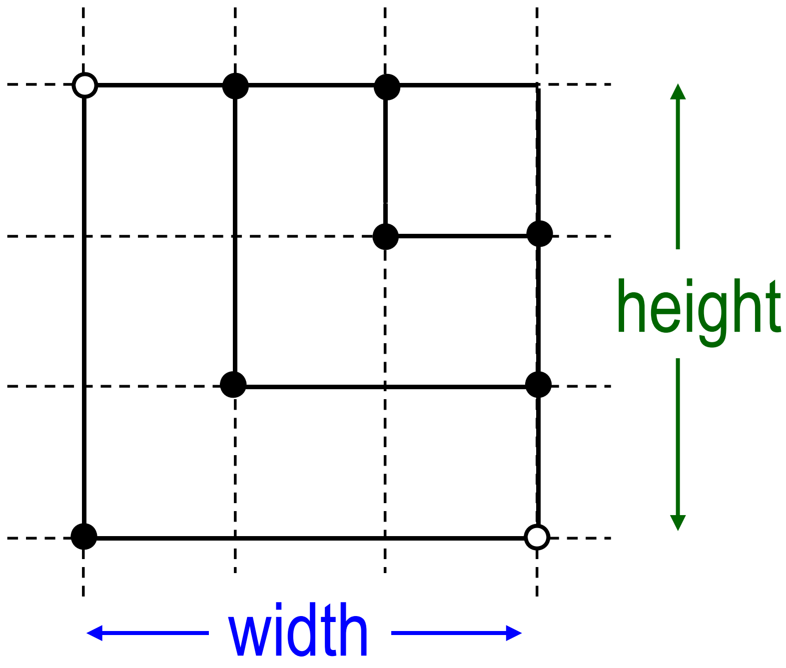
An optimal orthogonal drawing of a SP graph G of $\Delta \leqq 3$ can be found in linear time.

Conclusions

bend(G) $\leqq \lceil n/3 \rceil$ for biconnected SP graphs G of $\Delta \leqq 3$

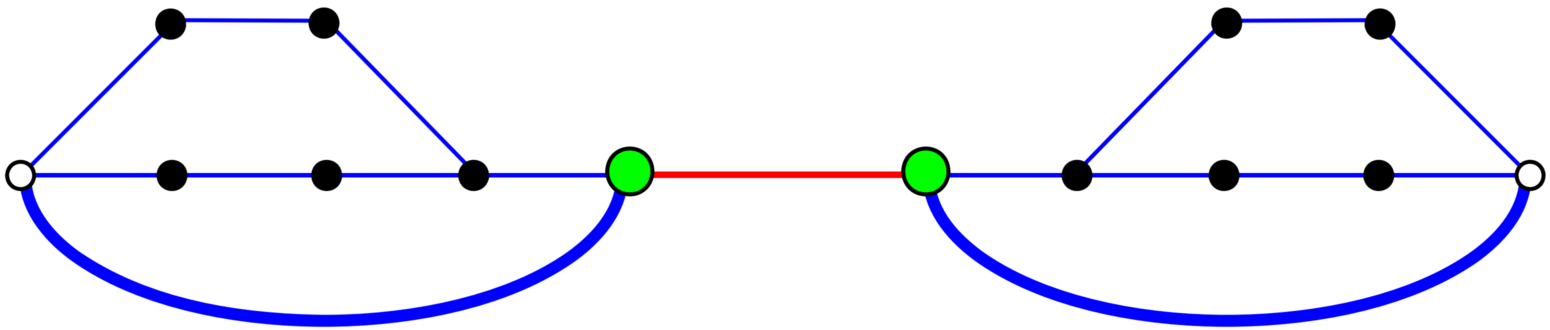
Grid size $\leqq 8n/9$

=width + height



Thank You

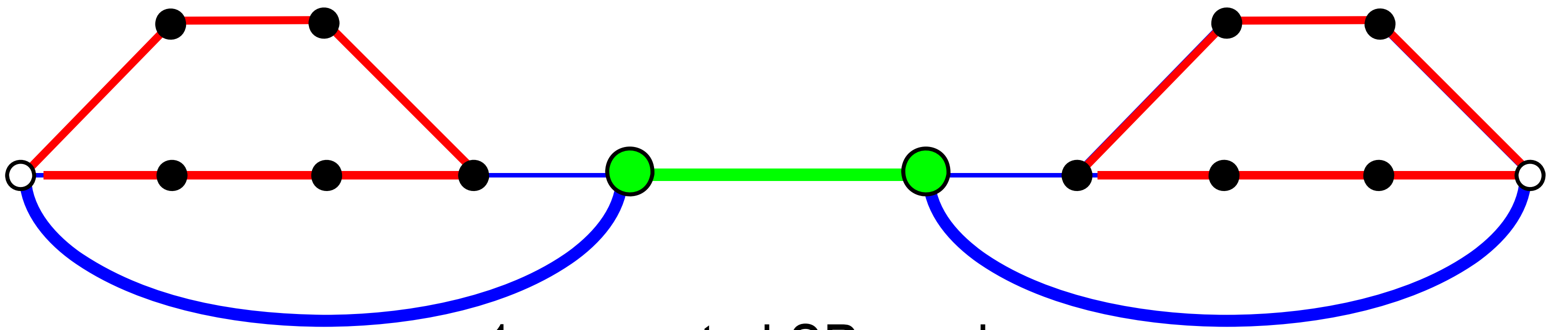
Optimal orthogonal drawing



planar graph

Optimal orthogonal drawings ?

Optimal orthogonal drawing



1-connected SP graph

∃ a one-bend orthogonal drawing ?

a 2-bend orthogonal drawing

one bend

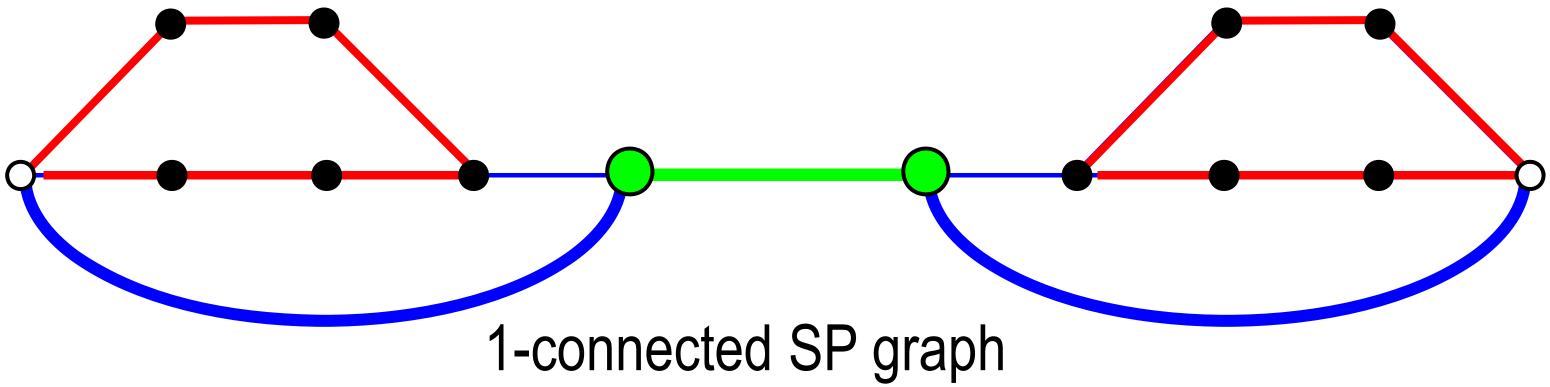
not optimal

Is this optimal ?

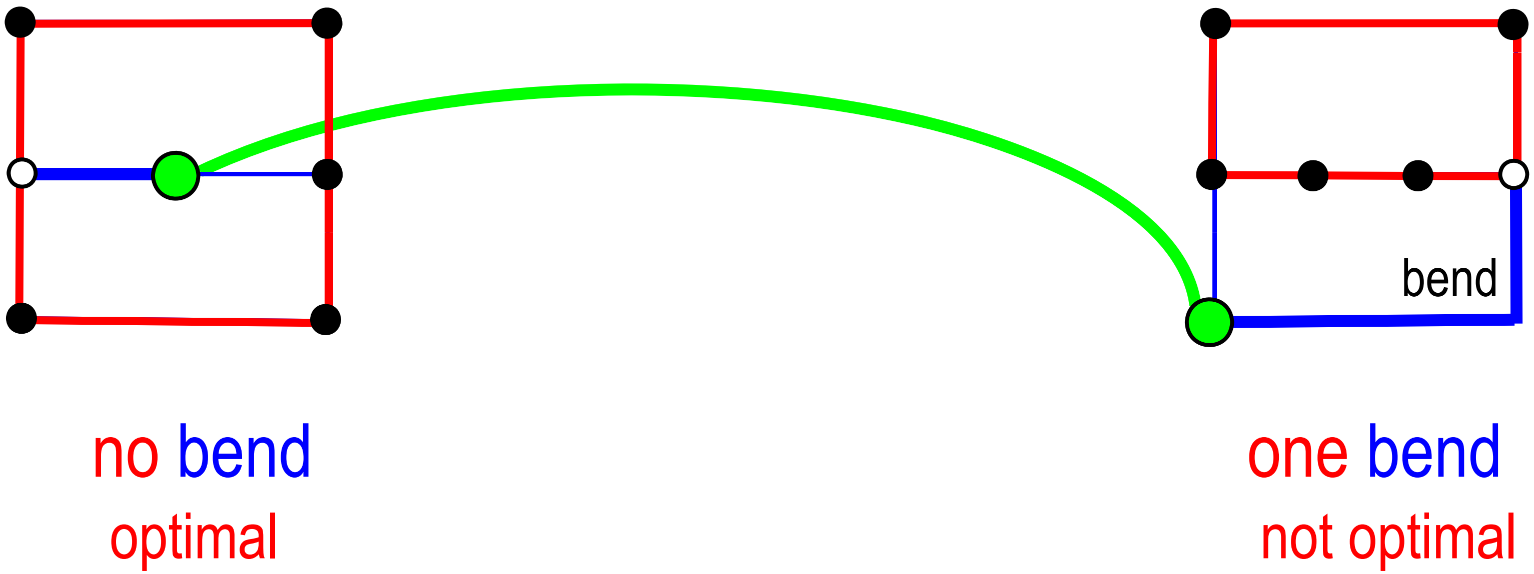
one bend

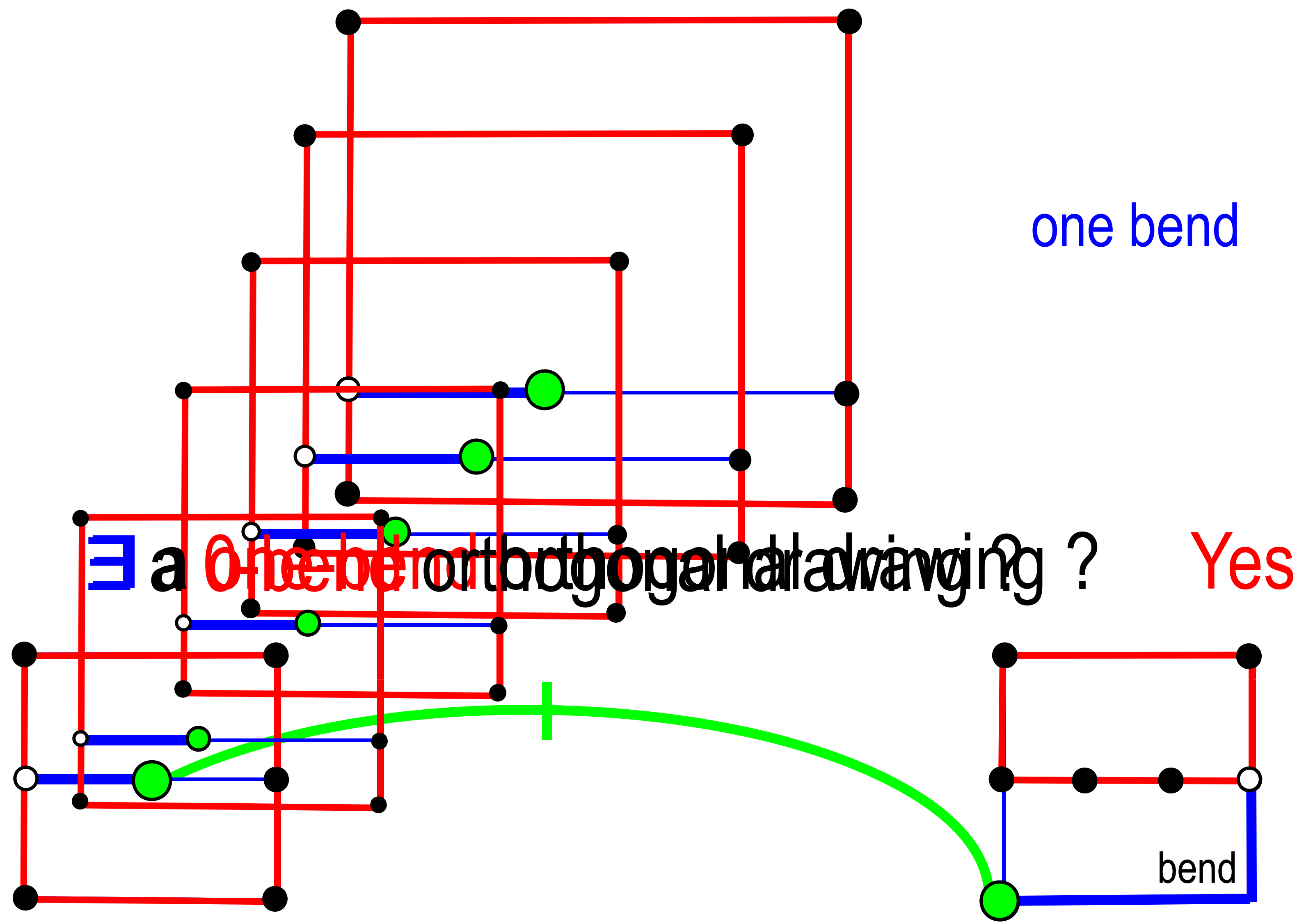
not optimal

Optimal orthogonal drawing



∃ a one-bend orthogonal drawing ?



