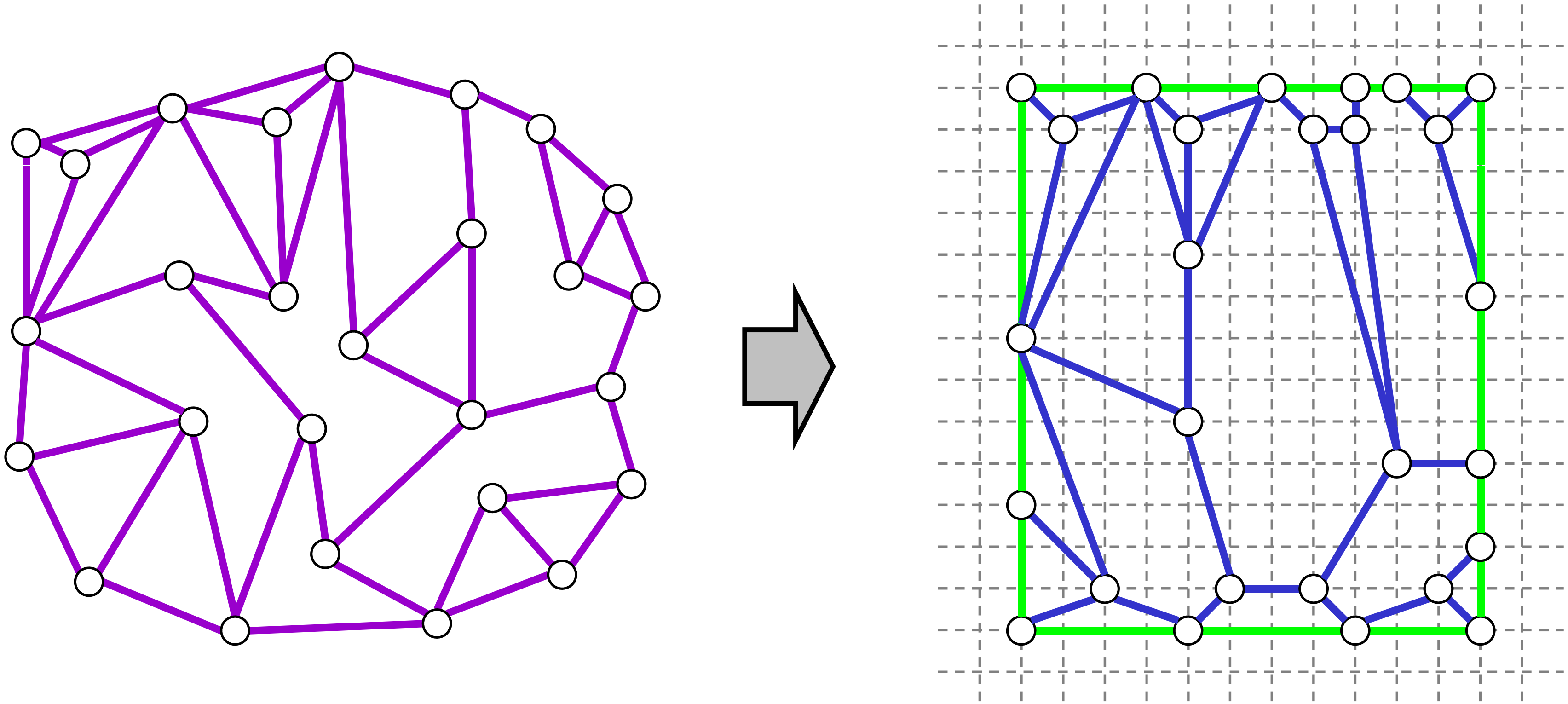
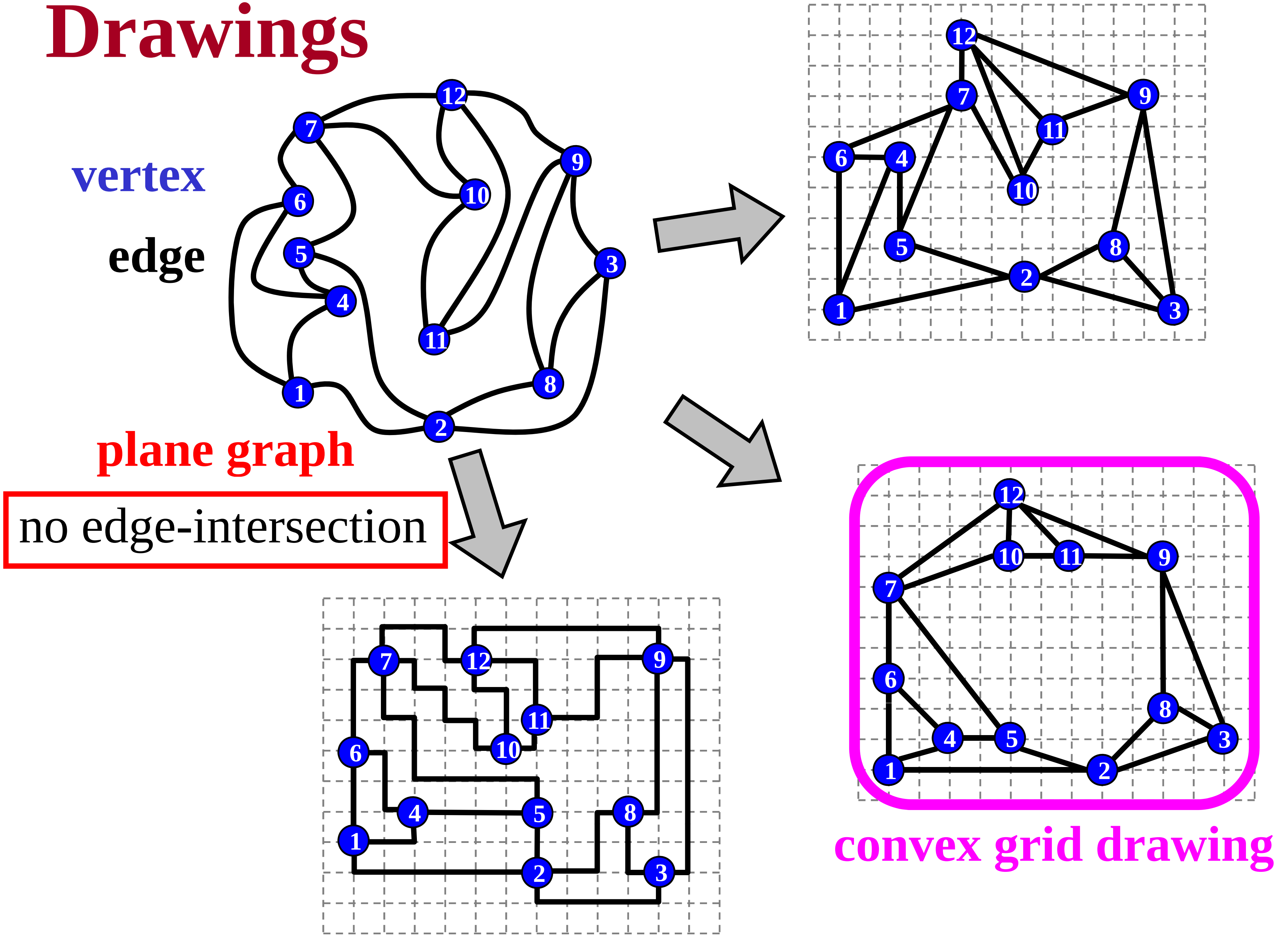
Convex Grid Drawings of Plane Graphs with Rectangular Contours

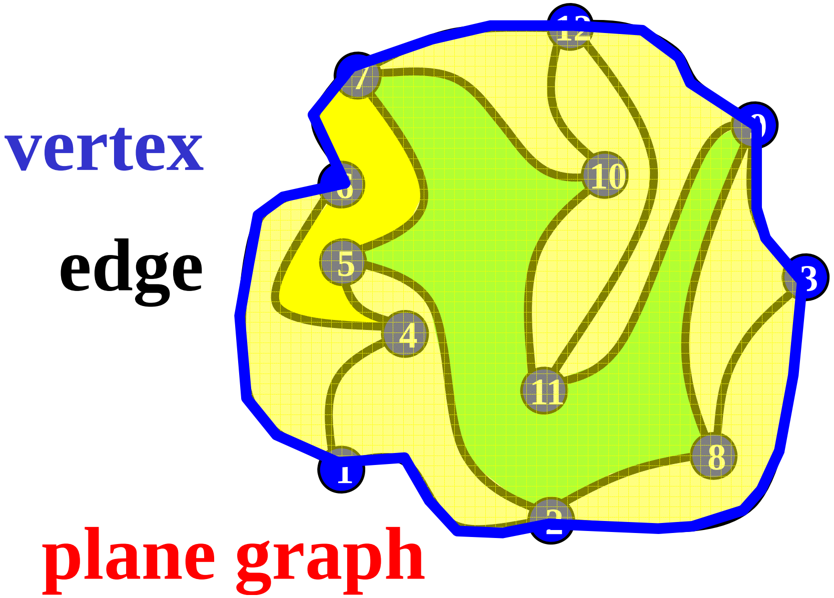


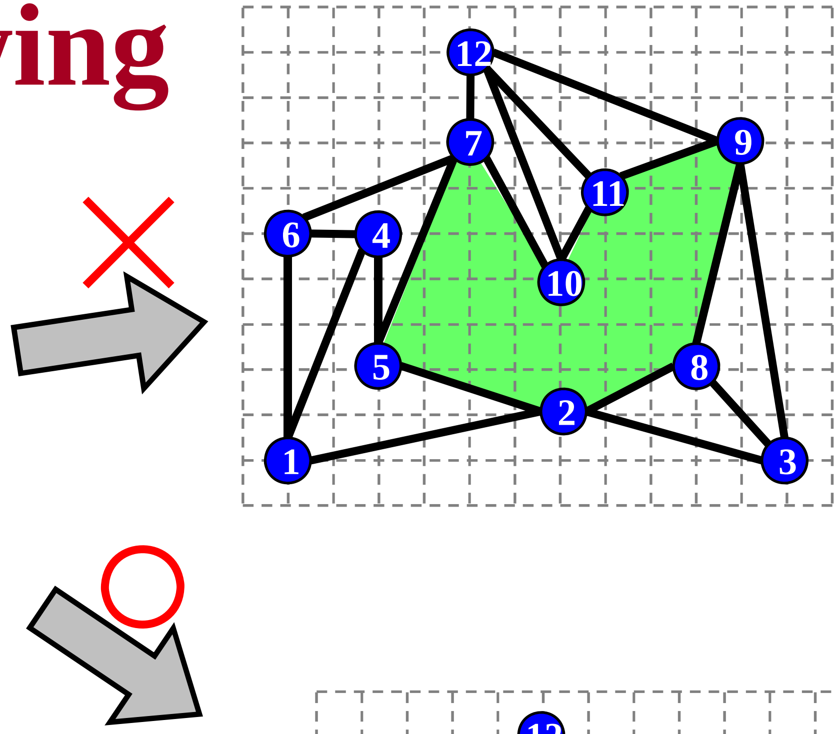
Akira Kamada (Tohoku University)
Kazuyuki Miura (Fukushima University)
Takao Nishizeki (Tohoku University)