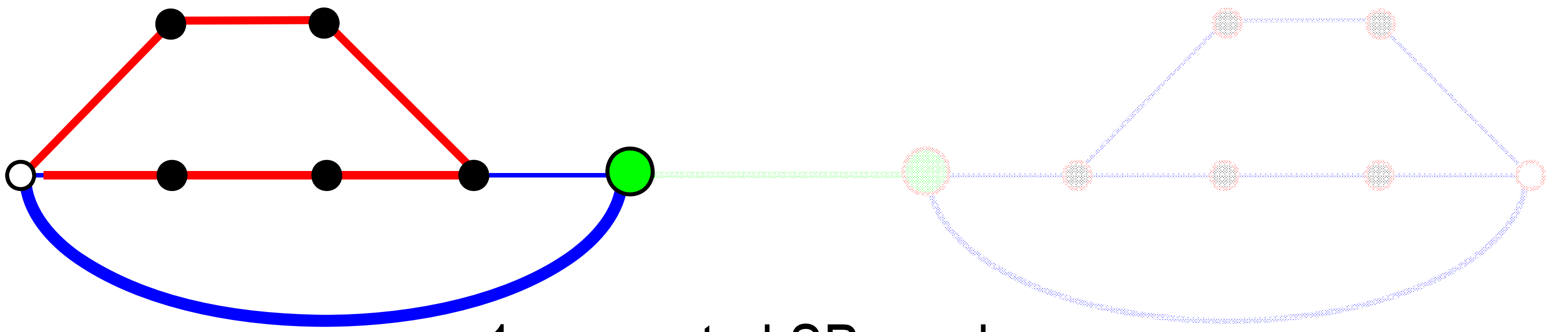


no bend
optimal

Is this optimal ?

one bend
not optimal

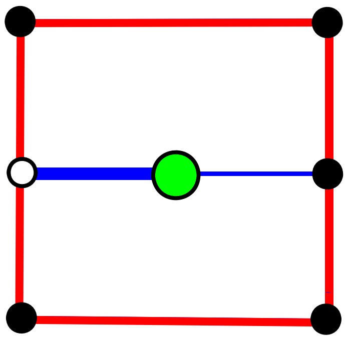
Optimal orthogonal drawing



1-connected SP graph

∃ a 0-bend orthogonal drawing ?

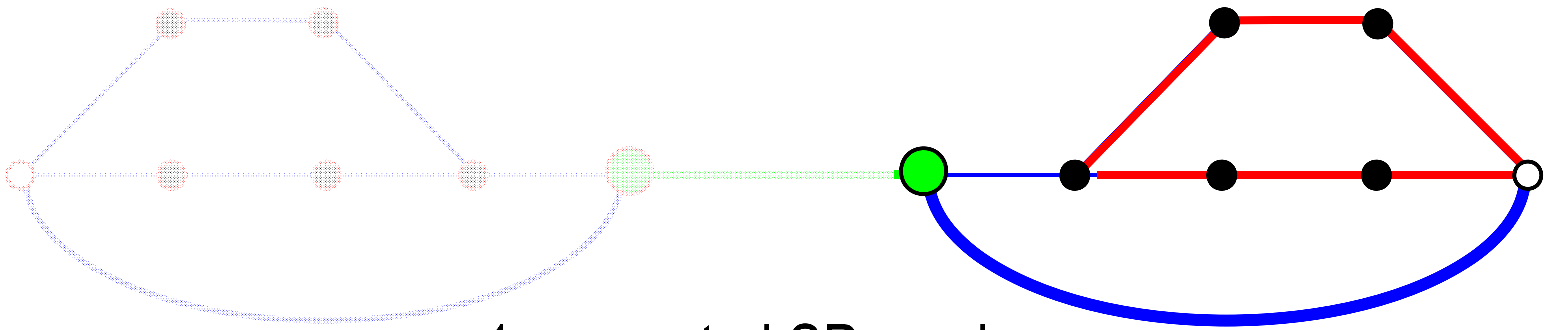
0 bend



optimal

orthogonal drawings

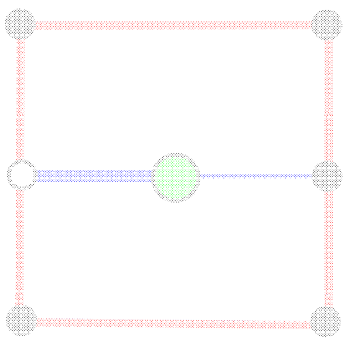
Optimal orthogonal drawing



1-connected SP graph

∃ a 0-bend orthogonal drawing ?

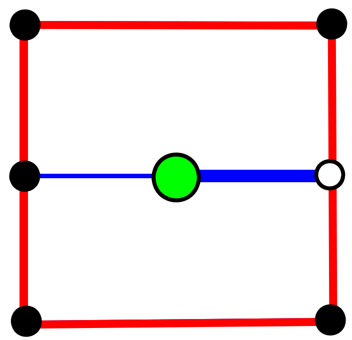
0 bend



optimal

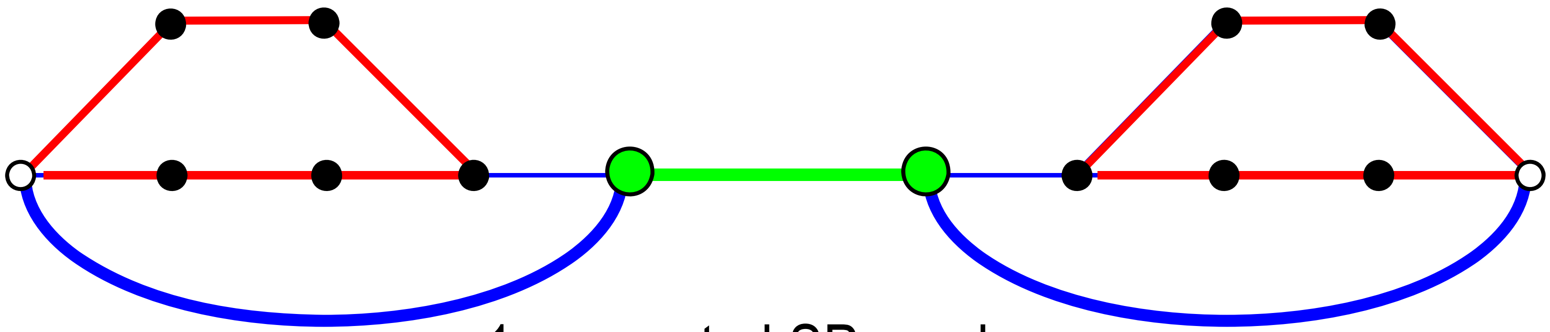
orthogonal drawings

0 bend



optimal

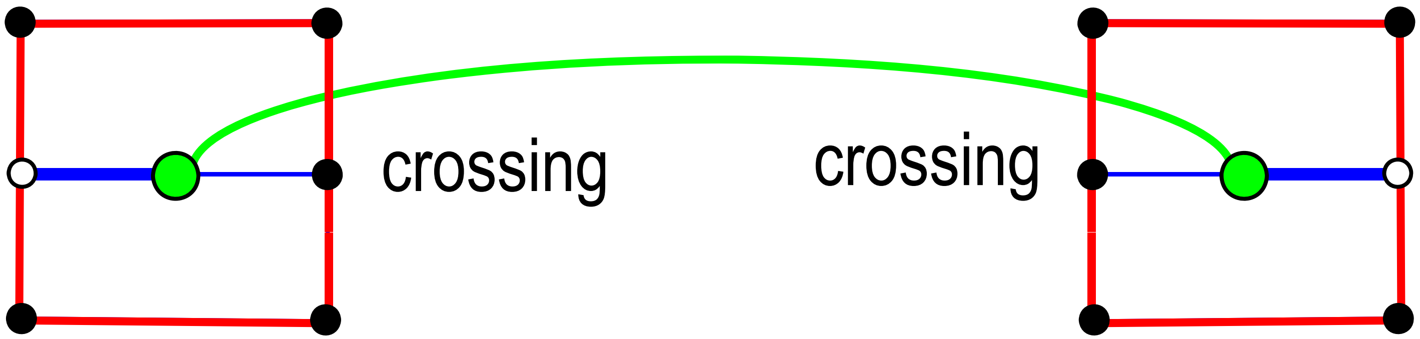
Optimal orthogonal drawing



1-connected SP graph

∃ a 0-bend orthogonal drawing ?

No

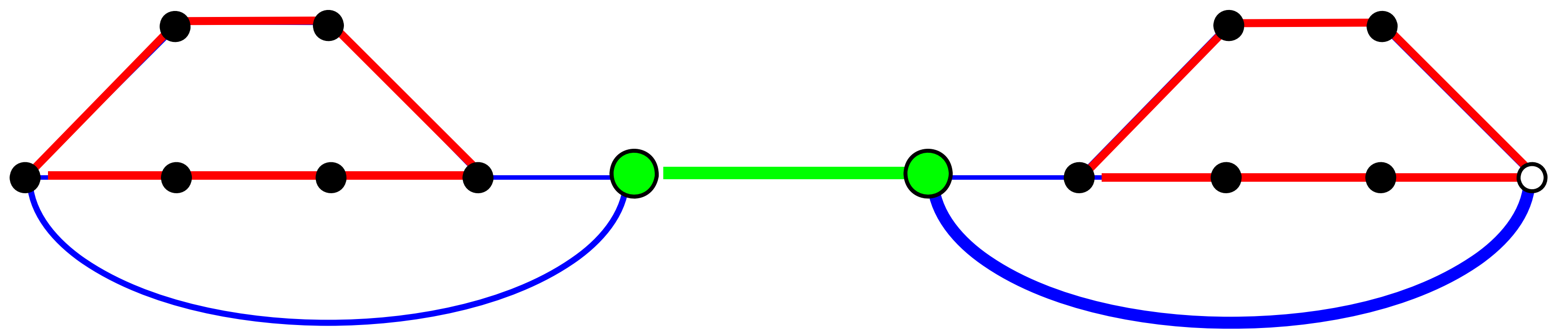


no bend

optimal

no bend

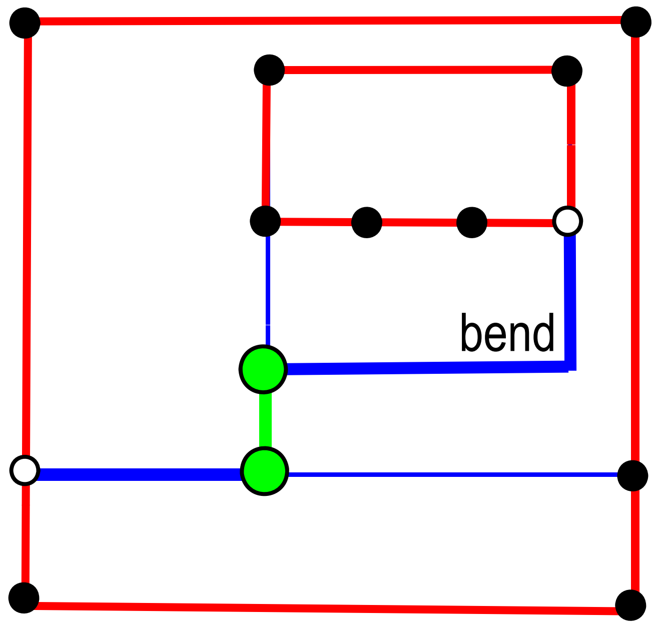
optimal

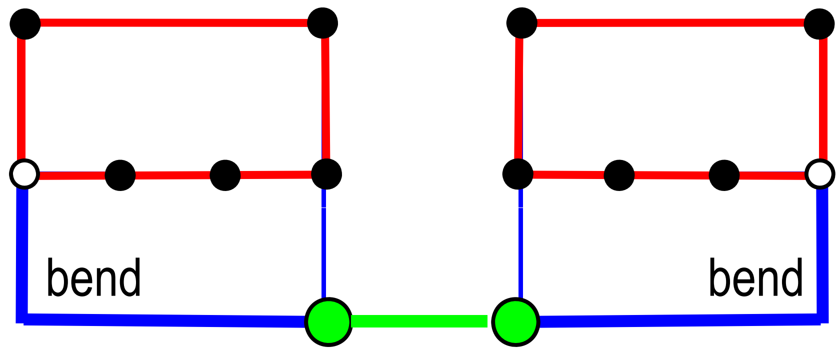


one bend

optimal

1-connected SP graph





two bends

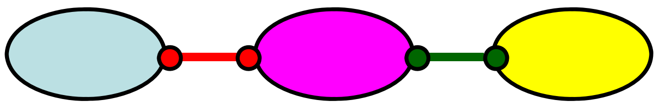
Conclusions

Theorem 1

An optimal orthogonal drawing of a biconnected SP graph G of $\Delta \leqq 3$ can be found in linear time.