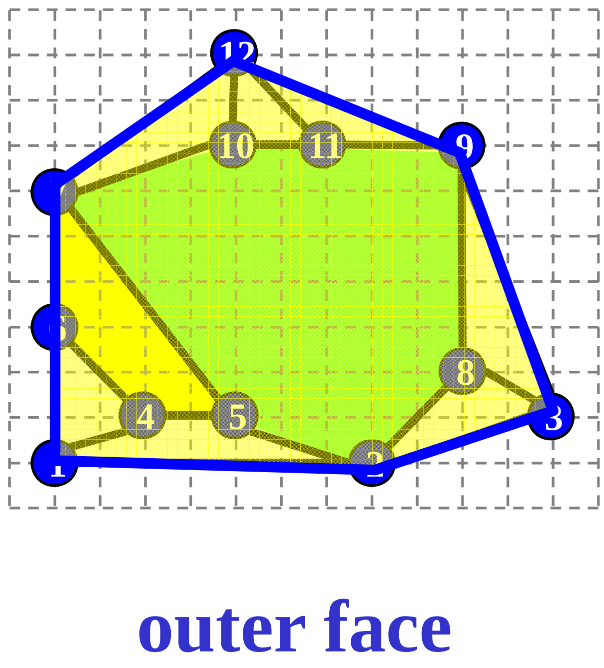
Drawings


vertex
edge
plane graph
no edge-intersection
convex grid drawing

Convex grid drawing





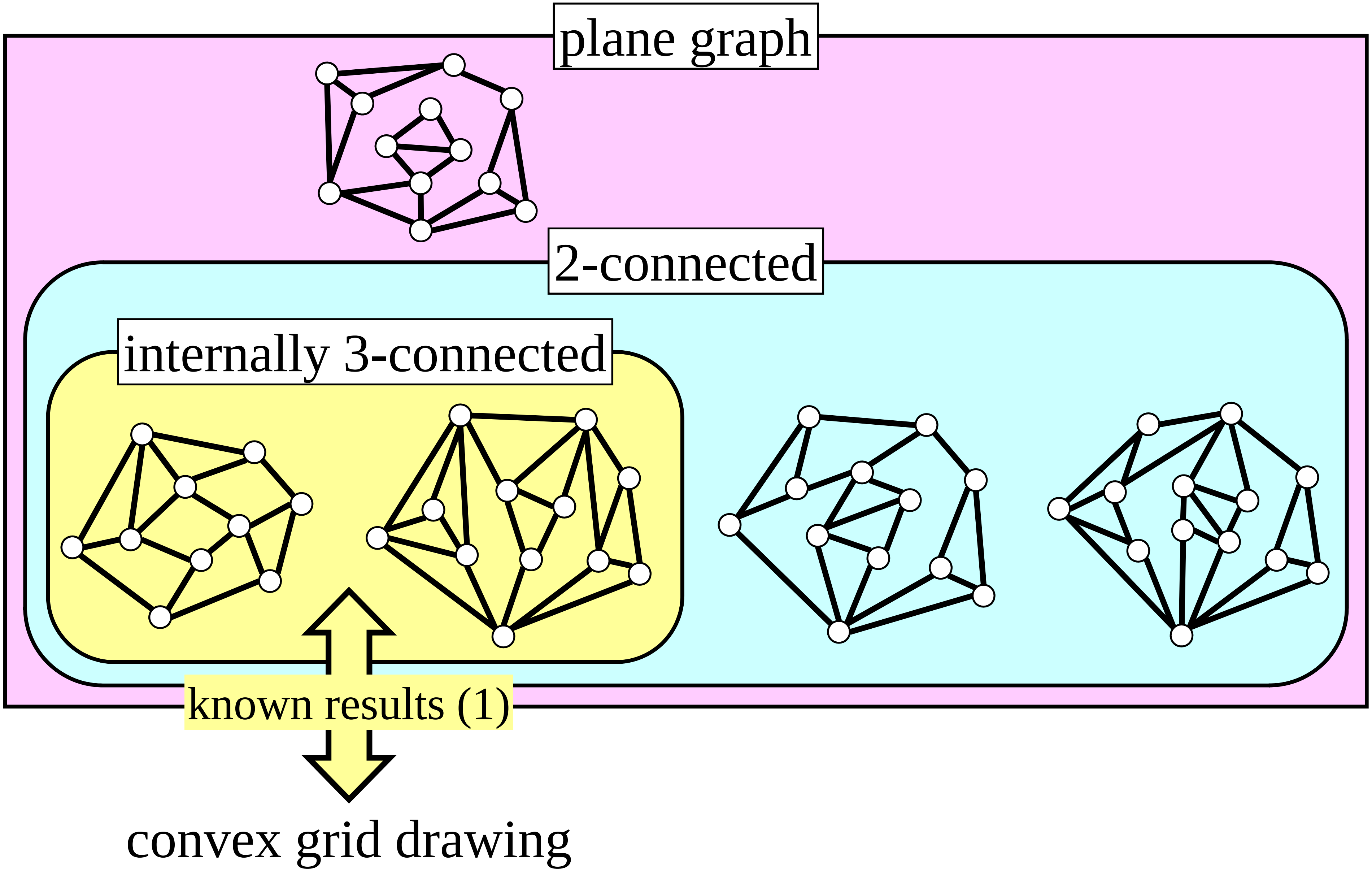


vertex ： grid point

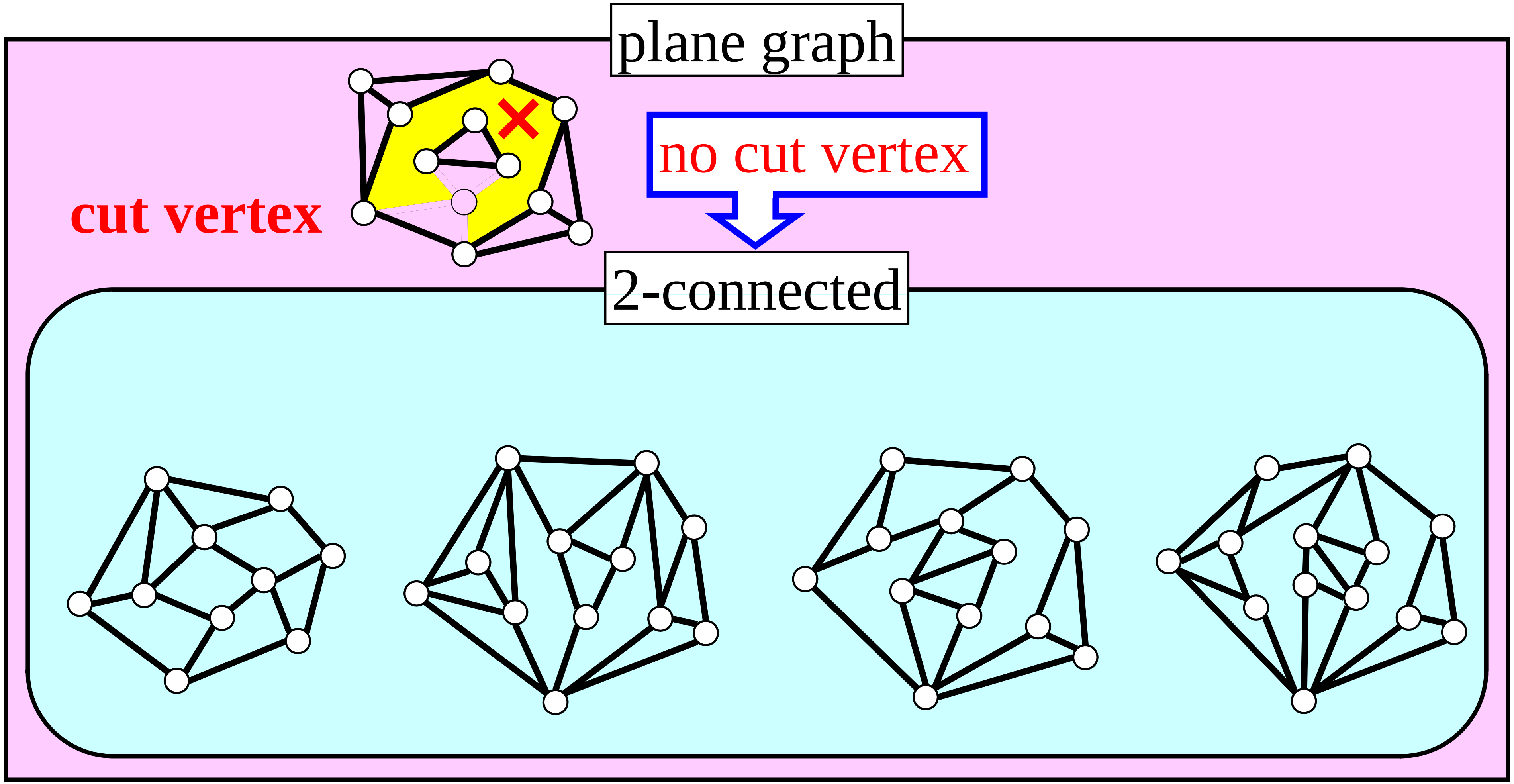
edge ： straight line segment
without edge-intersection

face ： convex polygon

Known results (1)



Known results (1)



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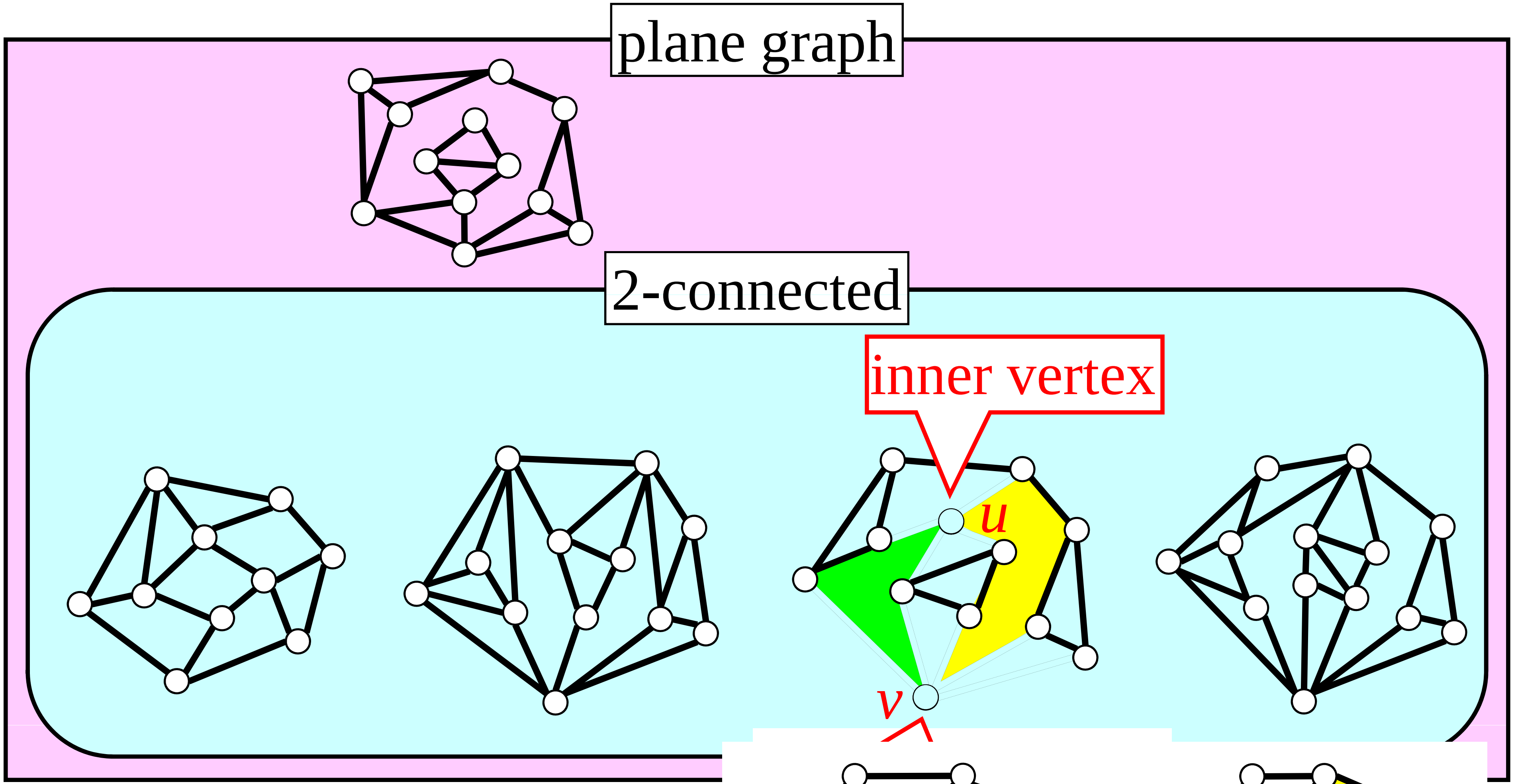
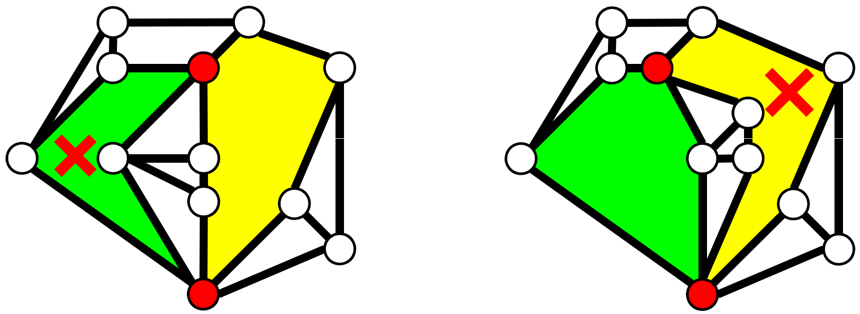


cannot be simultaneously drawn as convex polygons



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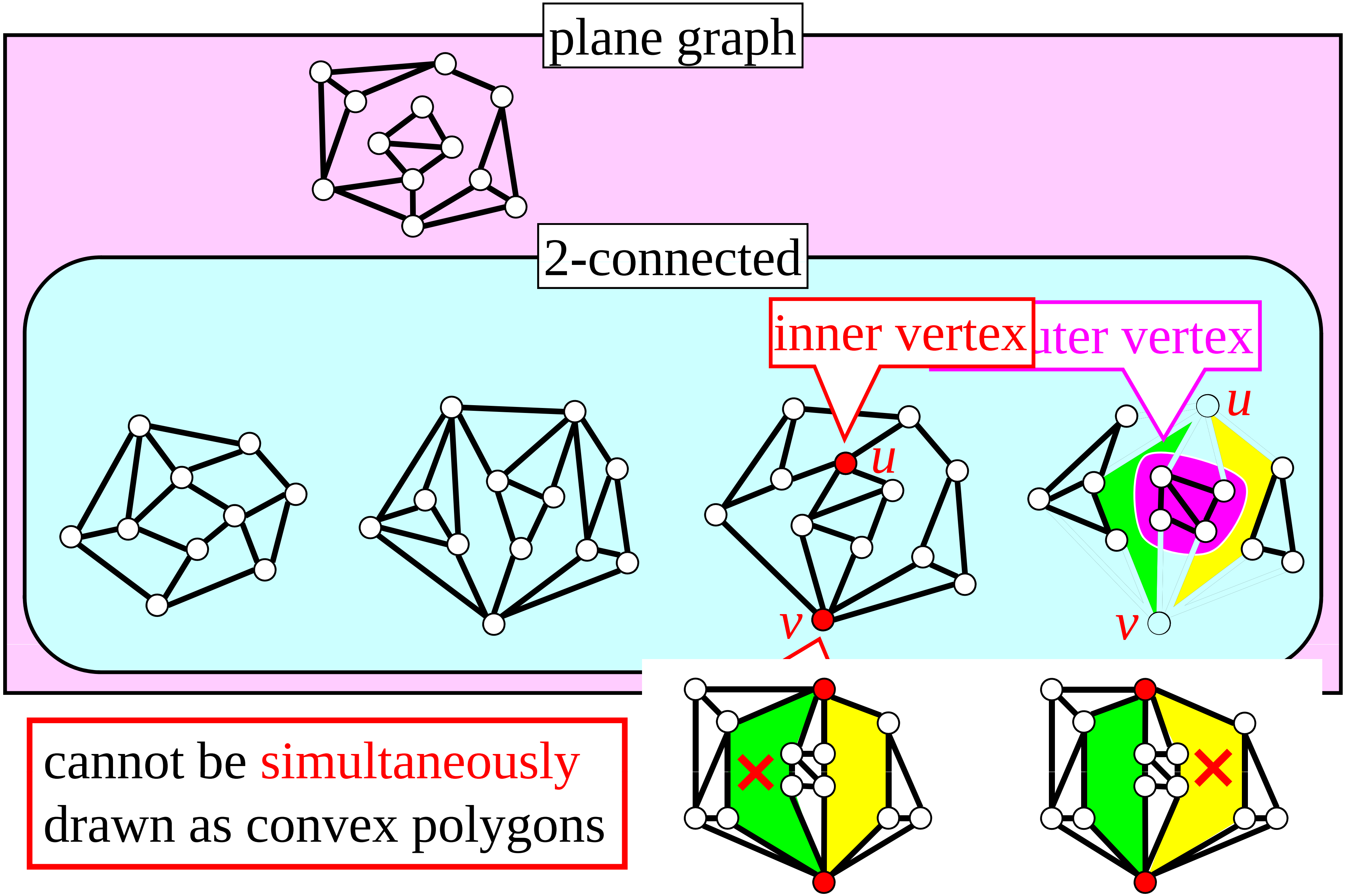
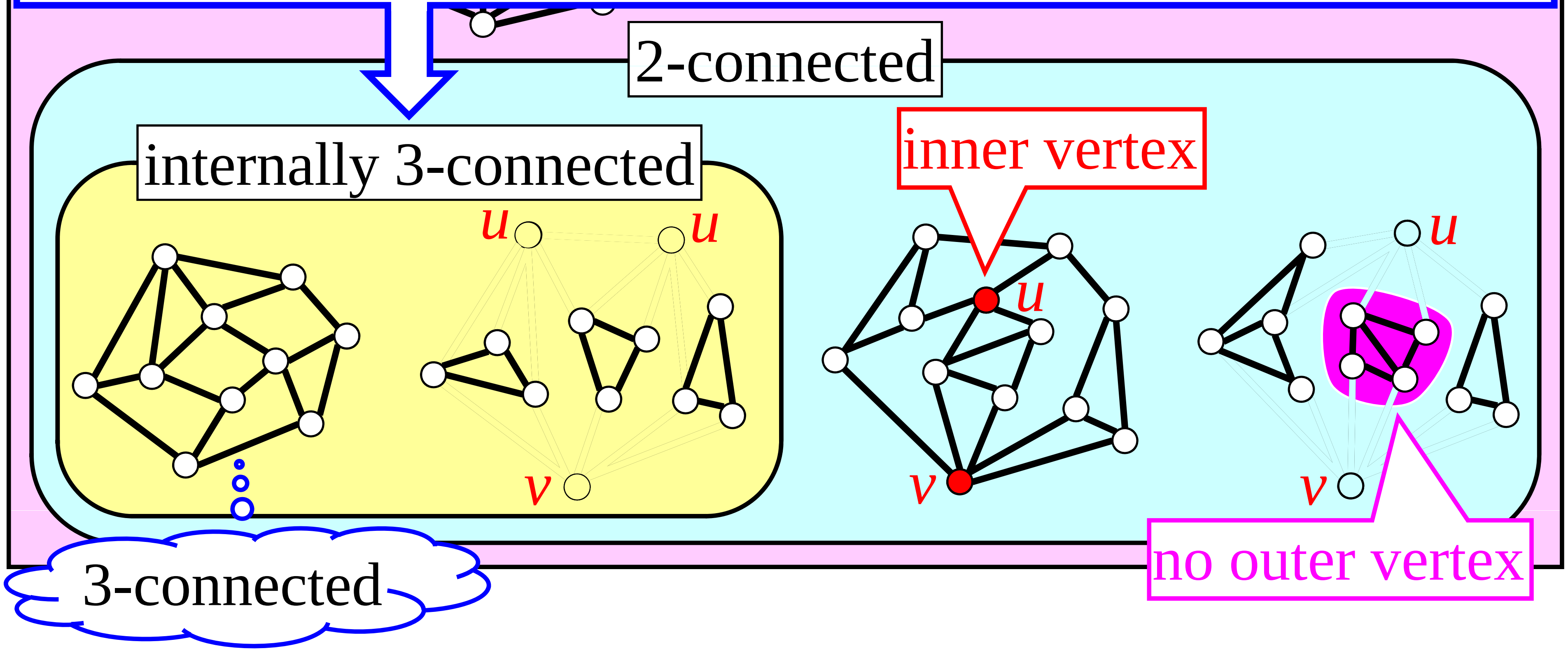


cannot be simultaneously drawn as convex polygons

Known results (1)

for any separation pair { u , v } of G ,

1) both u and v are outer vertices, and
2) each component of G – { u , v } contains an outer vertex



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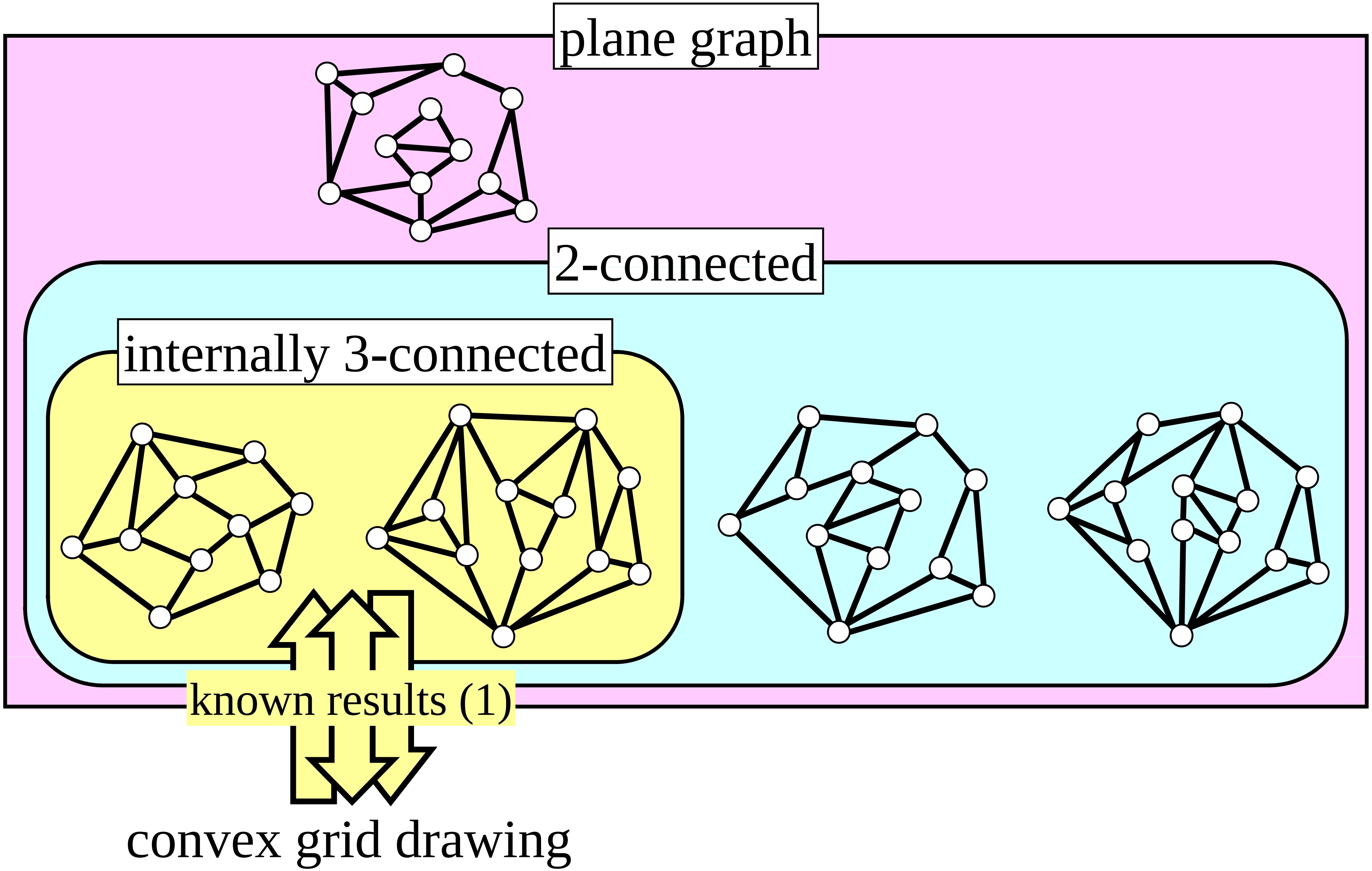
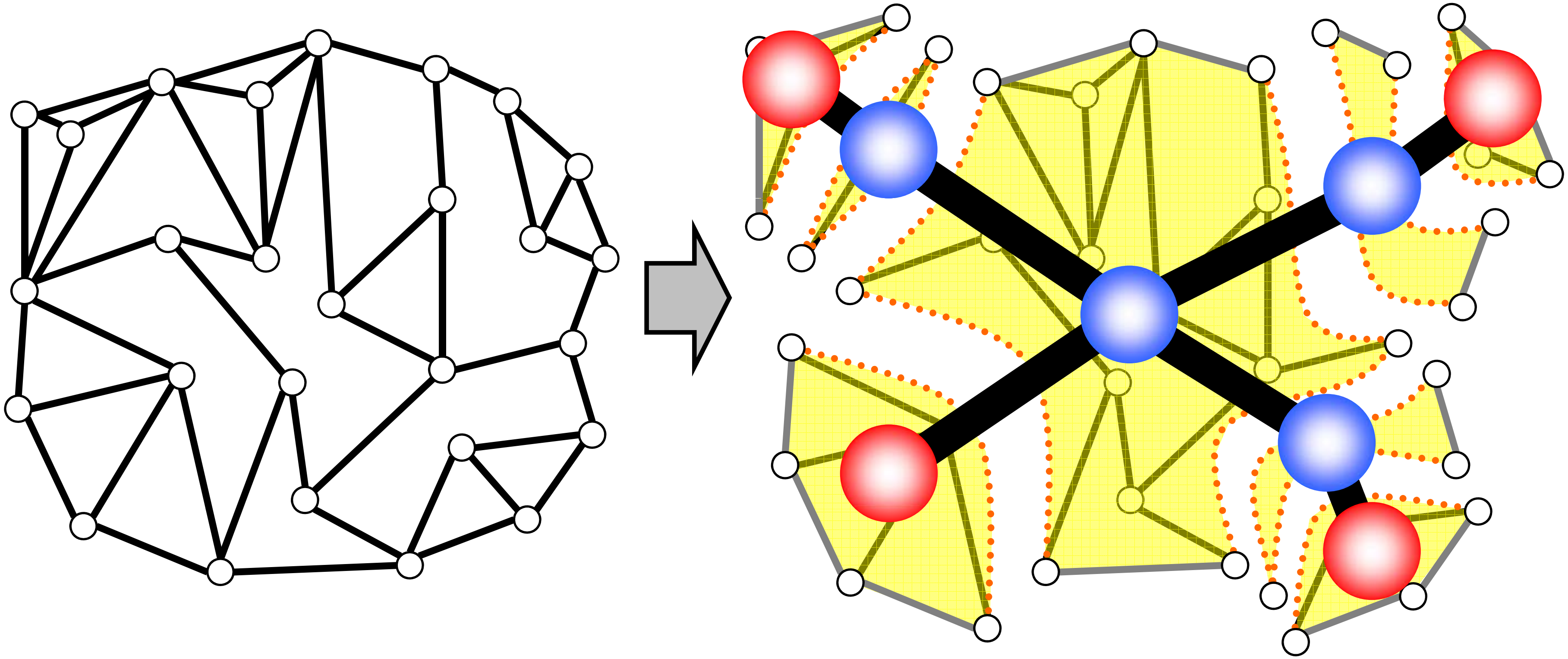
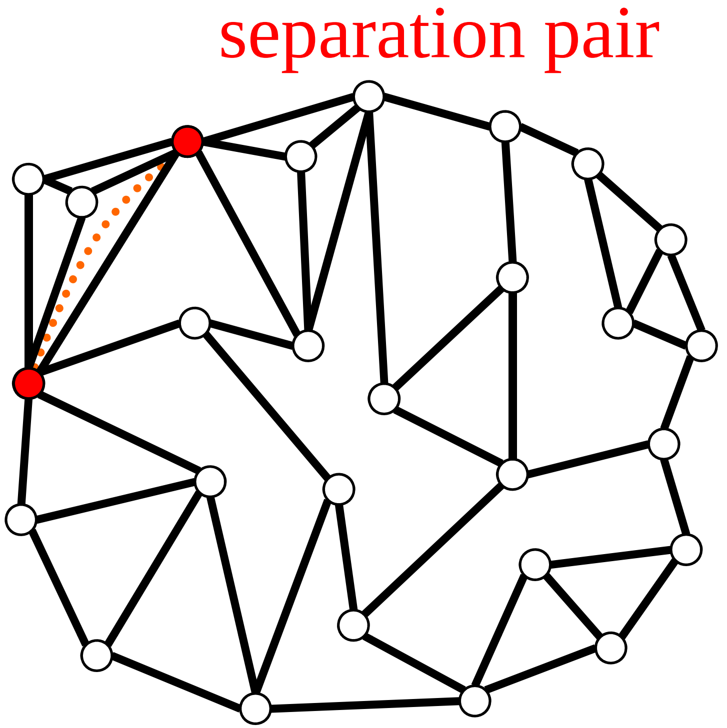