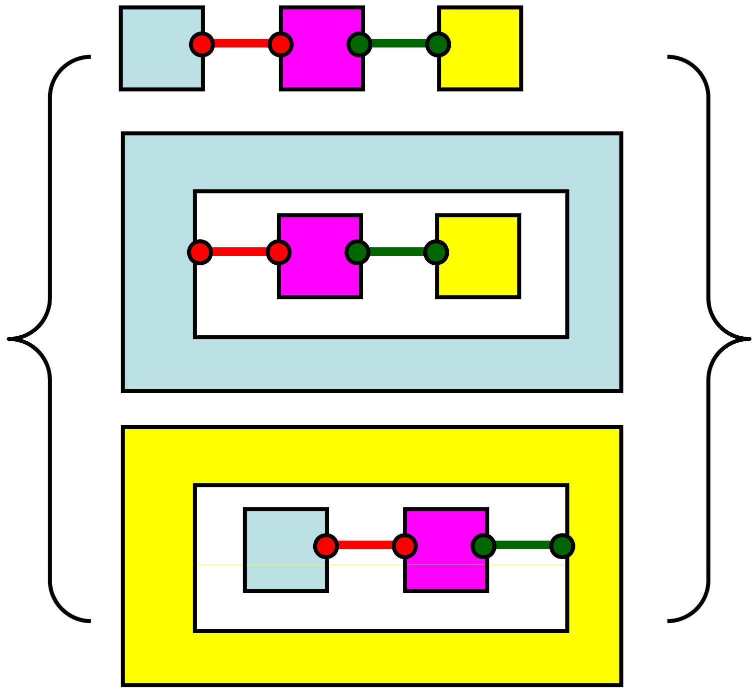
Our algorithm works well even if G is not biconnected.

min # of bends



1-connected SP graph

= min # of bends

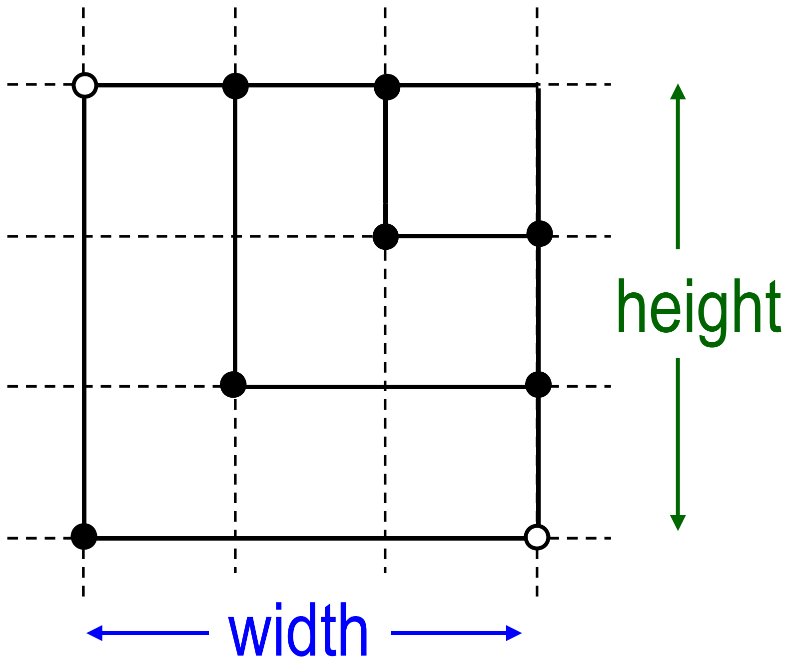


Conclusions

bend(G) $\leqq \lceil n/3 \rceil$ for biconnected SP graphs G of $\Delta \leqq 3$

Grid size $\leqq$ $8n/9$

=width + height

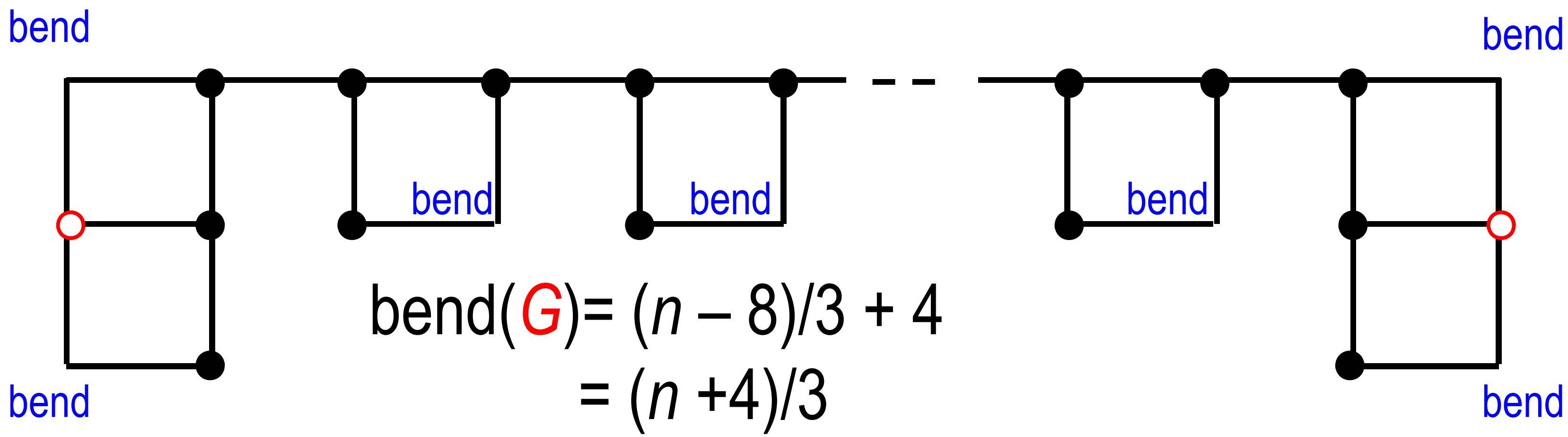


Conclusions

Our algorithm works well even if G is not biconnected.