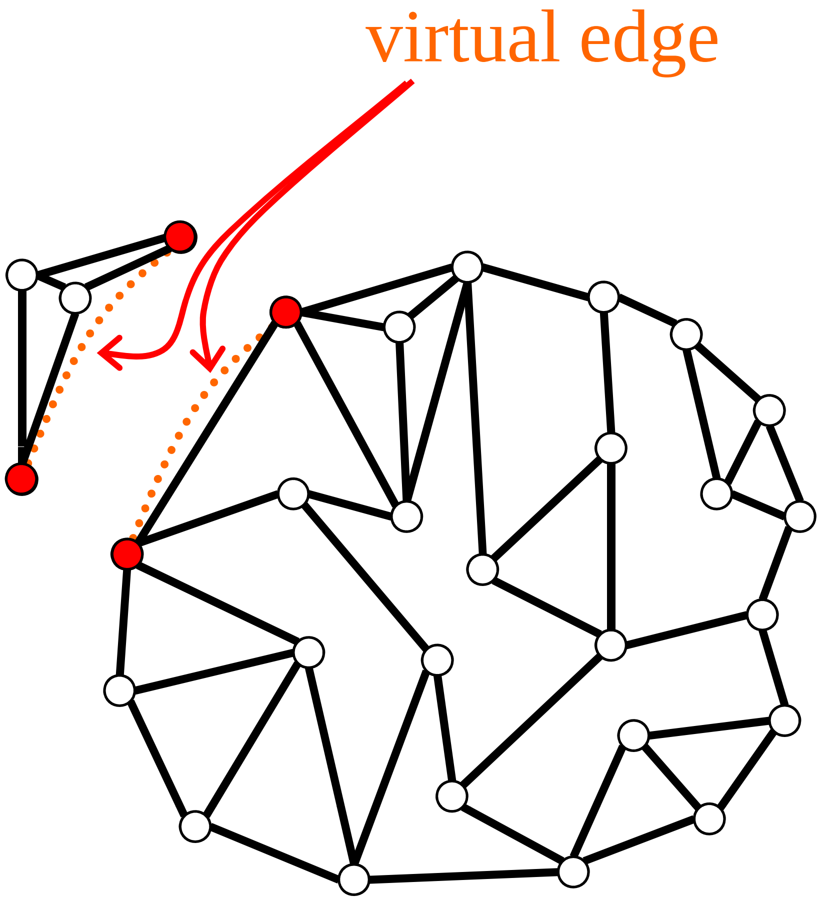
Decomposition tree



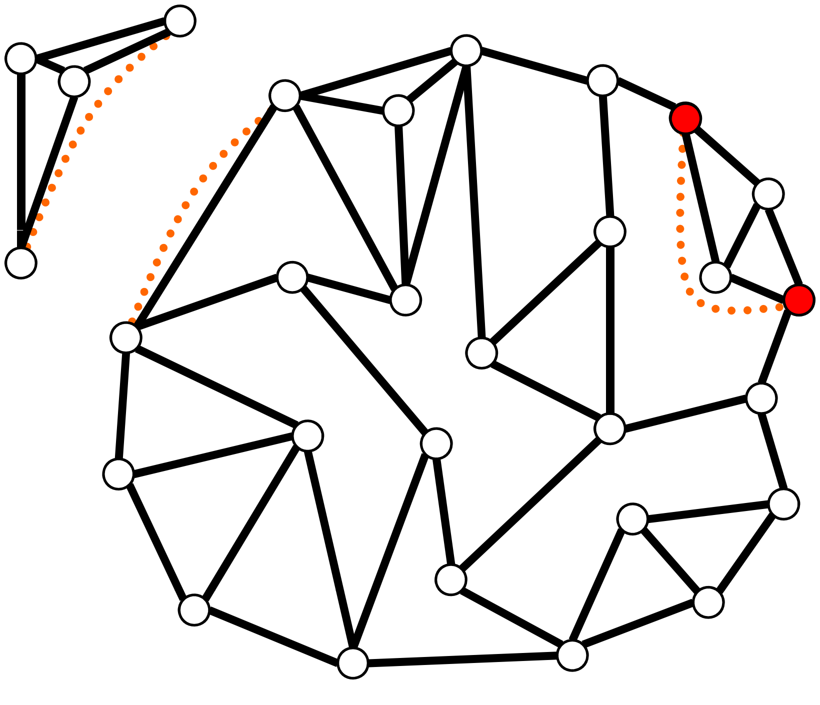
Decomposition tree



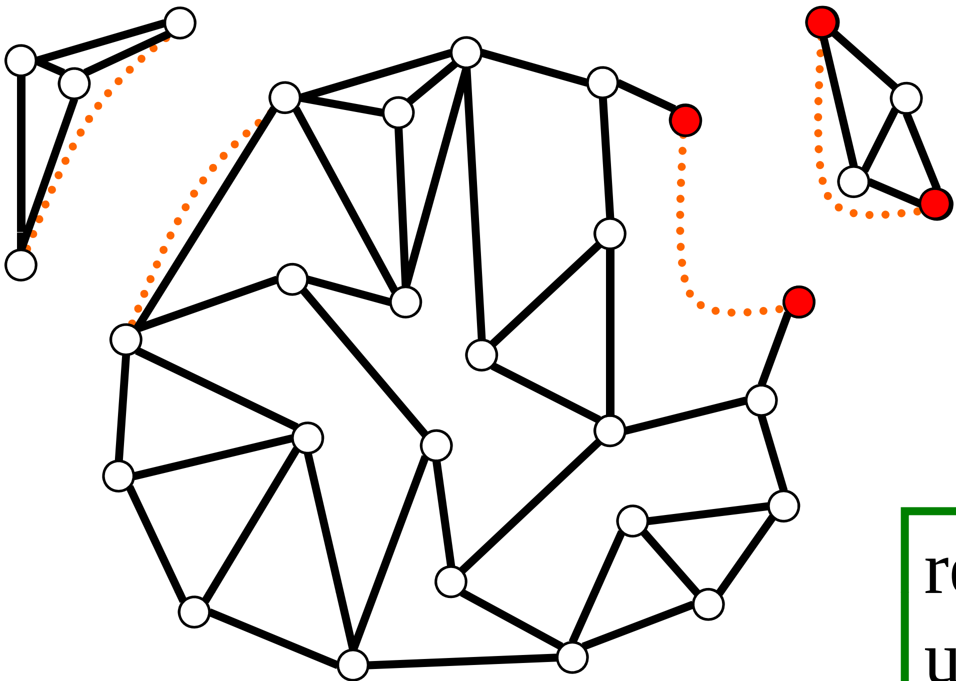
Decomposition tree



Decomposition tree

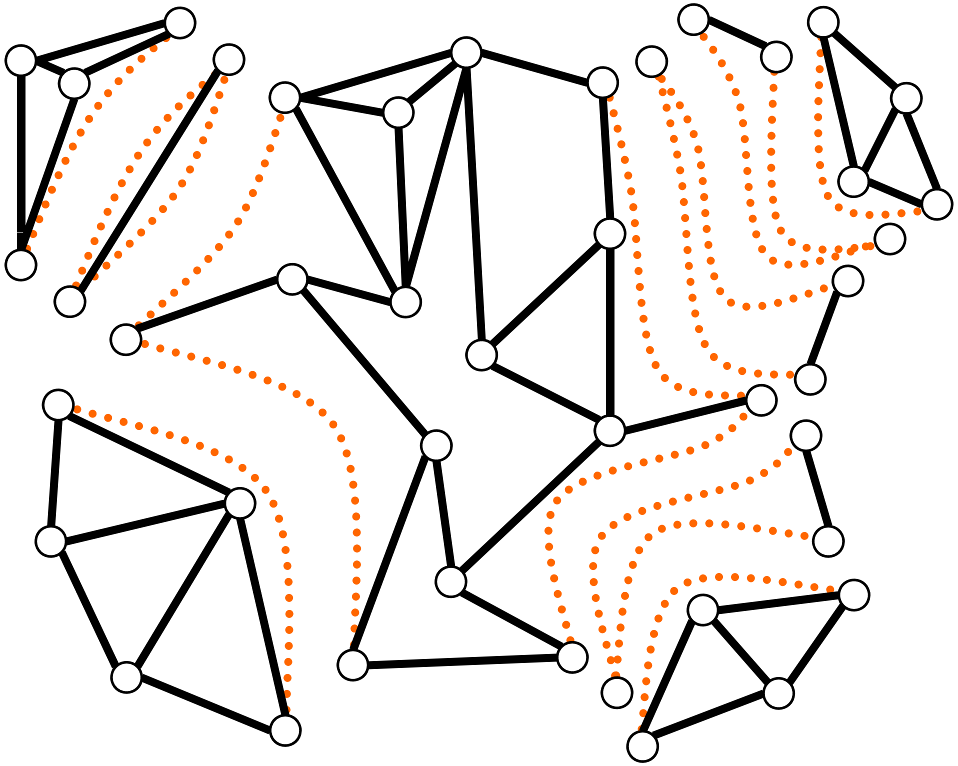


Decomposition tree



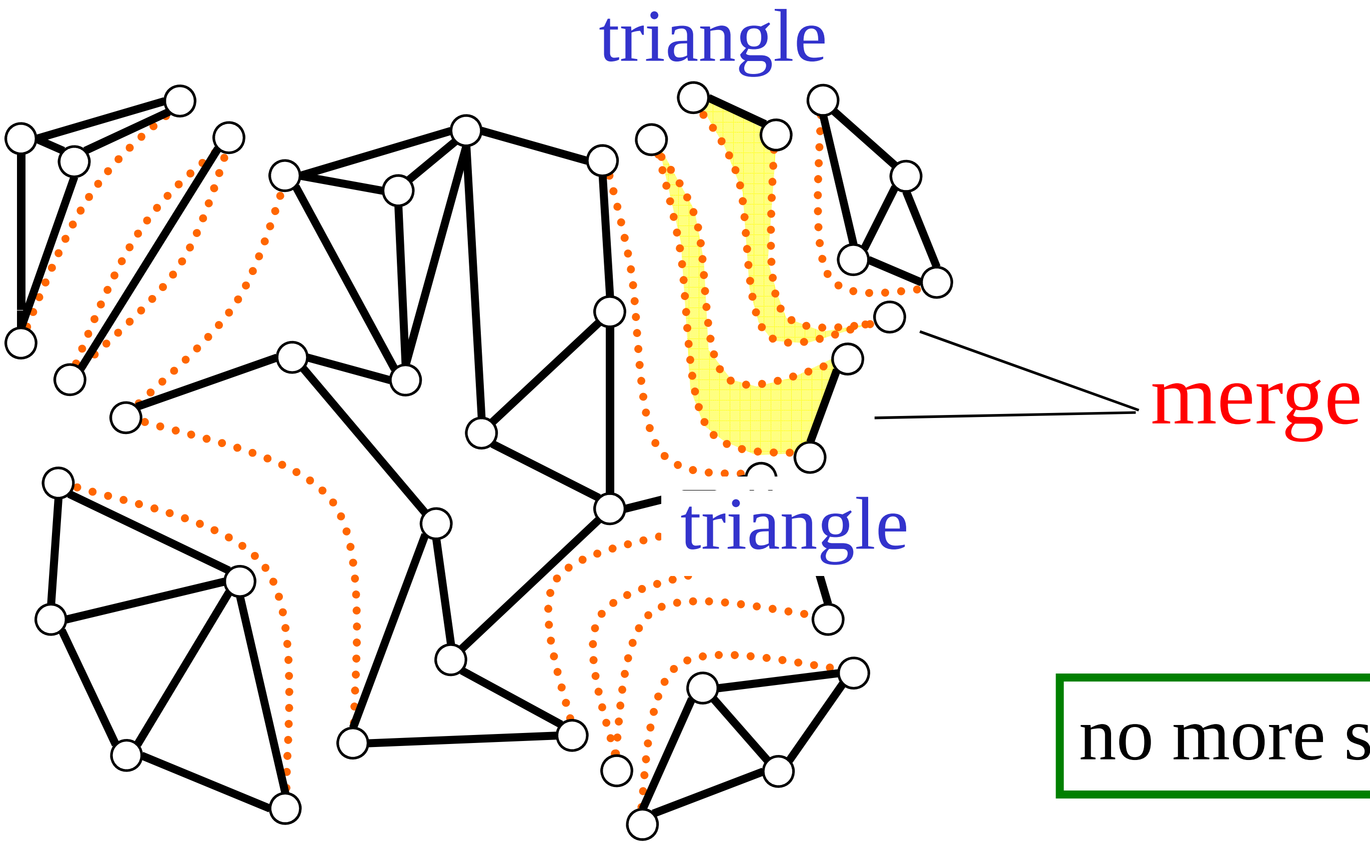
repeat this operation
until no more splits are possible

Decomposition tree



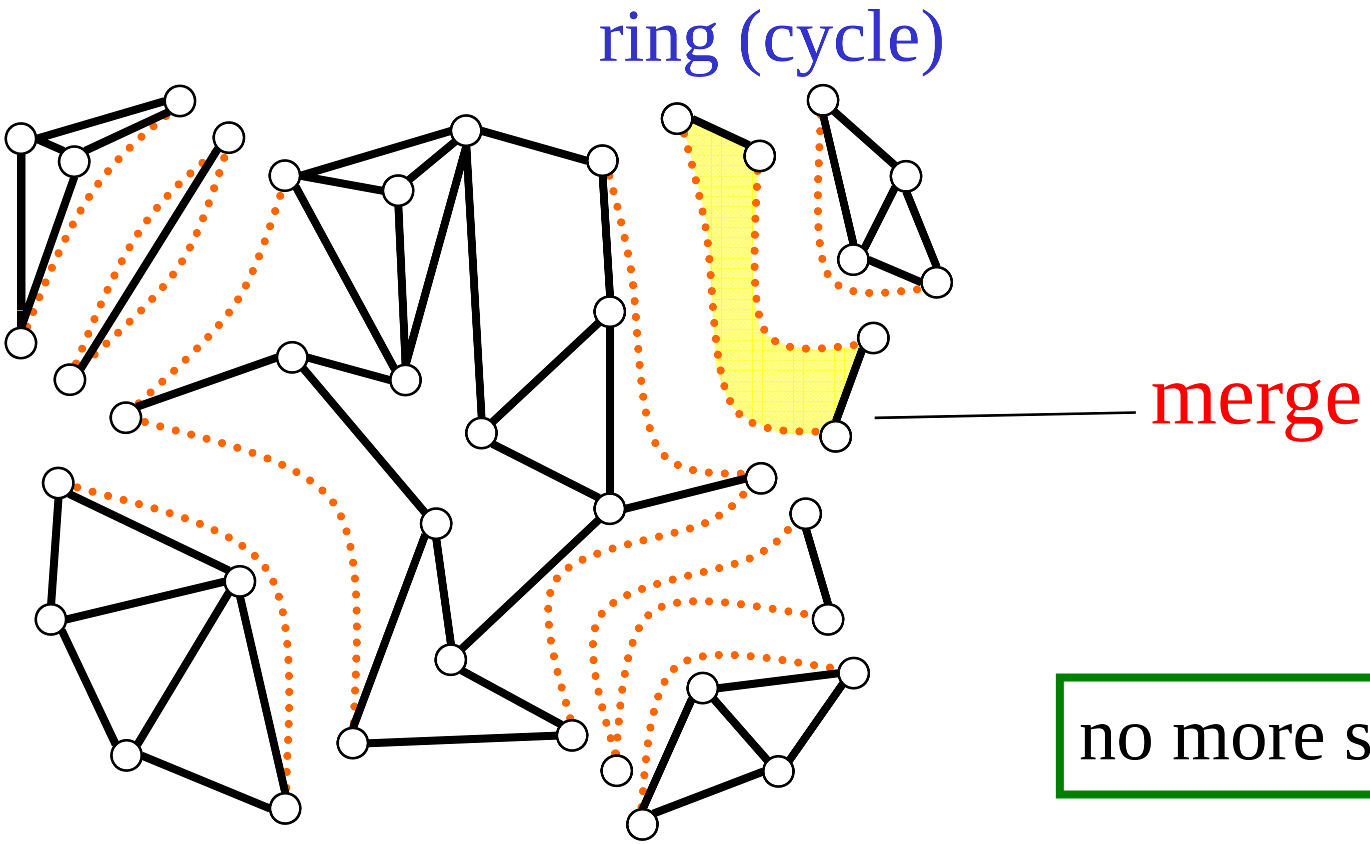
no more splits are possible

Decomposition tree



no more splits are possible

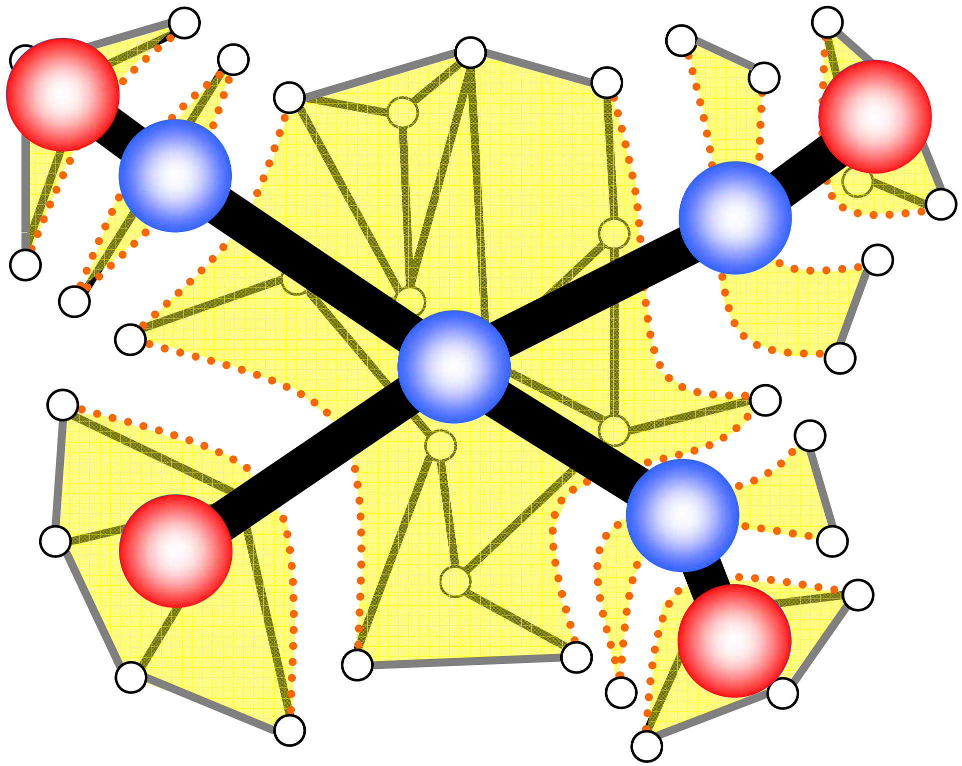
Decomposition tree



no more splits are possible

Decomposition tree

3-connected component decomposition tree [HT73]