$\mathrm{bend}(G) \leqq (n+4)/3$ for SP graphs G of $\Delta \leqq 3$

Best possible

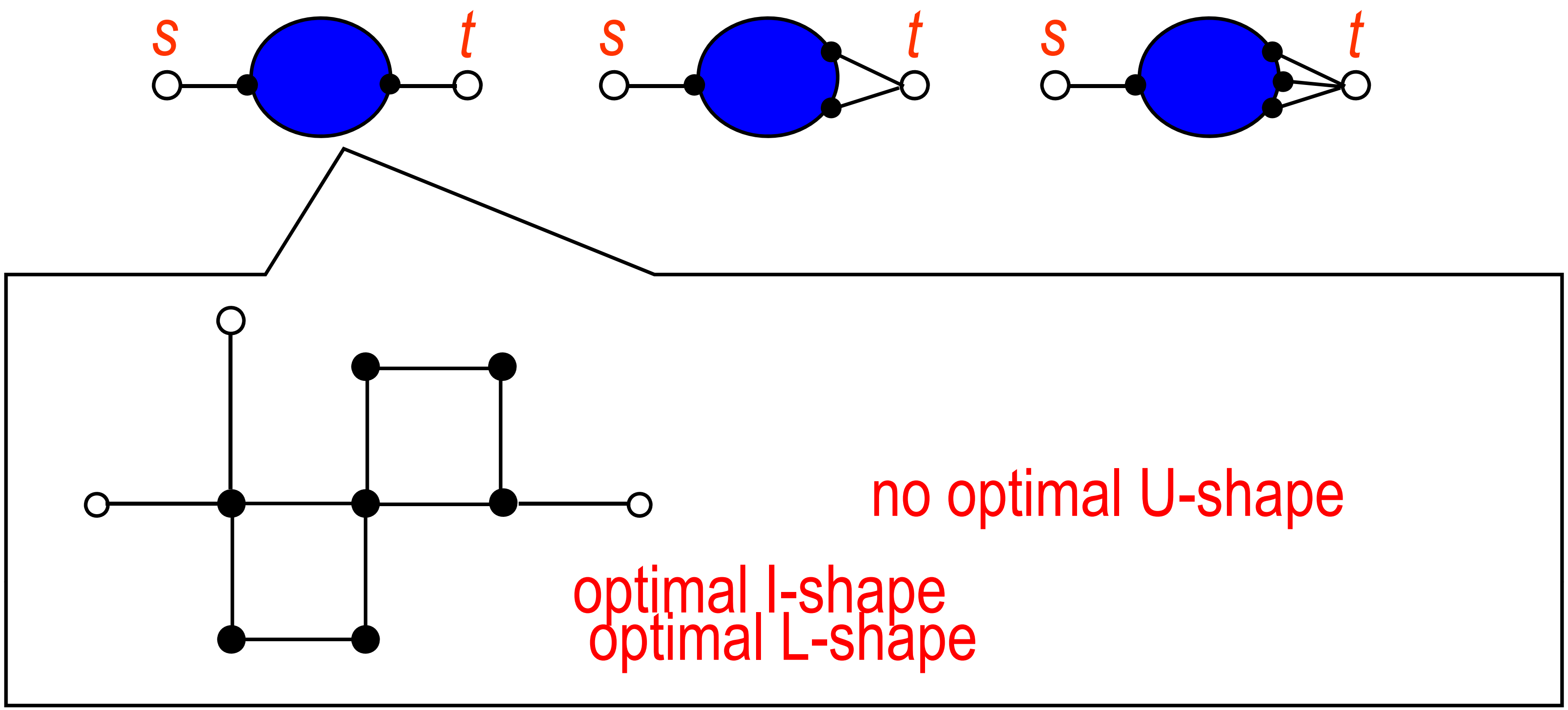


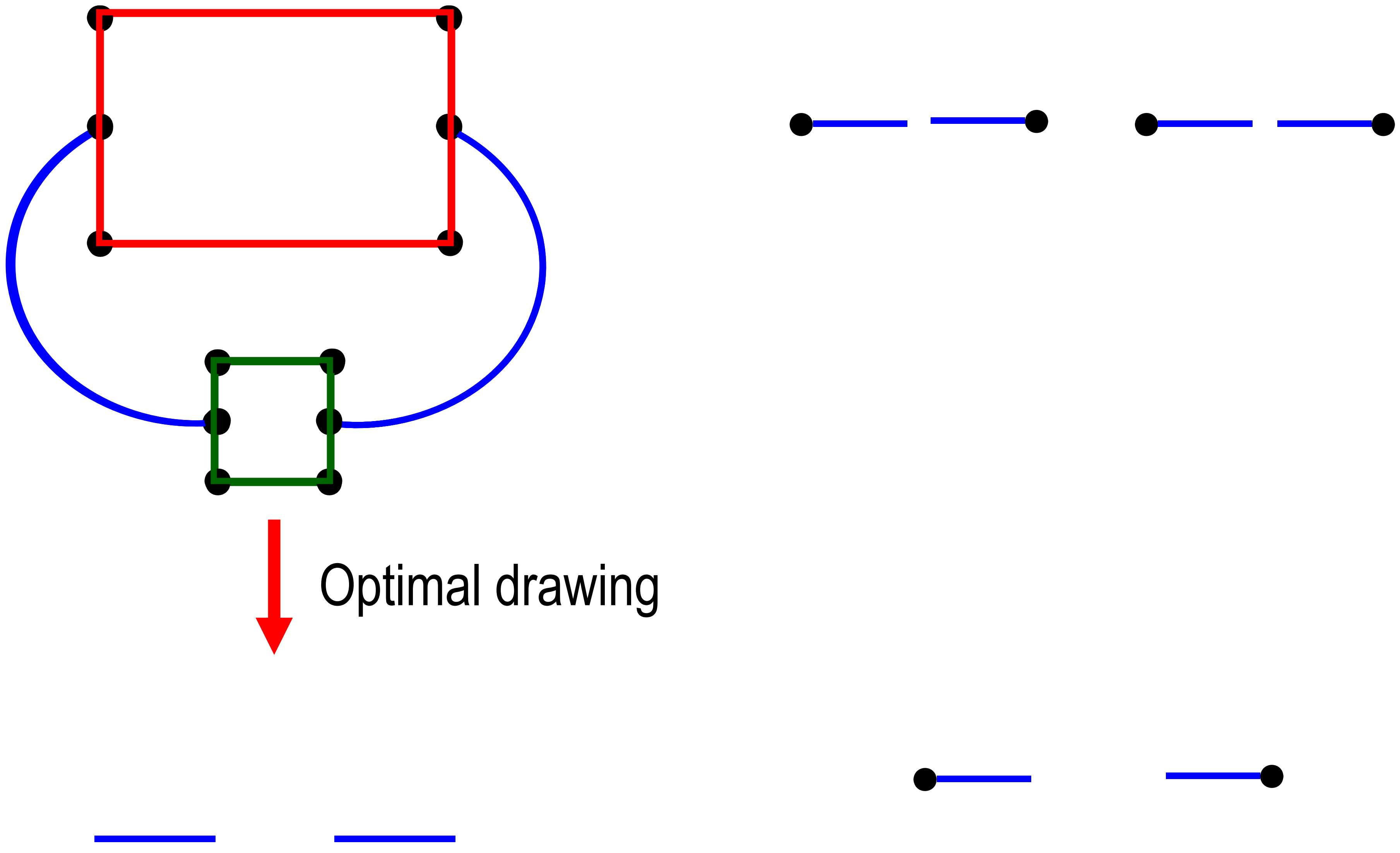
$$\mathrm{bend}(G) = (n-8)/3 + 4$$
$$= (n+4)/3$$

For series-parallel graphs *G* with *Δ*=4

Is there an O(*n*)-time algorithm to find an optimal orthogonal drawing of *G* ?

open

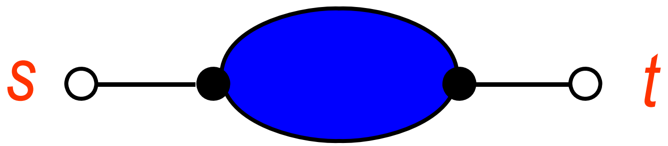


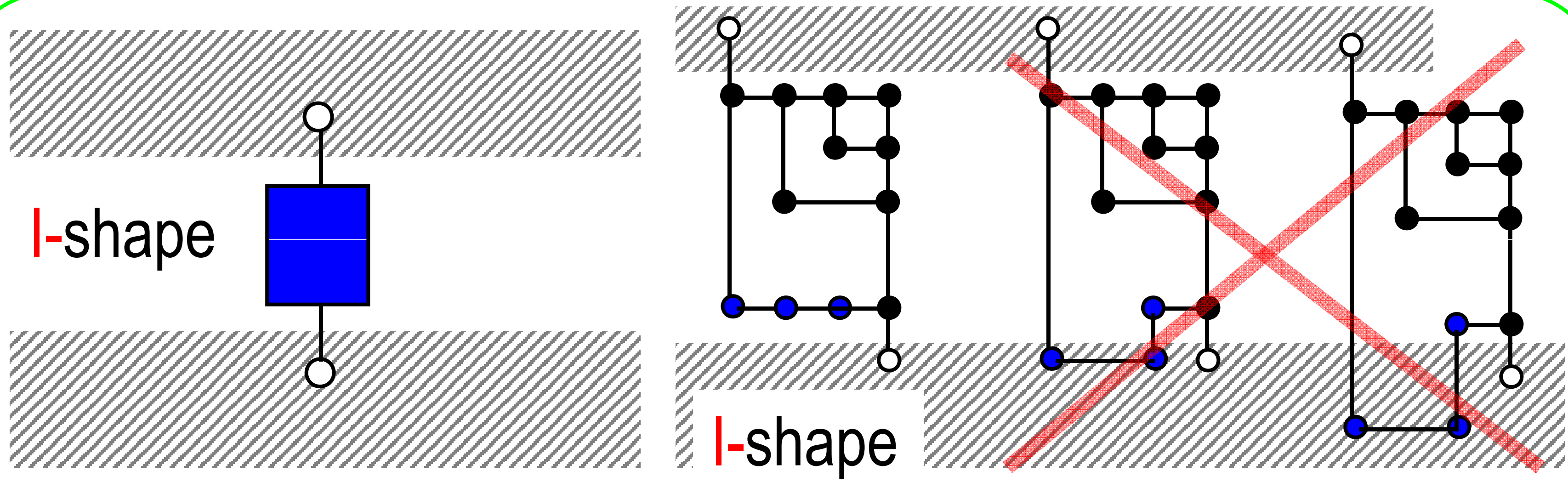

Optimal drawing

Our Main Idea

2-legged SP graph

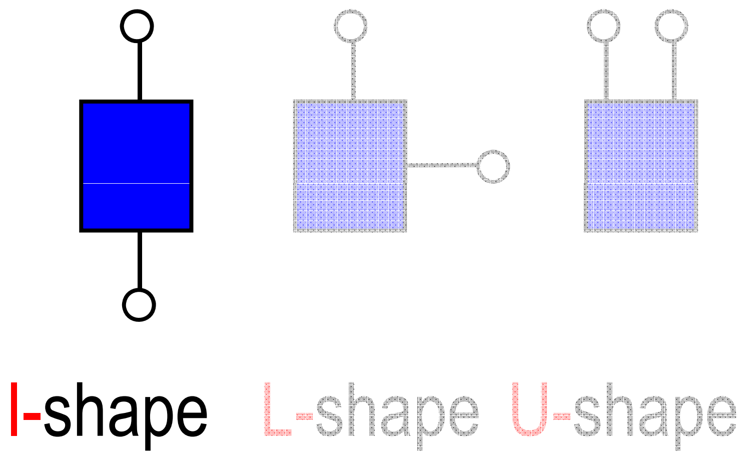
A SP graph G is 2-legged
if $n(G) \geqq 3$ and $d(s)=d(t)=1$ for the terminals s and t.



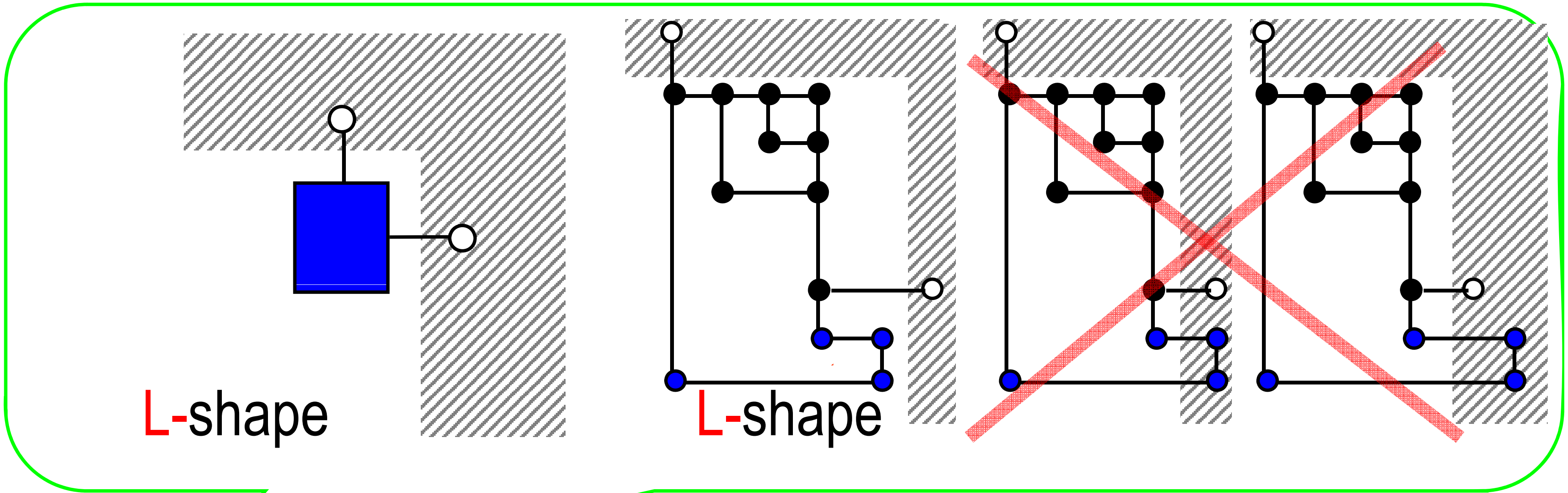


Definition of I- , L- and U-shaped drawings

- terminals are drawn on the outer face;
- the drawing except terminals intersects neither the north side nor the south side

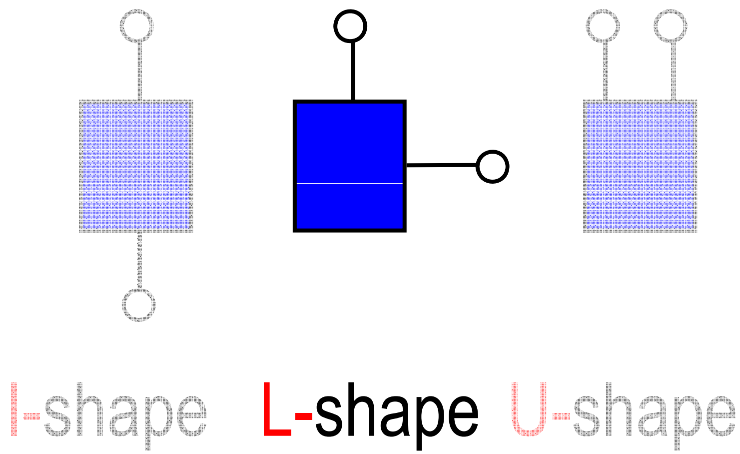


Our Main Idea

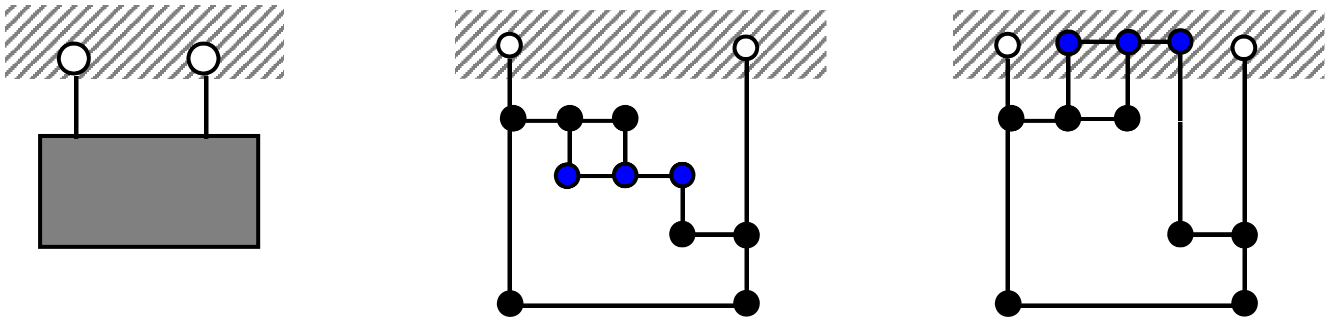


Defin… , L- and U-shaped drawings

- terminals are drawn on the outer face;
- the drawing except terminals intersects neither the north side nor the east side