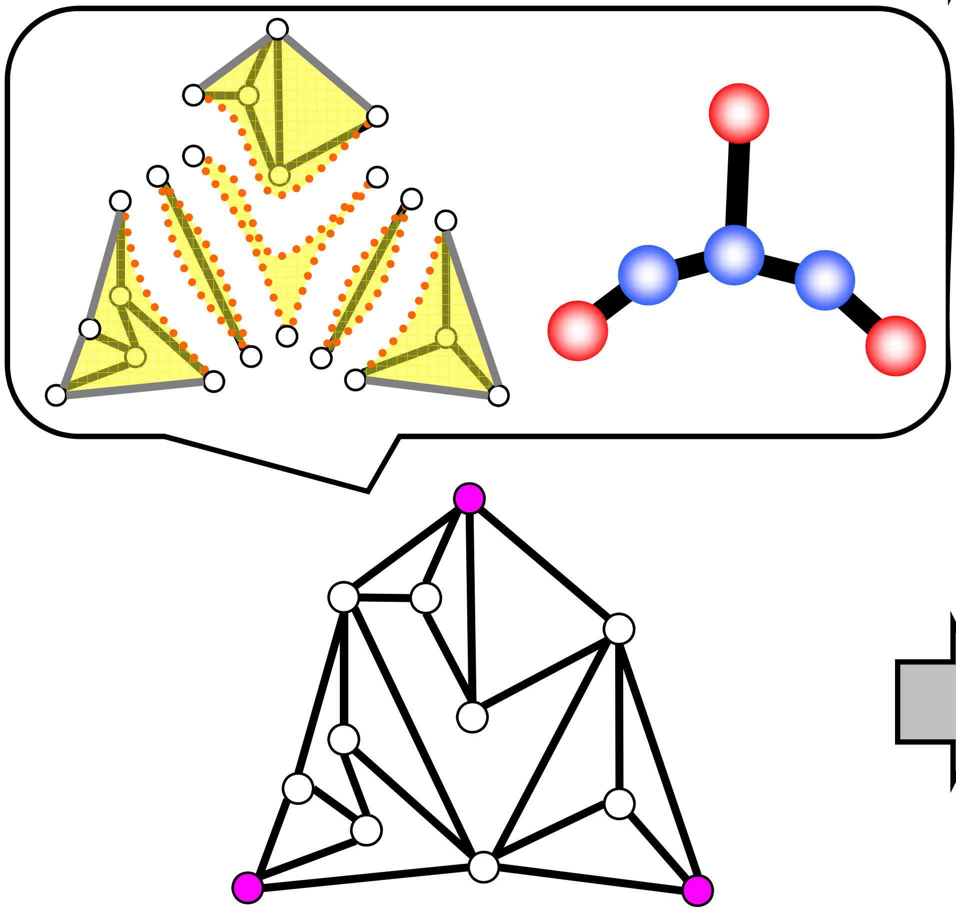


Known results (2)

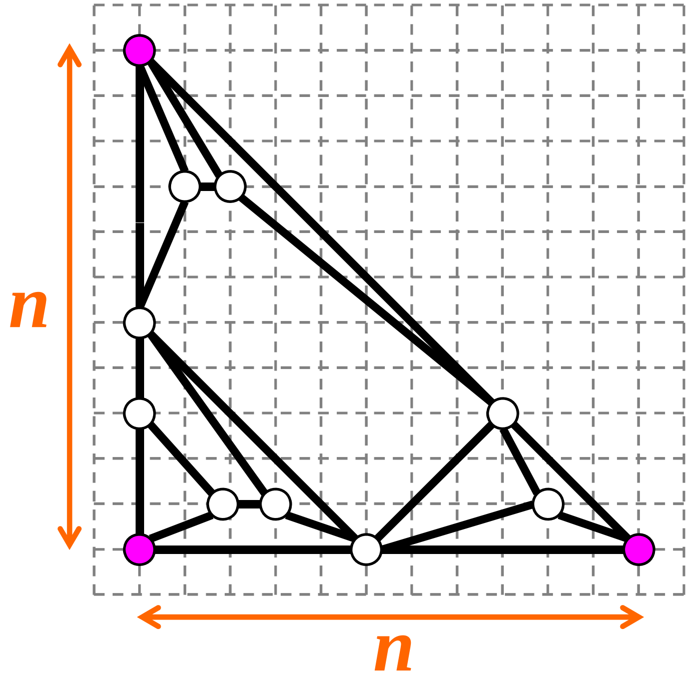
leaves ≤ 3

convex grid drawing
[CK97], [MAN05]

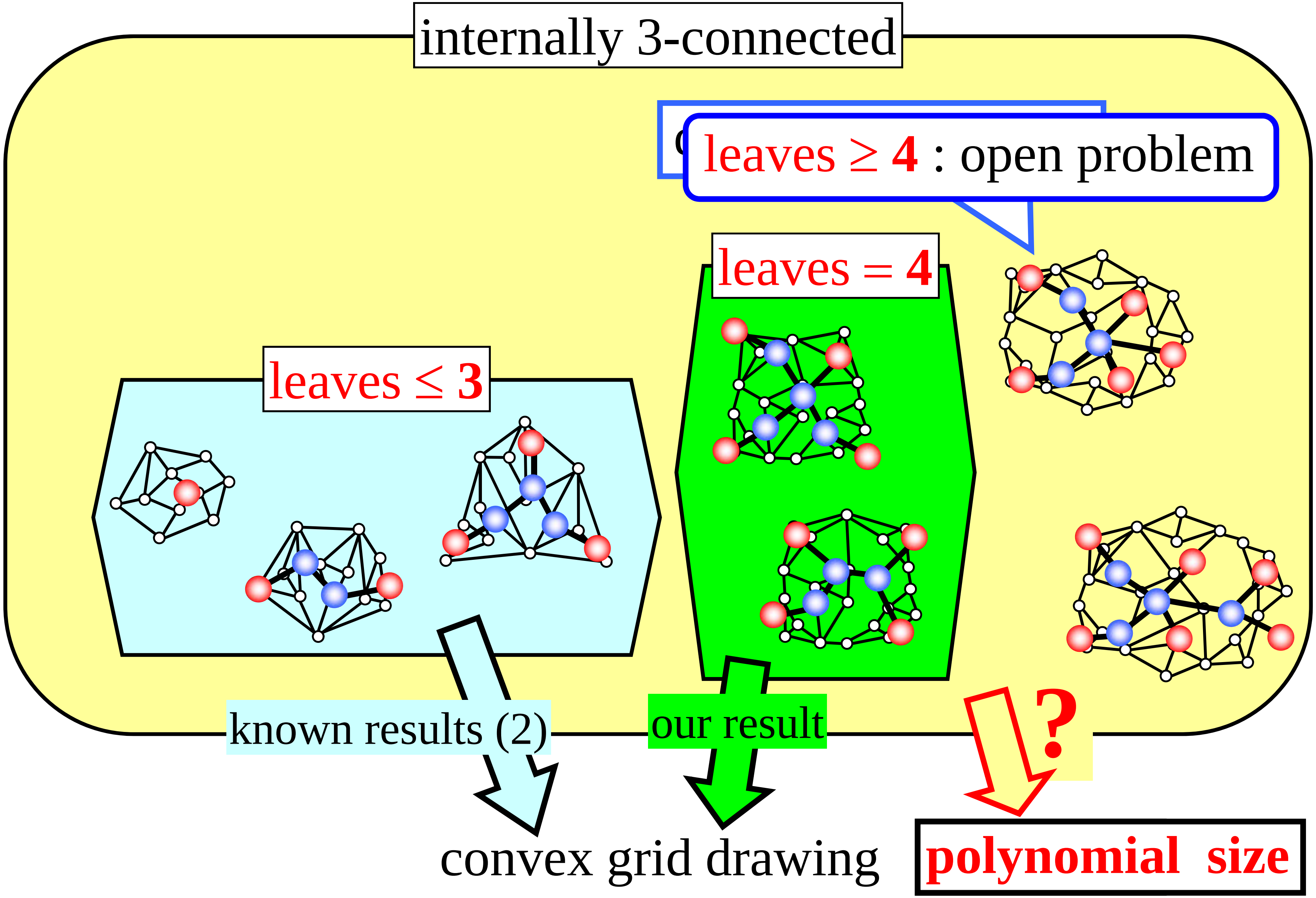
triangular contour



n vertices

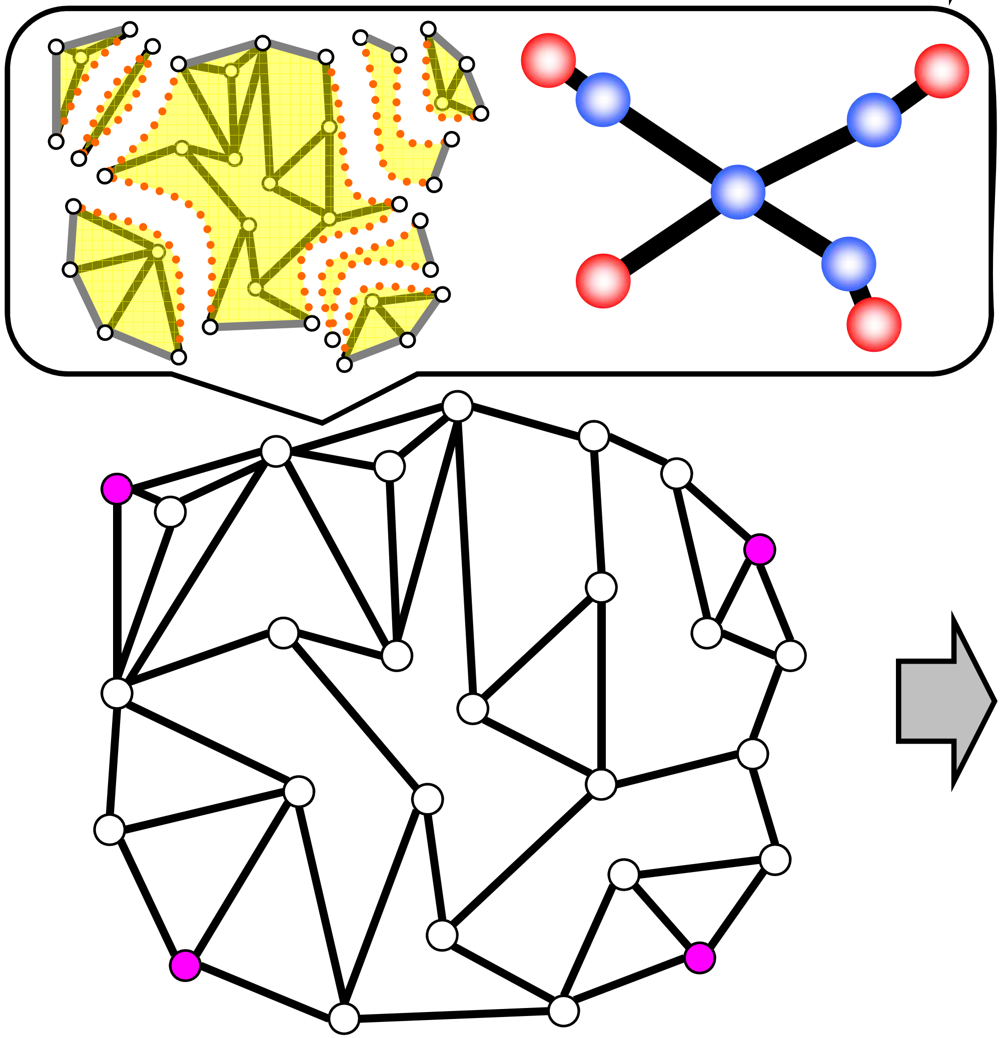


Known results (2)



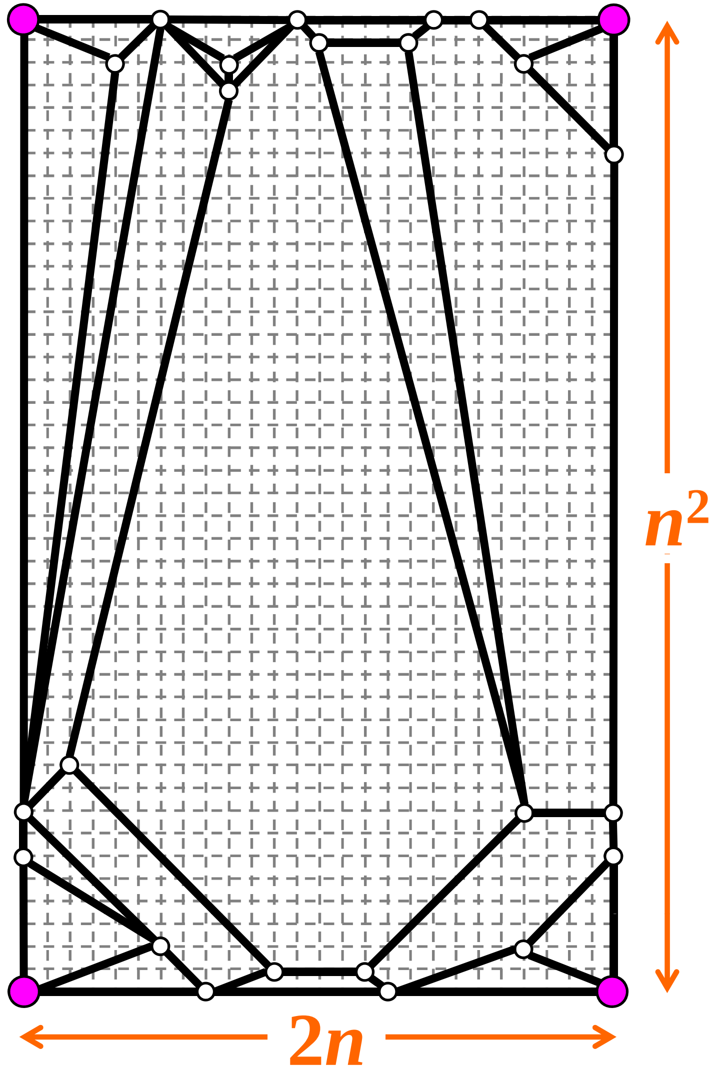
Our result

leaves = 4



rectangular contour

convex grid drawing



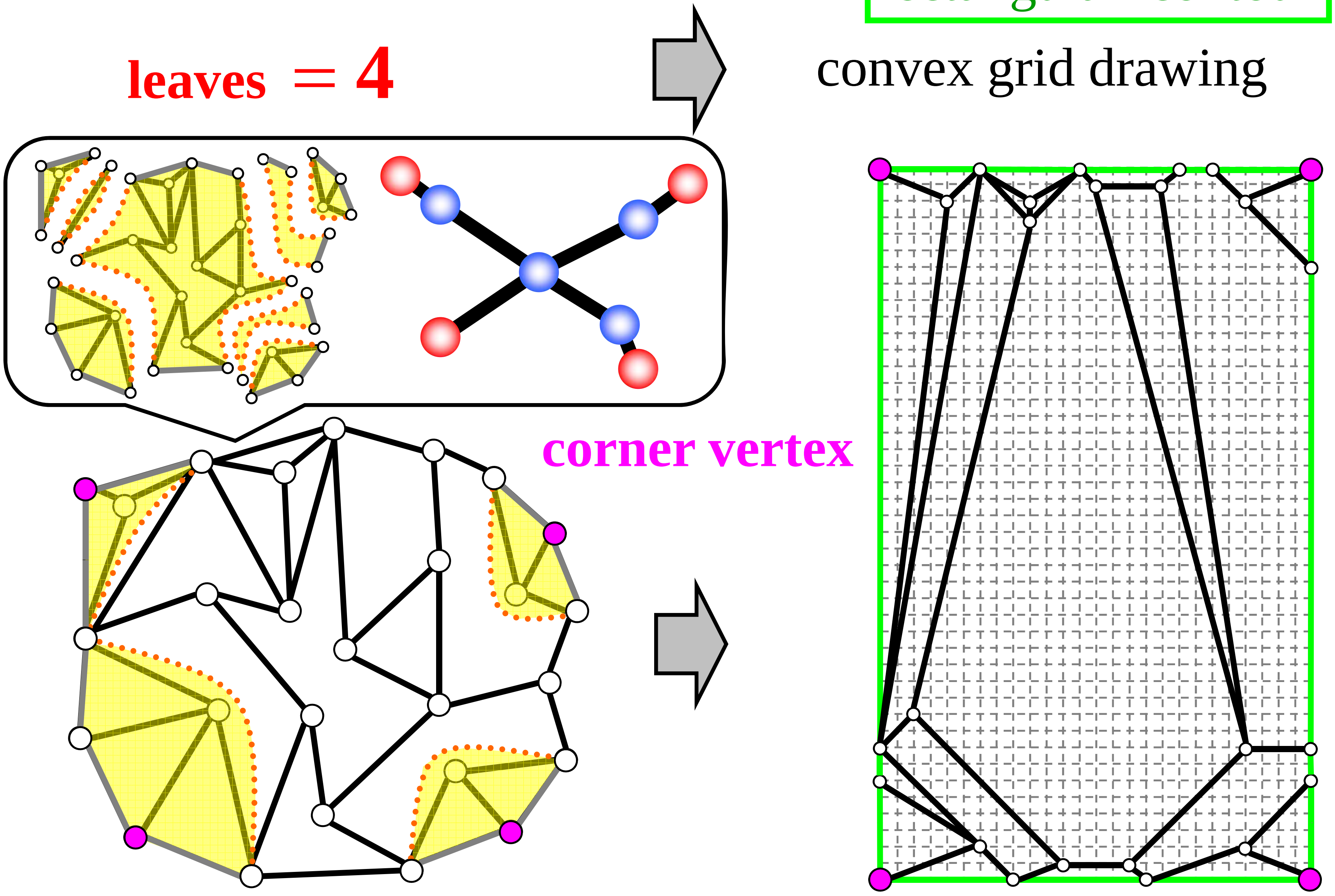
n vertices

Our result

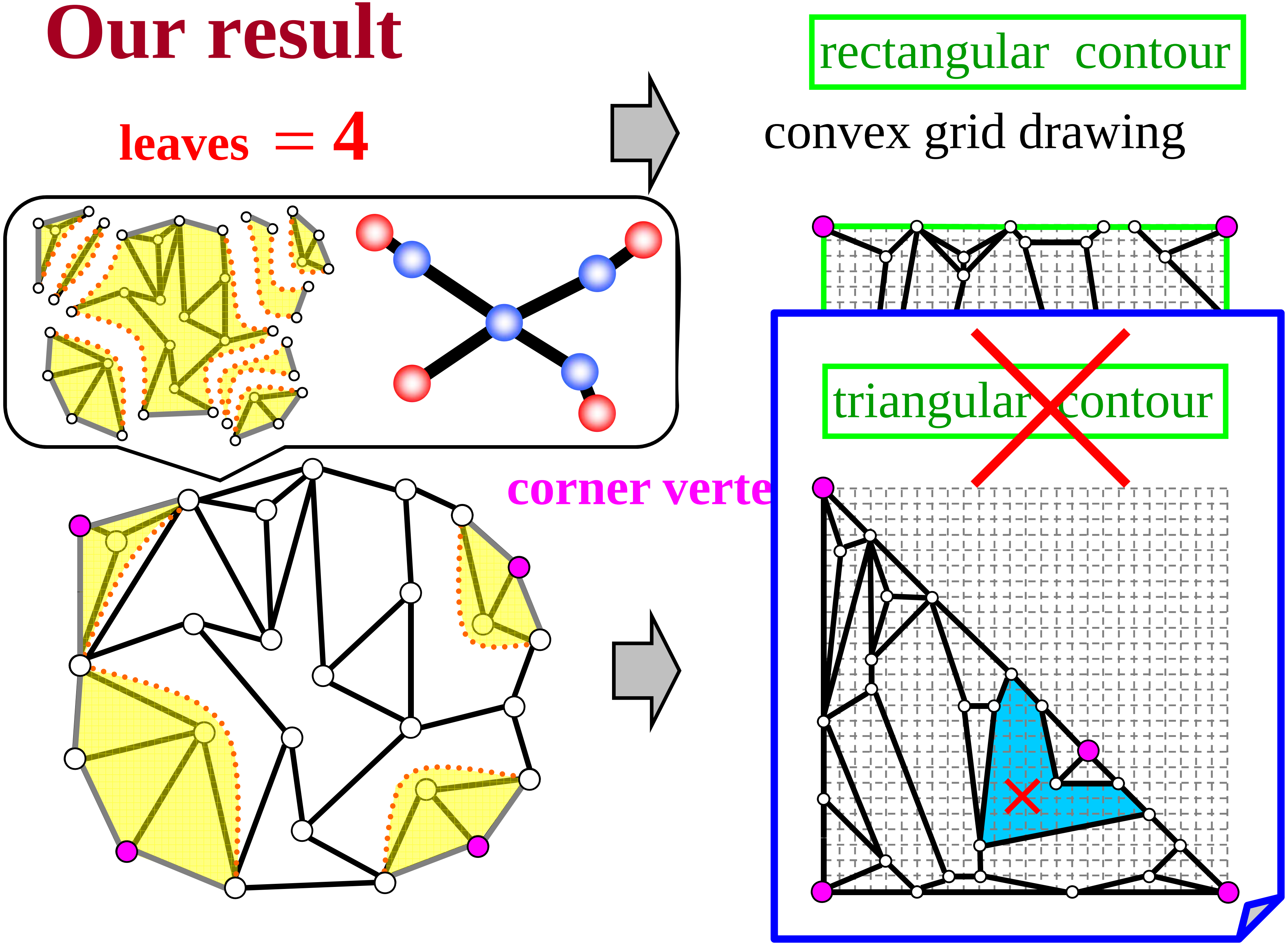
leaves = 4

rectangular contour

convex grid drawing



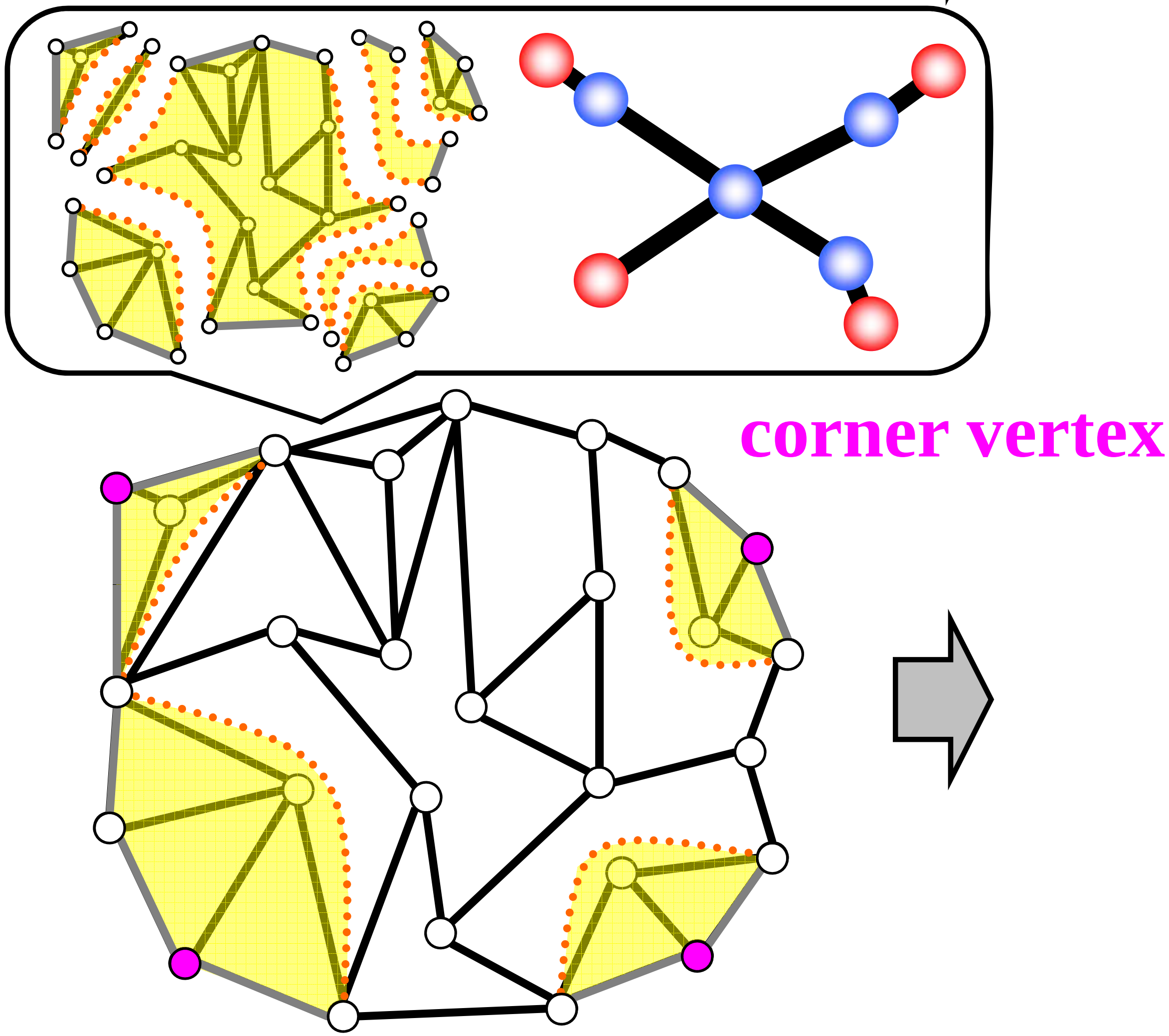
Our result

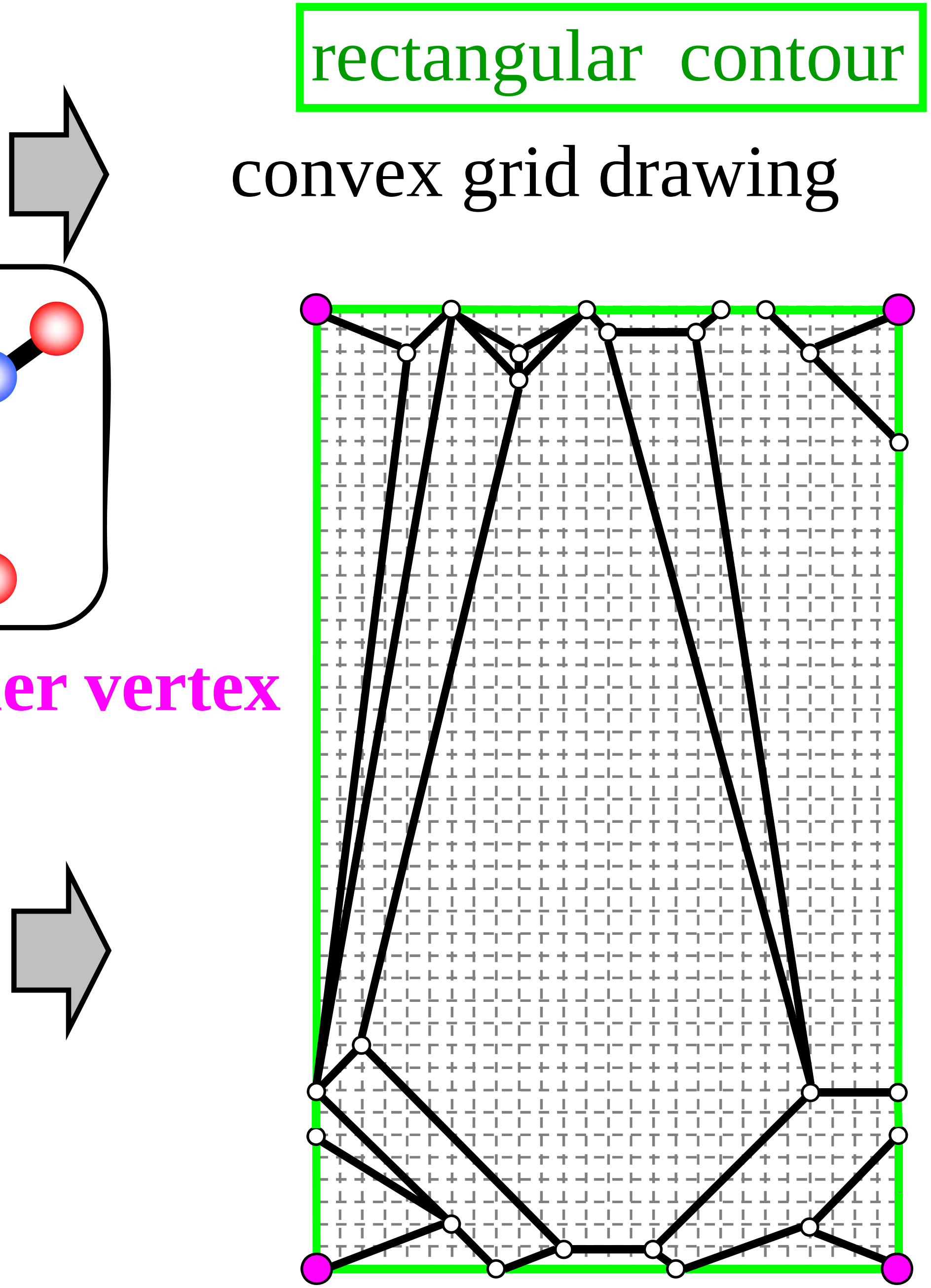


leaves = 4
rectangular contour
convex grid drawing
triangular contour
corner verte