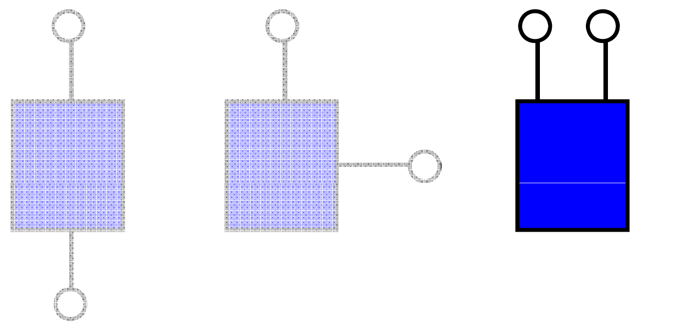
Our Main Idea



U-shape   U-shape   not U-shape

Defi[illegible] I-, L- and U-shaped drawings

- terminals are drawn on the outer face;
- the drawing except terminals doesn't intersect the north side



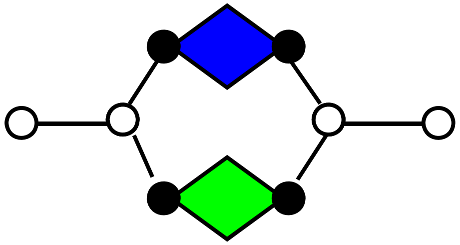
I-shape   L-shape   U-shape

Lemma 2

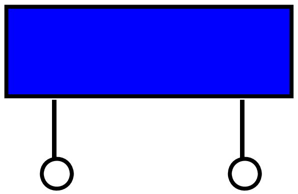
The following (a) and (b) hold for a 2-legged SP graph *G* of $\Delta \leqq 3$ unless *G* has a diamond:

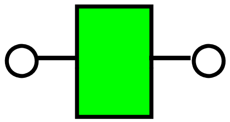
(a) *G* has three optimal I-, L- and U-shaped drawings

(b) such drawings can be found in linear time.



optimal U-shaped drawings

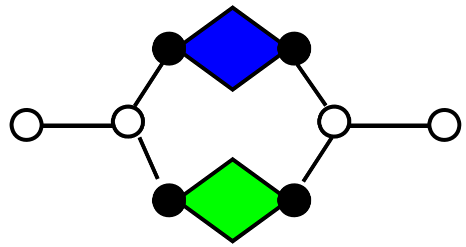




optimal I-shaped drawings

Lemma 2

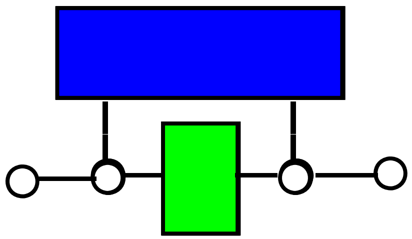
The following (a) and (b) hold for a 2-legged SP graph *G* of $\Delta \leqq 3$ unless *G* has a diamond:



(a) *G* has three optimal I-, L- and U-shaped drawings

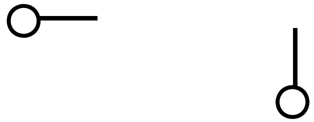
(b) such drawings can be found in linear time.

optimal



I-shape

optimal



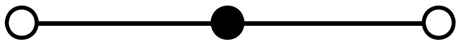
L-shape

optimal

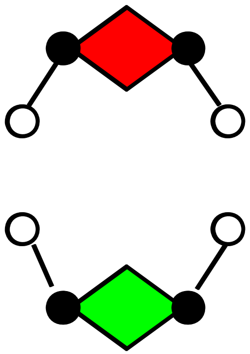
U-shape

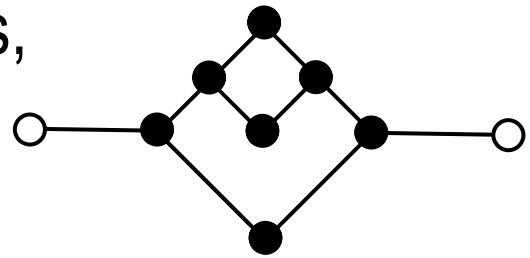
Definition of Diamond Graph

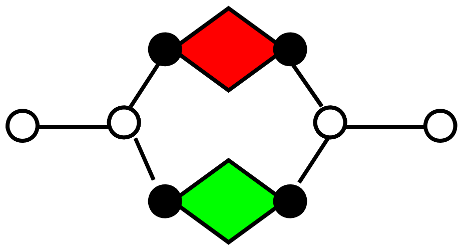
A Diamond graph is recursively defined as follows:

(a)  is a diamond graph.

a path with three vertices

(b) if  are diamond graphs,



then  is a diamond graph

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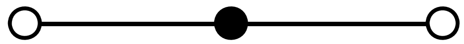
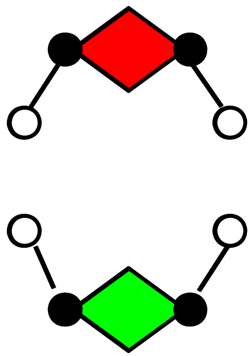
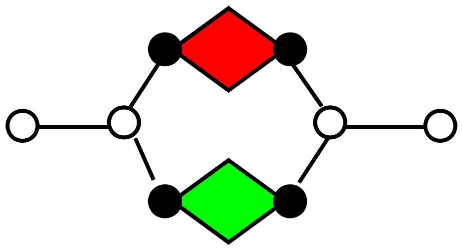
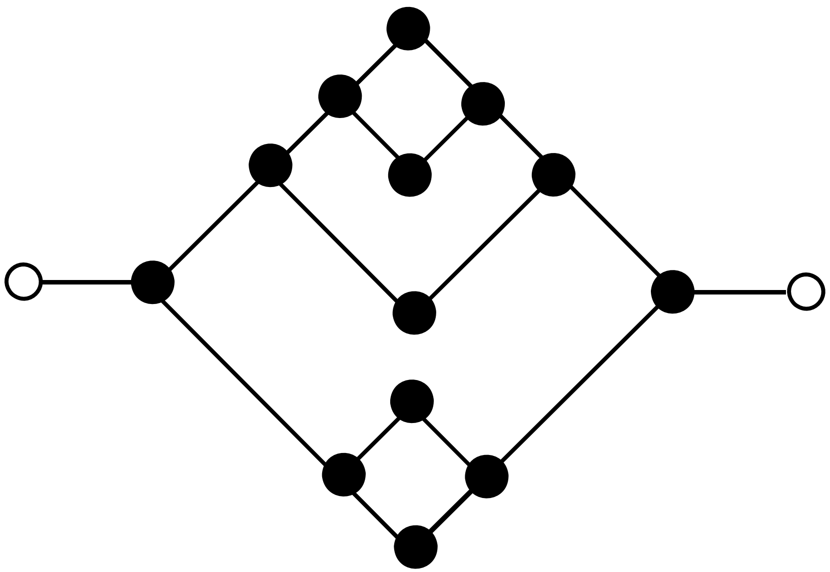
(a) 

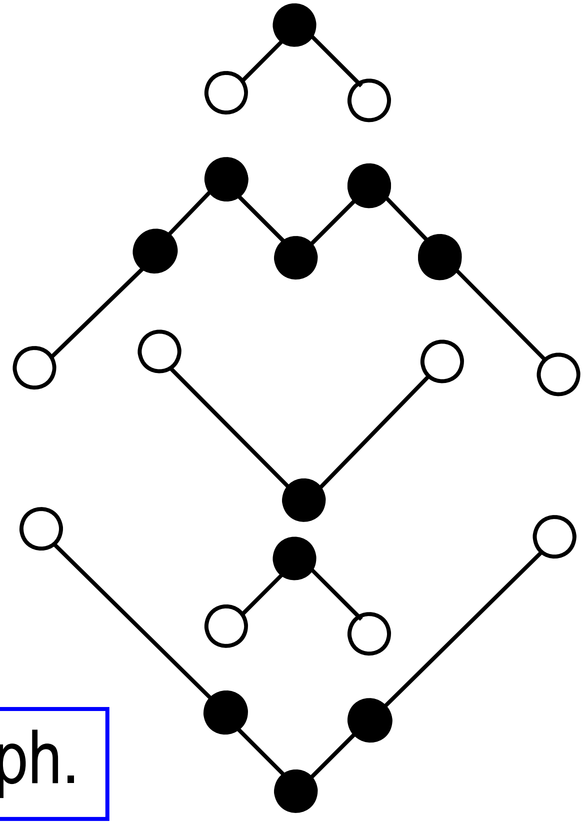
a path with three edges is a diamond graph.

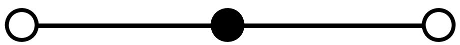
(b) if  are diamond graphs,

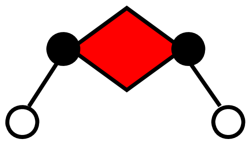
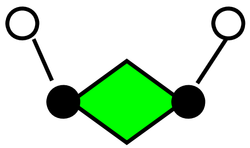
then  is a diamond graph

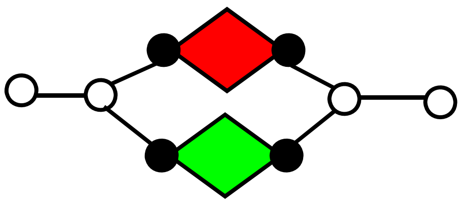
Diamond Graph



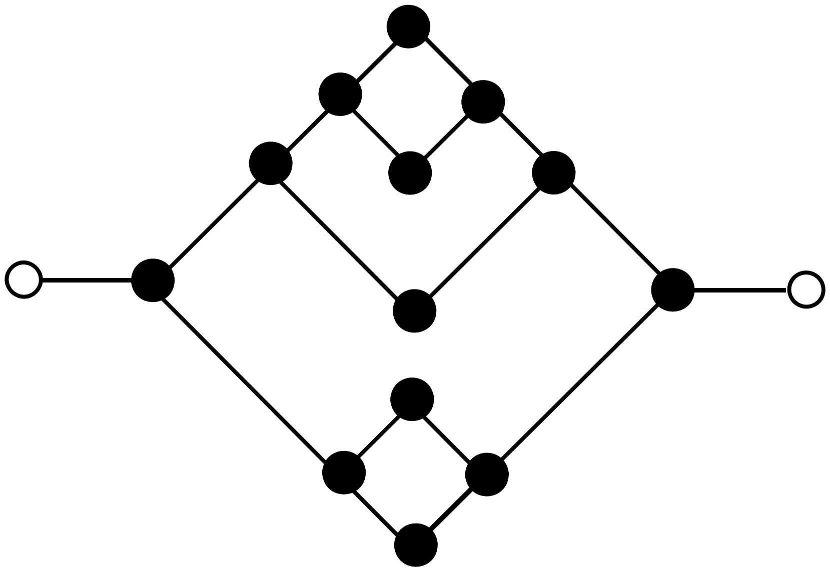


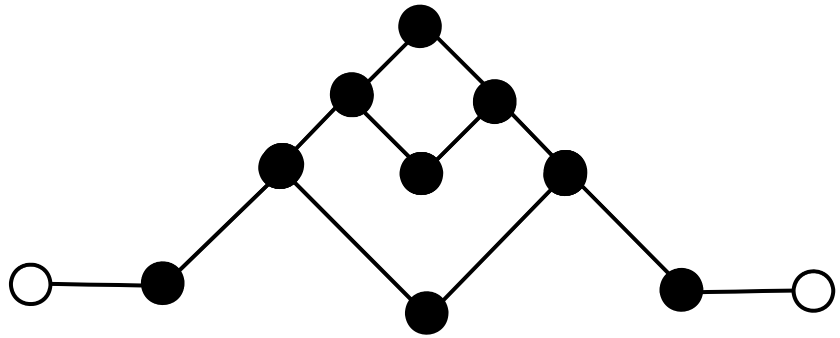
(a)  is a diamond graph.

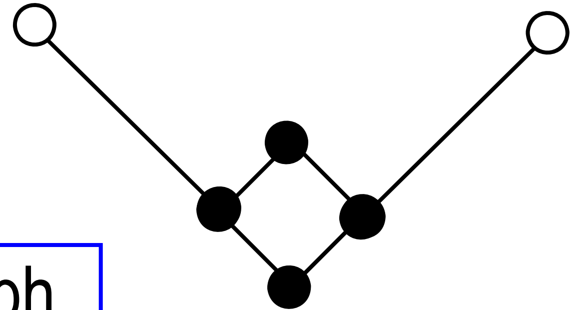
(b) If  and  are diamond graphs,

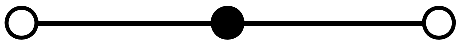
then  is a diamond graph

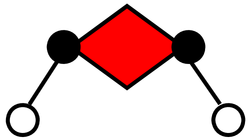
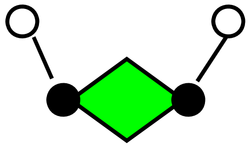
Diamond Graph

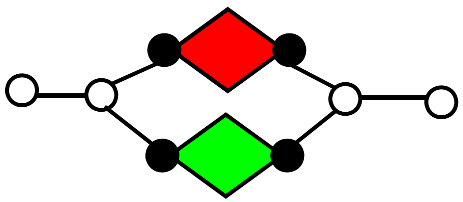






(a)  is a diamond graph.