Our result

leaves = 4





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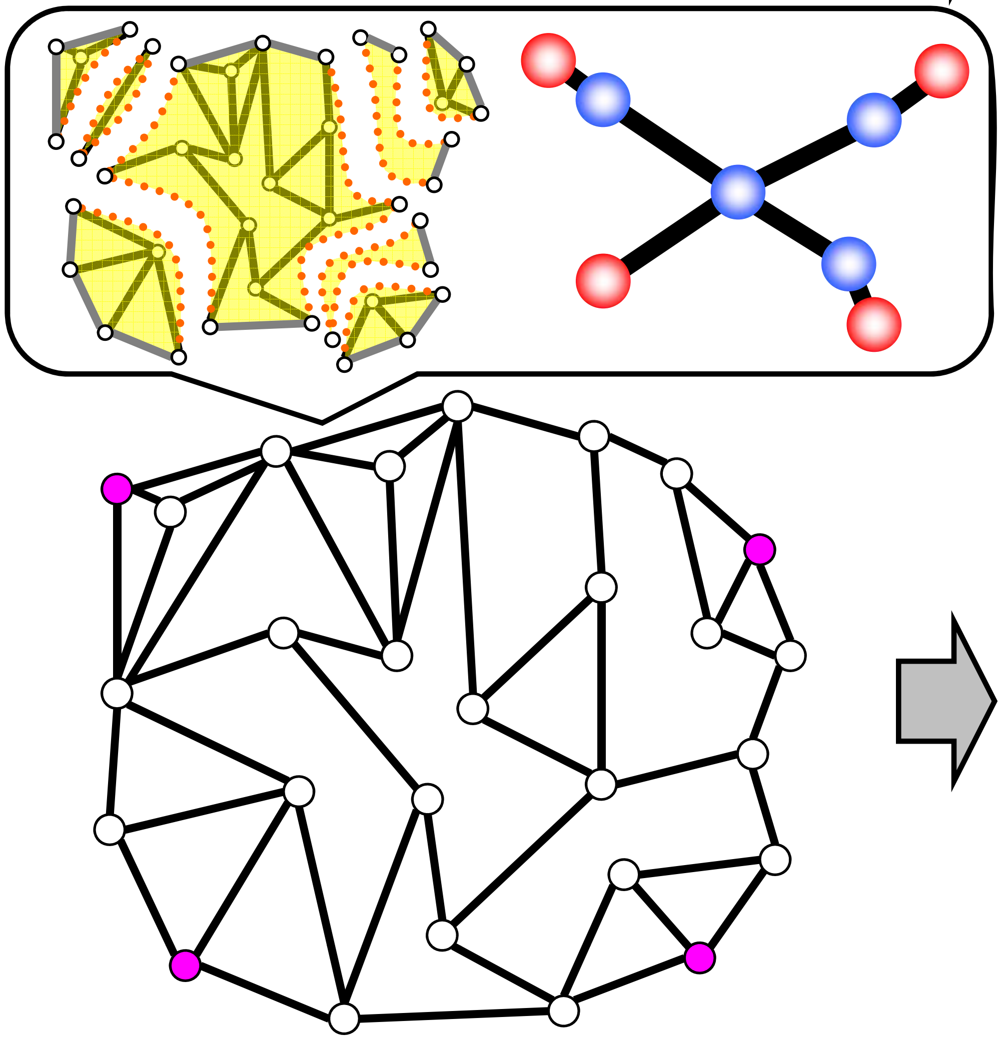
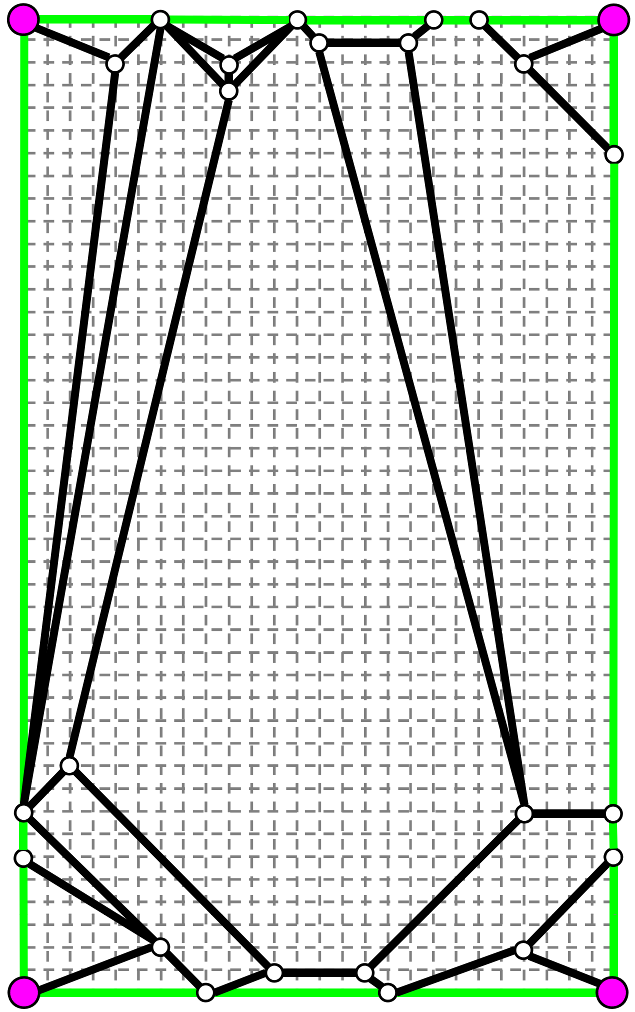
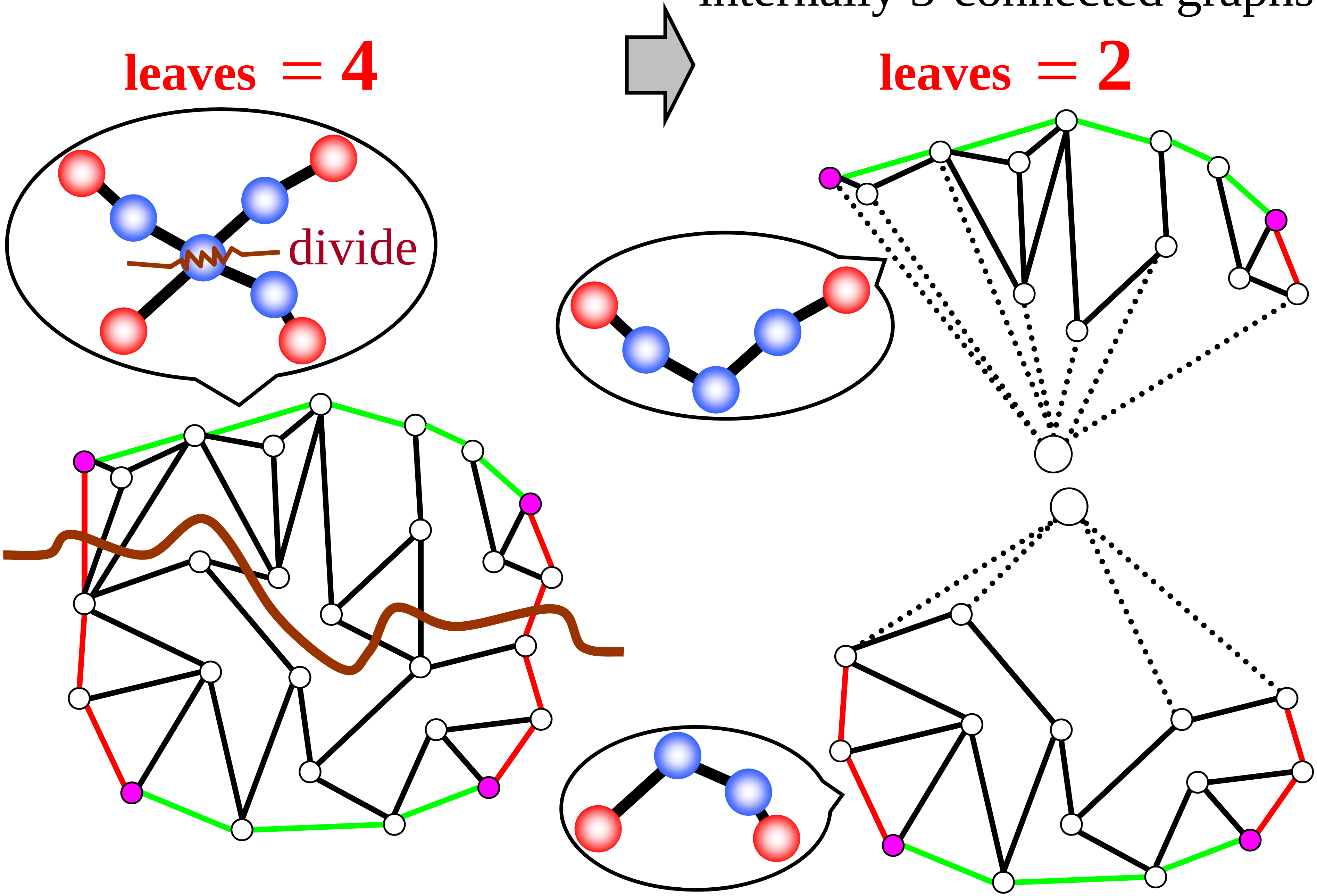


convex grid drawing



Main idea

internally 3-connected graphs

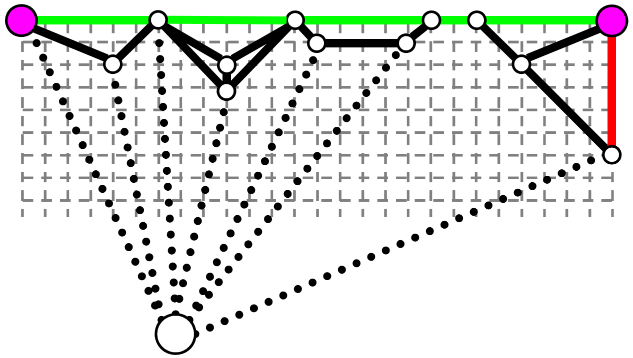


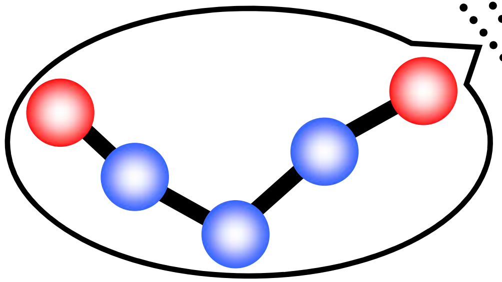
Main idea

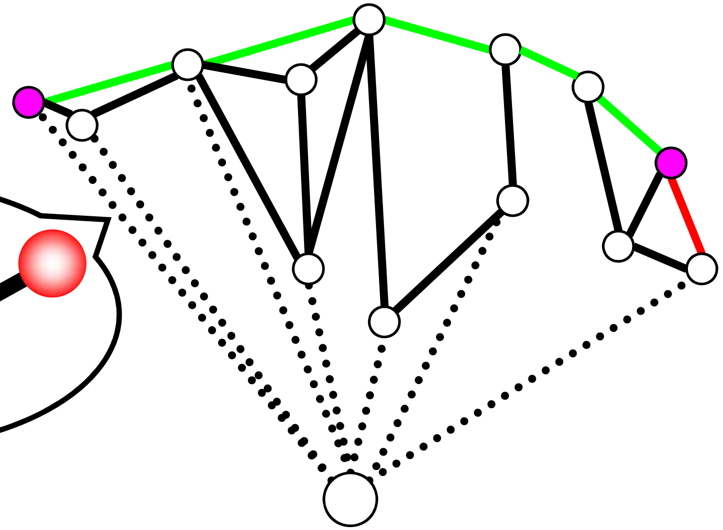
internally 3-connected graphs

inner convex grid drawings

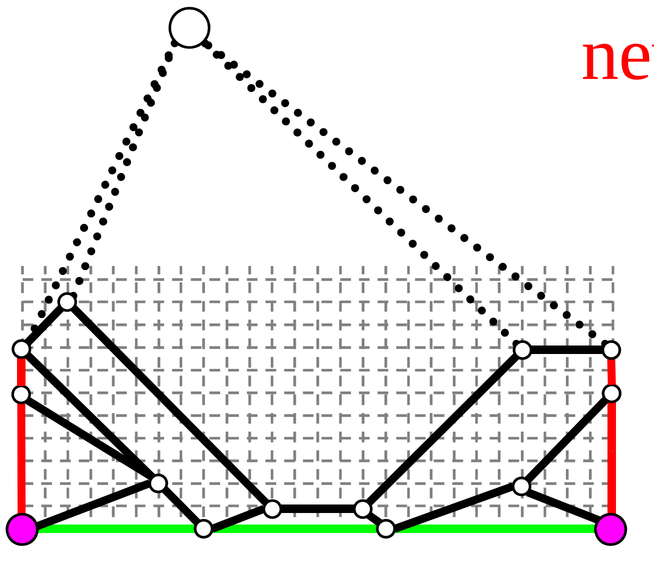
leaves = 2

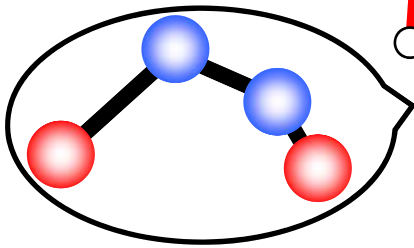


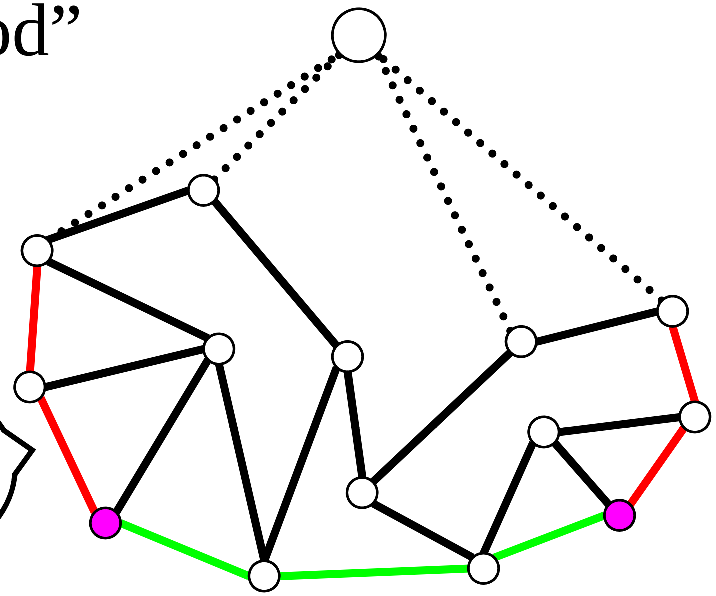




new “shift method”