(b) If  and  are diamond graphs,

then  is a diamond graph

Lemma 1

If G is a diamond graph, then

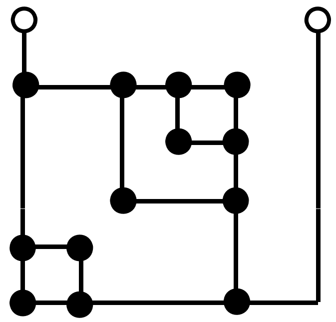
(a) G has both

a no-bend I-shaped dra

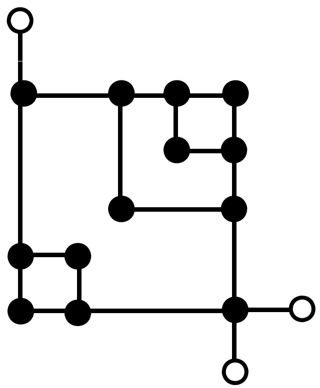
and

a no-bend L-shaped dra

(b) every no-bend drawing is eithe aped or L-shaped.



1-bend
U-shaped
drawing



∃ no-bend U-shaped drawing ?

NO

Lemma 1

If *G* is a diamond graph, then

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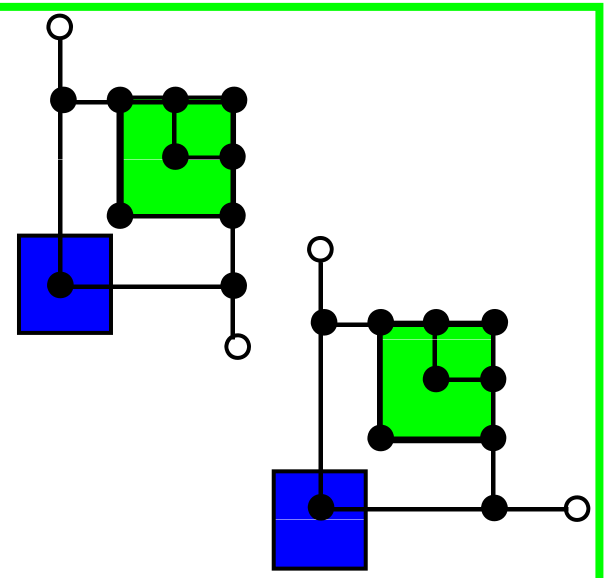
a no-bend I-shaped drawing

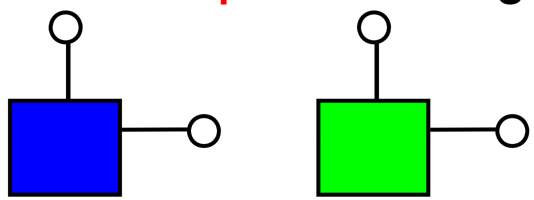
and

a no-bend L-shaped drawing



(b) every no-bend drawing is either I-shaped or L-shaped.

no-bend I-shaped drawings



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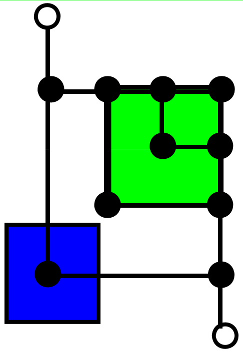
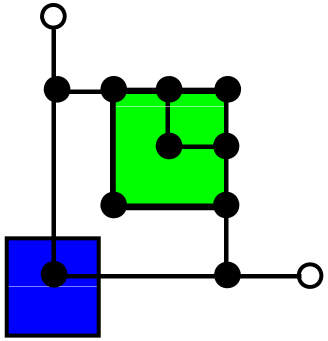
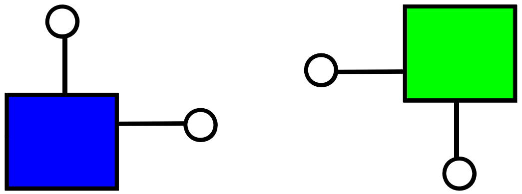




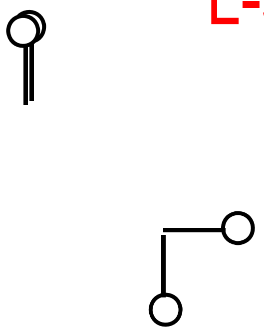
(b) every no-bend drawing is either I-shaped or L-shaped.

Such drawings can be found in linear time.

no-bend I-shaped drawings



L-shaped



Lemma 2

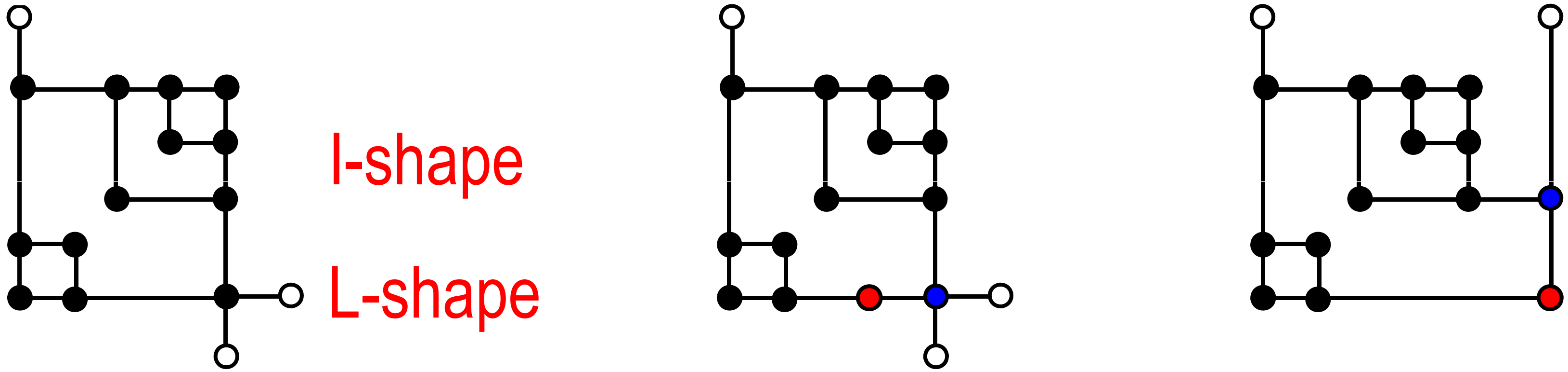
The following (a) and (b) hold for a 2-legged SP graph G of $\Delta \leqq 3$ unless G is a diamond graph

(a) G has three optimal I-, L- and U-shaped drawings

(b) such drawings can be found in linear time.

diamond graph

not a diamond graph



no-bend drawing

L-shape

U-shape

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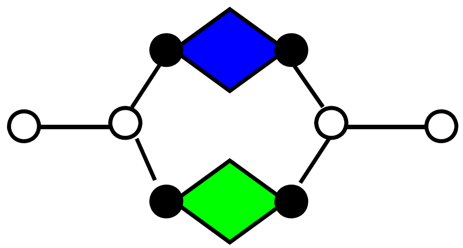
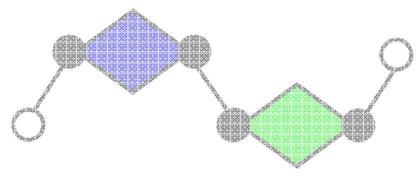
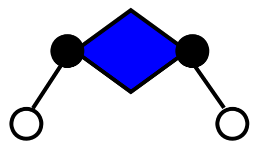
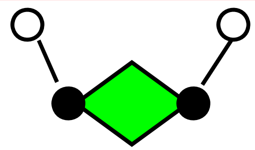
Proof: by an induction on # of vertices.



Parallel-connection



Series-connection

Suppose that the lemma holds true for  and 

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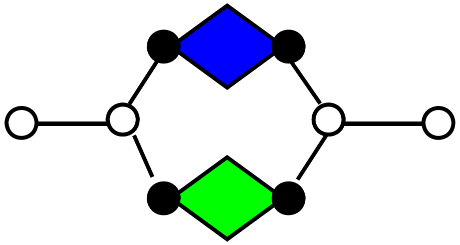
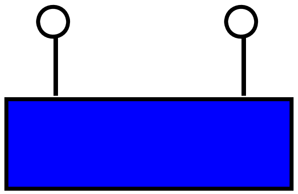
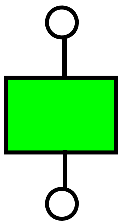


optimal U-shaped drawings





optimal I-shaped drawings

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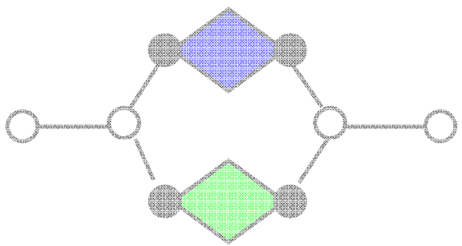
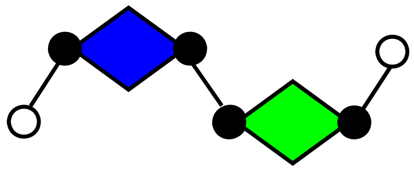
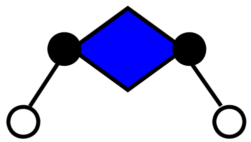
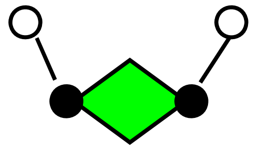


Parallel-connection



Series-connection

Suppose that the lemma holds true for  and 

Known results

In the fixed embedding setting:

n : # of vertices

For plane graph:

$O(n^2 \log n)$ time — R. Tamassia, 1987

Min-cost flow problem

Known results

In the fixed embedding setting:

n : # of vertices

For plane graph:

$O(n^2 \log n)$ time — R. Tamassia, 1987

$O(n^{7/4}\sqrt{\log n})$ time — A. Garg, R. Tamassia, 1997

improved

Known results

In the fixed embedding setting:

For plane graph:

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In the variable embedding setting:

NP-complete for planar graphs of $\Delta \leqq 4$

A. Garg, R. Tamassia, 2001

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In the fixed embedding setting:

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In the variable embedding setting:

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A. Garg, R. Tamassia, 2001

If $\Delta \leqq 3$, $O(n^5 \log n)$ time — D. Battista, *et al*. 1998

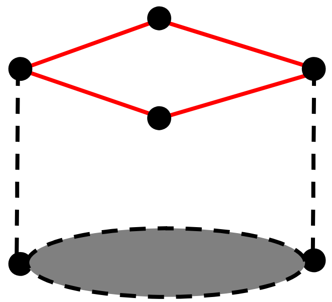
Lemma 3

Every biconnected SP graph G of $\Delta \leqq 3$ has one of the following three substructures:

(a) a diamond C

(b) two adjacent vertices u and v s.t. $d(u)=d(v)=2$

(c) a complete graph K_3.



(a) G

Lemma 1 (Our Main Idea)

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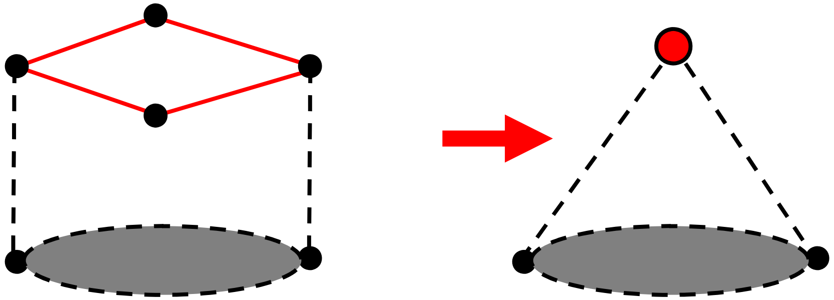
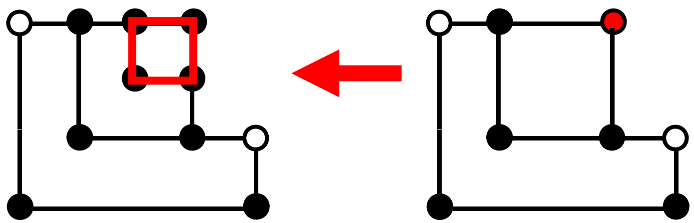
bend(G)=bend(G')



G $\quad$ G'

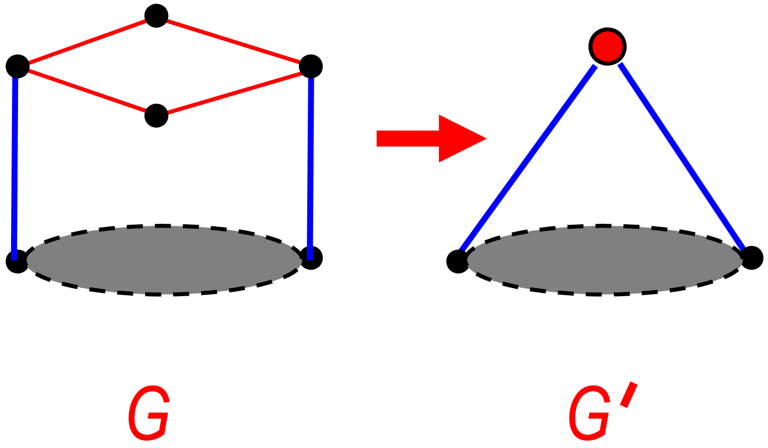
bend(G)=bend(G')=0



G $\quad$ G'

Proof

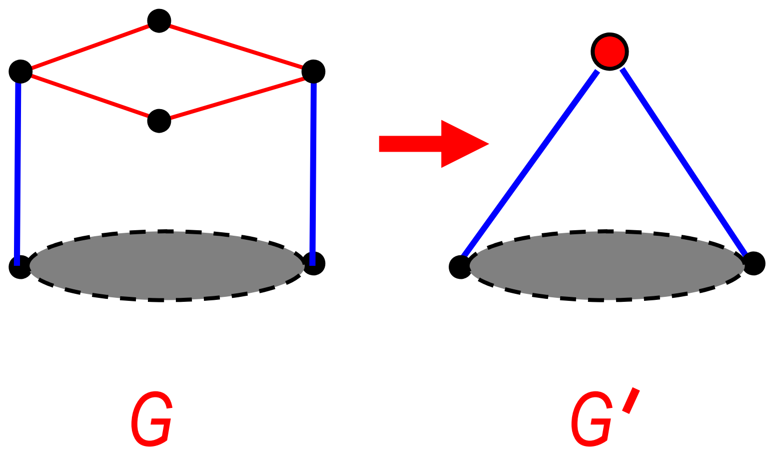
bend(G)=bend(G')

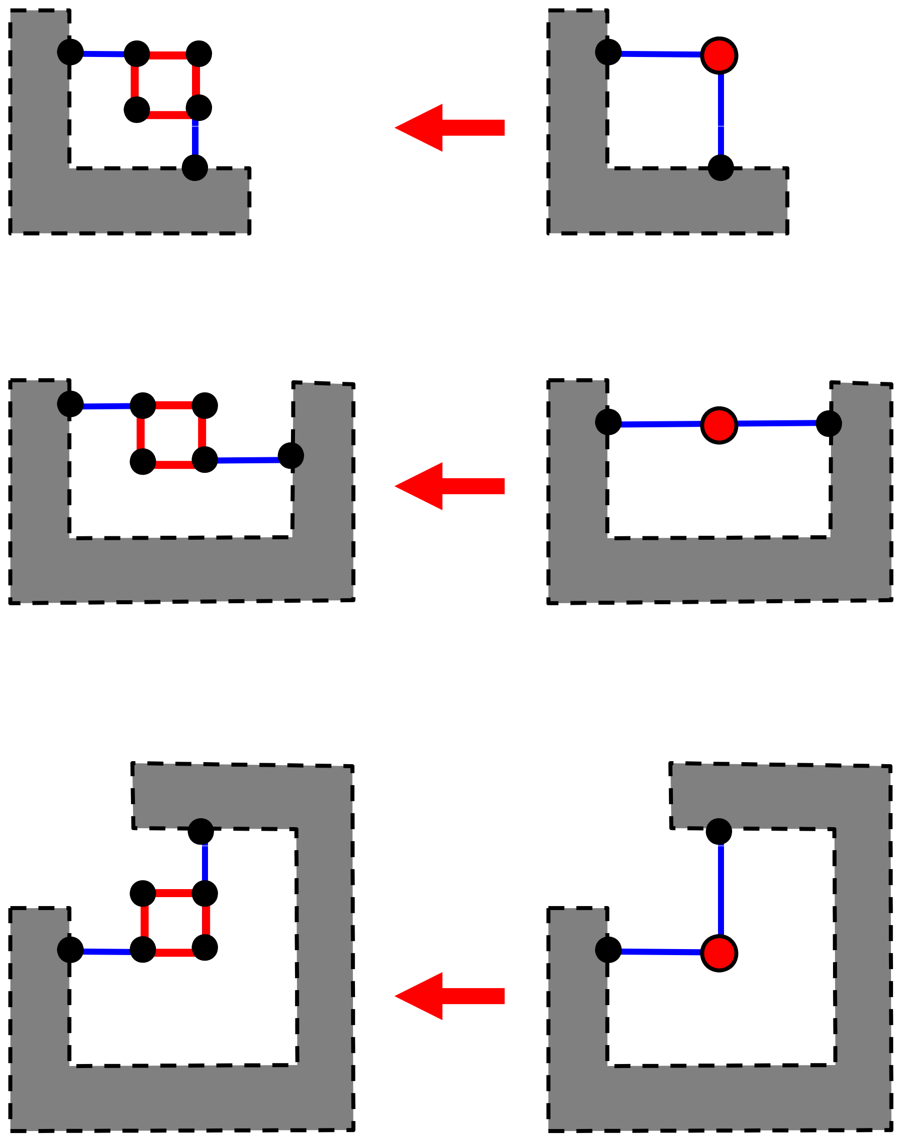


Proof

Given an optimal drawing of G'

bend(G) ≦ bend(G')

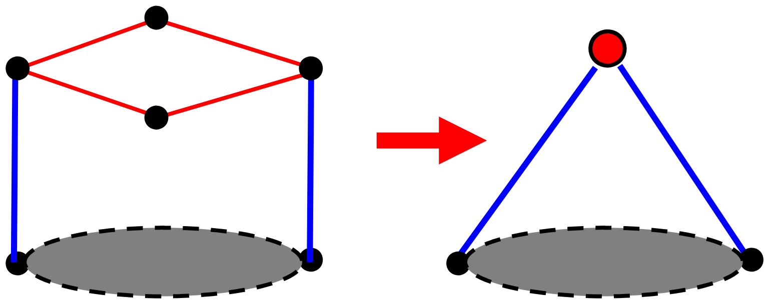




Proof

Given an optimal drawing of G

$\mathrm{bend}(G) \geqq \mathrm{bend}(G')$



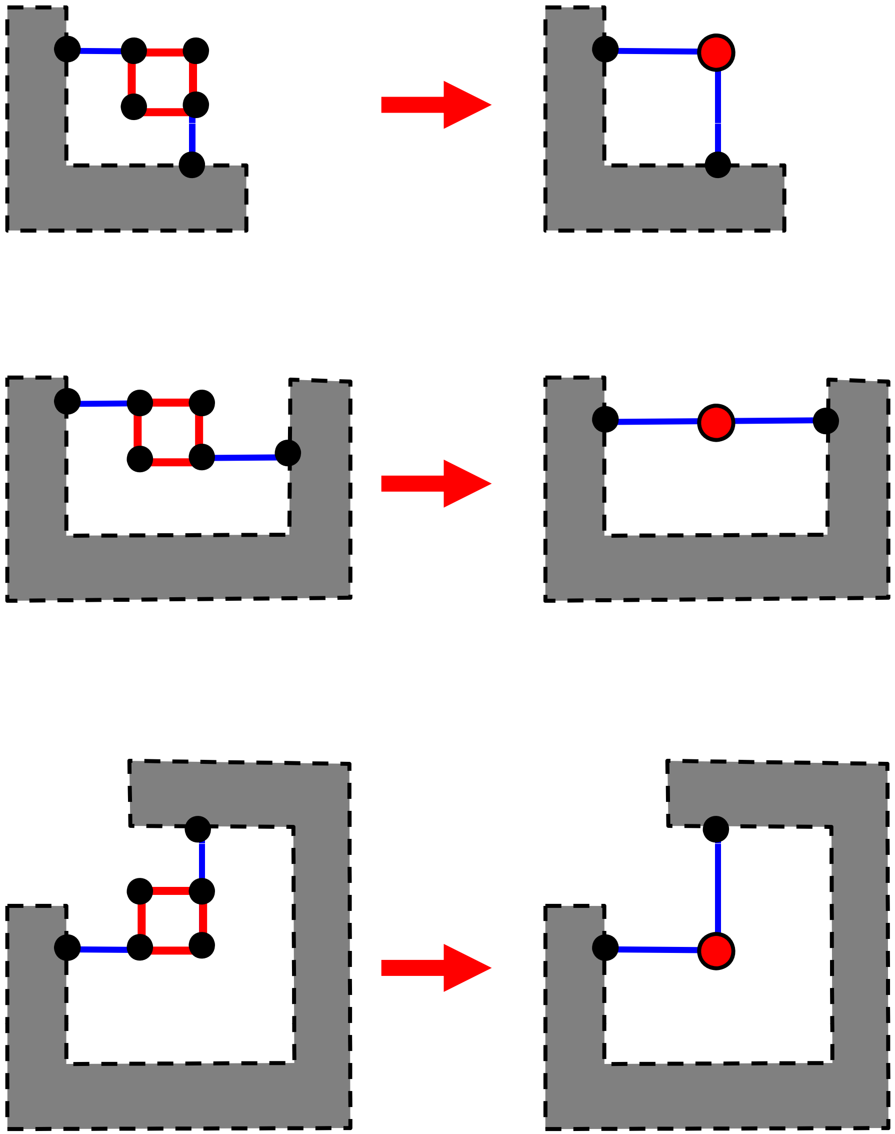
G G'

Case 1: bend( ) = 0

Case 2: bend( ) = 1

omitted

Case 3: bend( ) $\geq$ 2



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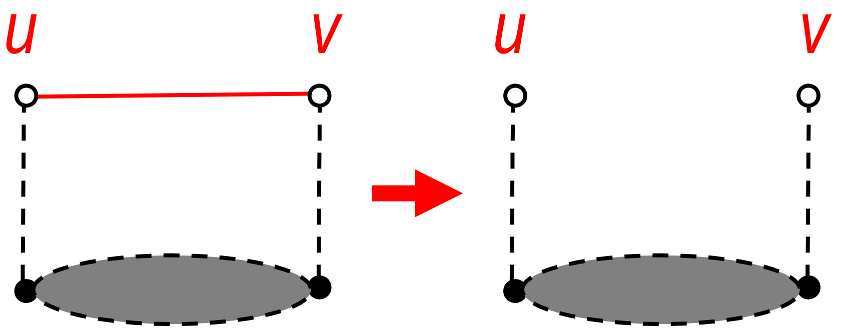
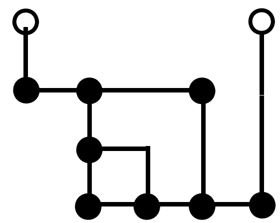


G No diamonds G'

∃ an optimal U-shape drawing



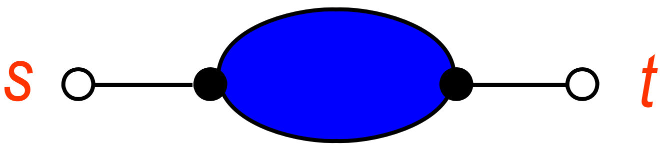
G'

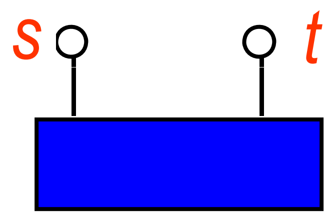
Lemma 2

?

For each SP graph *G′*
If n ≧ 4, Δ≦3 and d(s)=d(t)=1,
then ∃ an optimal U-shape drawing

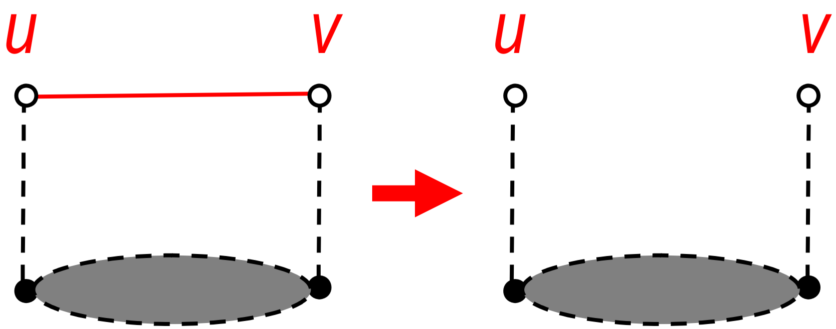
optimal U-shaped drawing



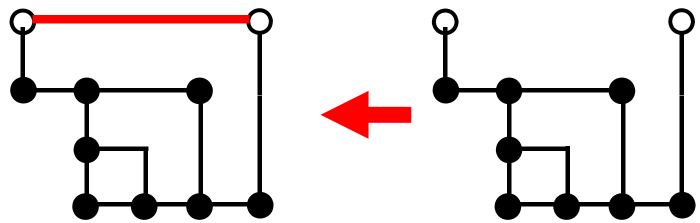


bend(*G*)=bend(*G′*)

∃ an optimal U-shape drawing

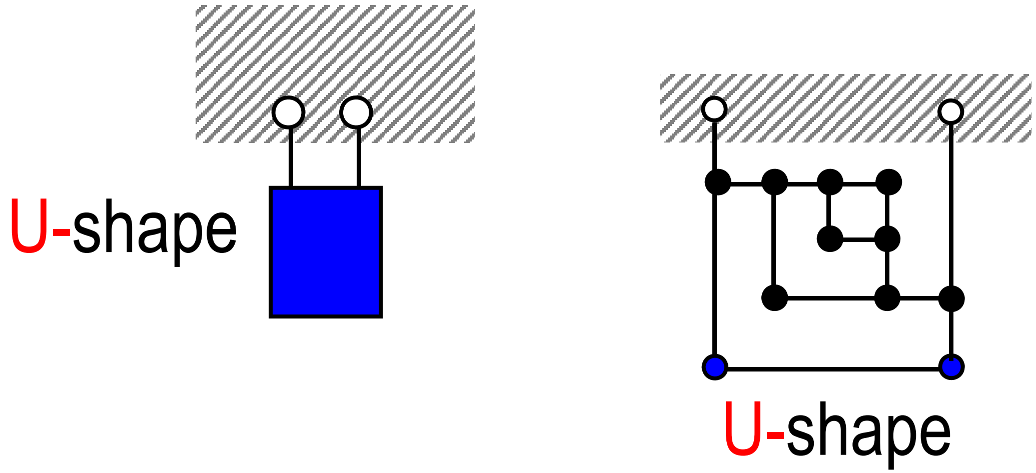


G No diamonds *G′*



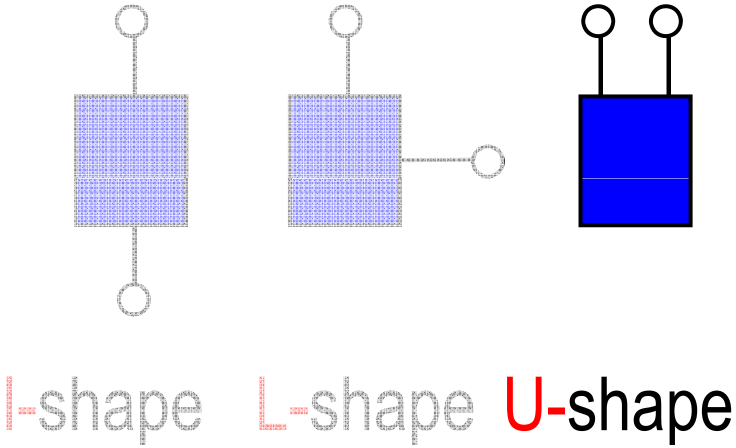
G *G′*

Our Main Idea



Definition of I-, L- and U-shaped drawings

- terminals are drawn on the outer face;
- the drawing except terminals doesn't intersect the north side



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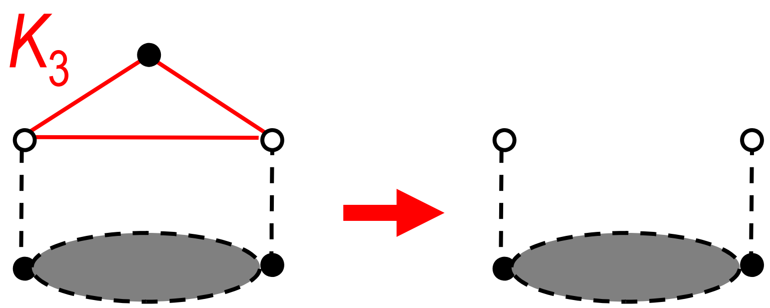
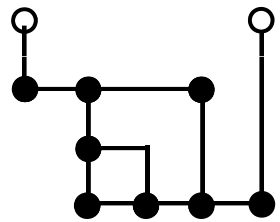
$\boxed{\text{(c) a complete graph } K_3.}$

bend(G)=bend(G')+1



G $\boxed{\text{No diamonds}}$ G'

$\boxed{\exists \text{ an optimal U-shape drawing}}$



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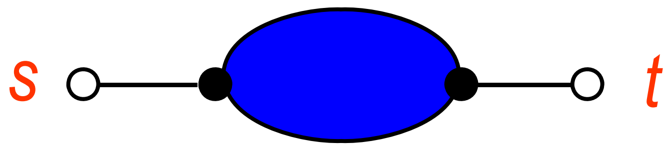
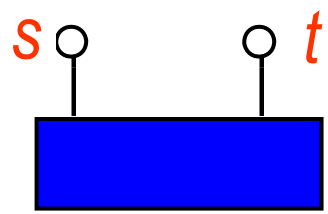
If n ≧ 4, Δ≦3 and d(s)=d(t)=1,

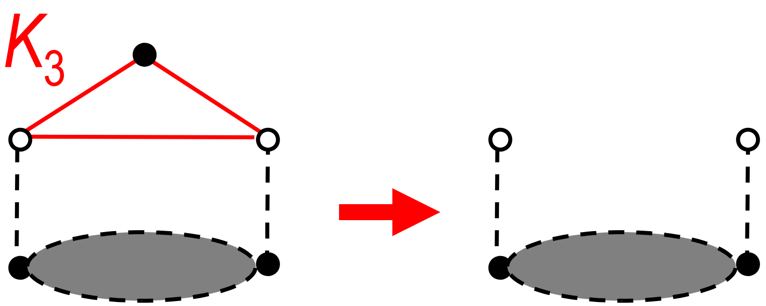
then ∃ an optimal U-shape drawing



optimal U-shaped drawing

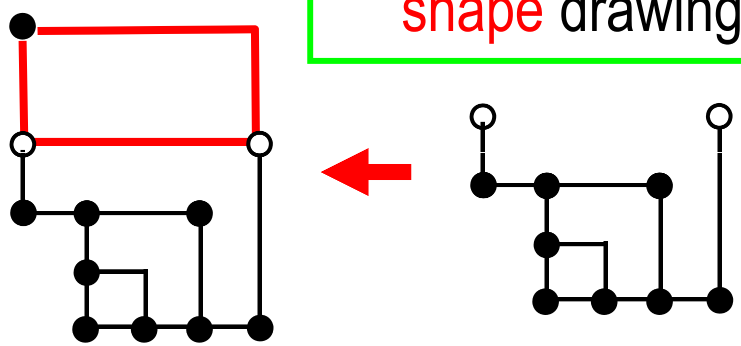


bend(G)=bend(G')+1



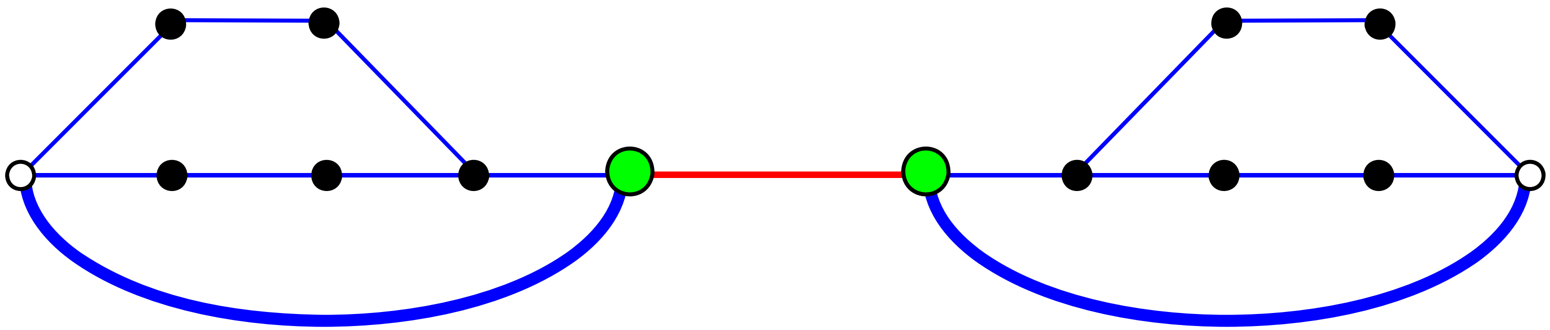
G No diamonds G'

∃ an optimal U-shape drawing



G G'

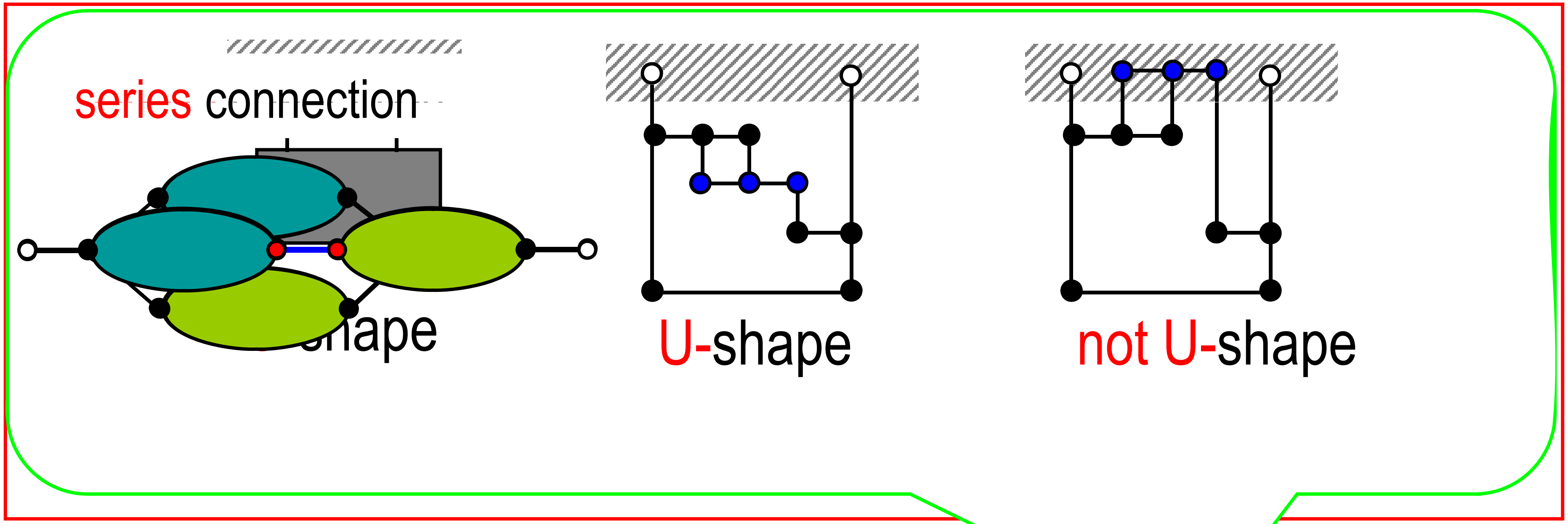
Optimal orthogonal drawing

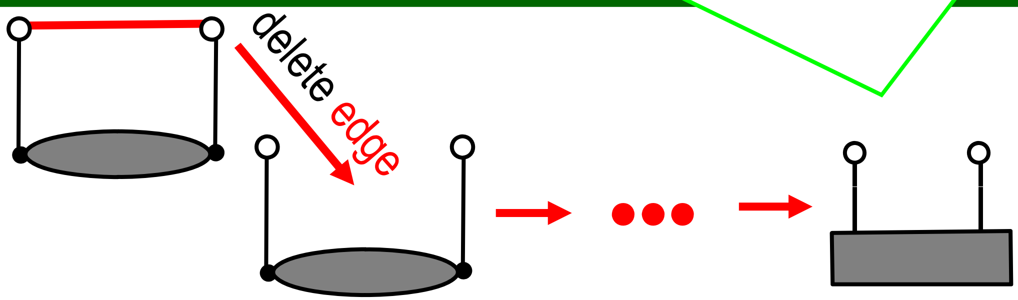


non -connected planar graph

Optimal orthogonal drawings ?

Case (b): ∃





Find an
opt & U-shape