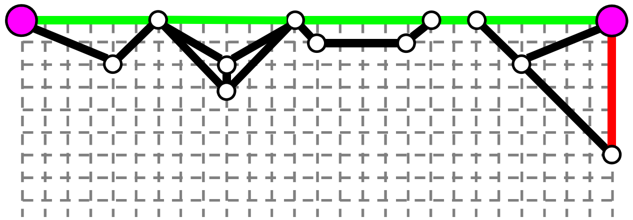


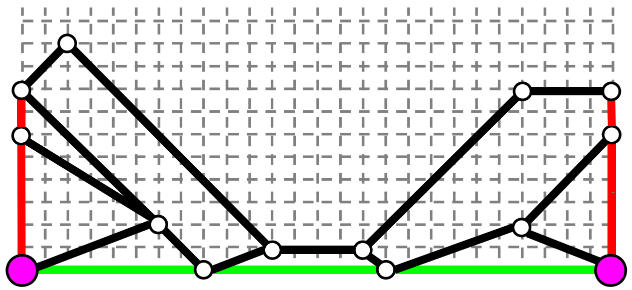


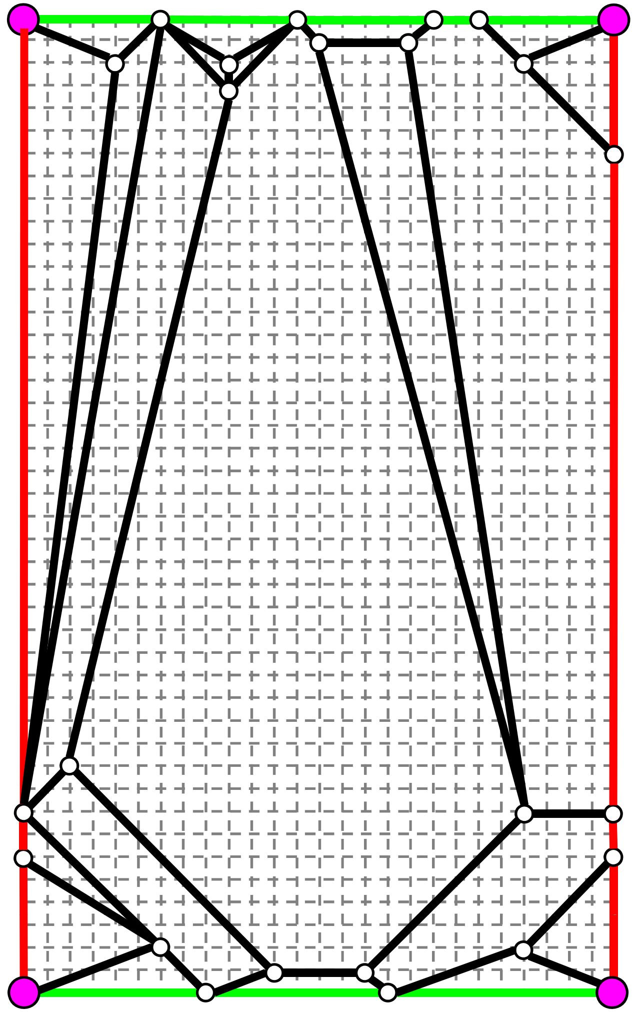


Main idea

inner convex grid drawings → convex grid drawing

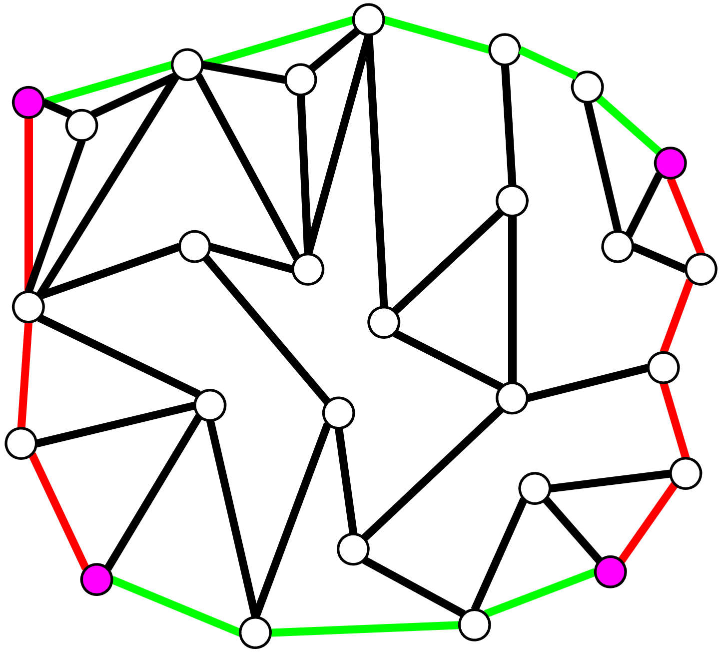






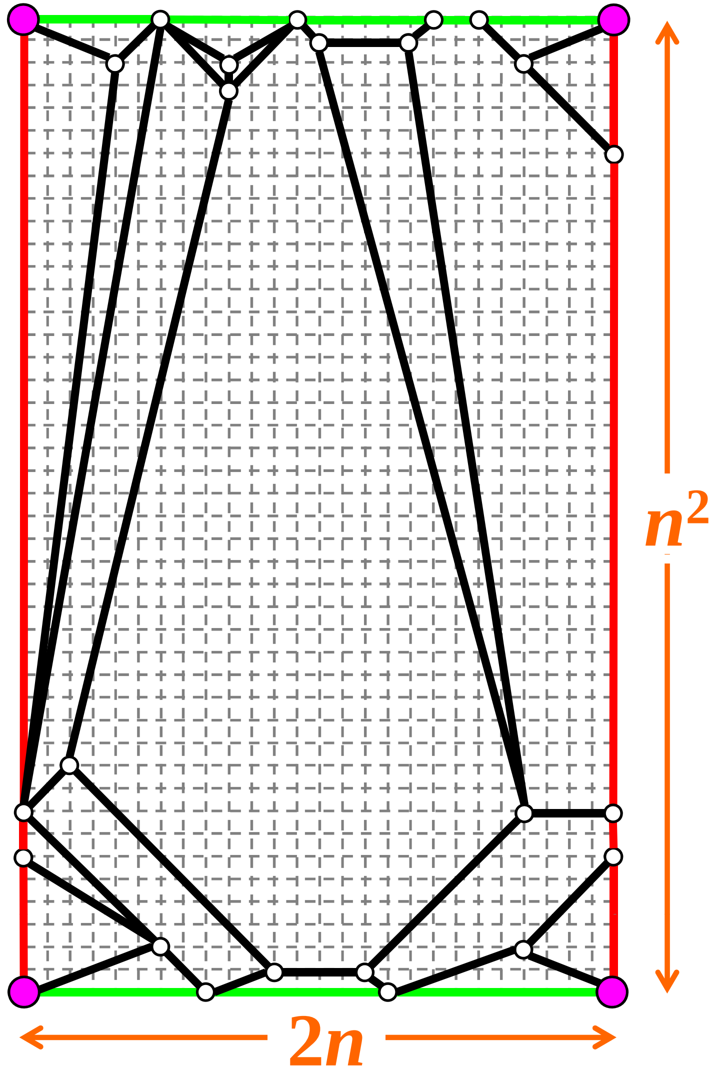
Main idea

input graph



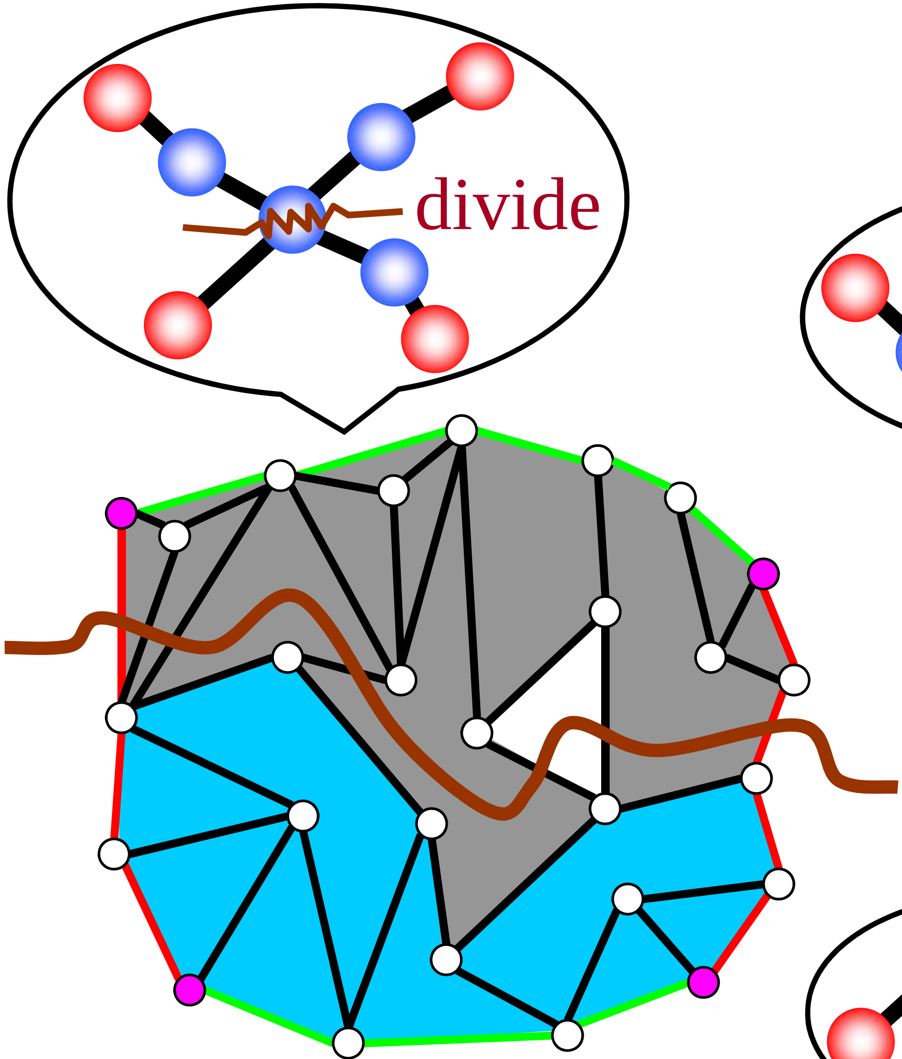
n vertices

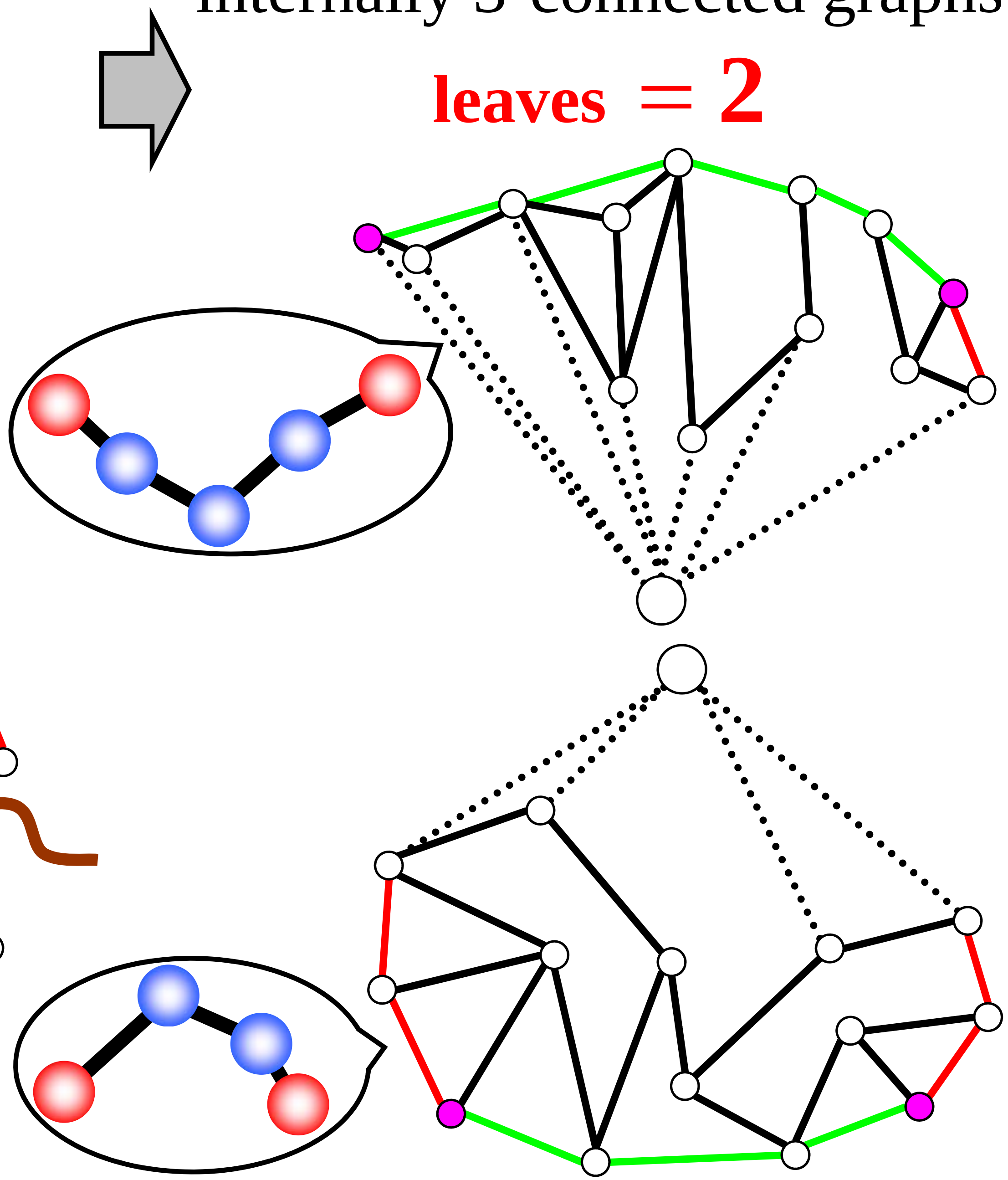
convex grid drawing



Algorithm

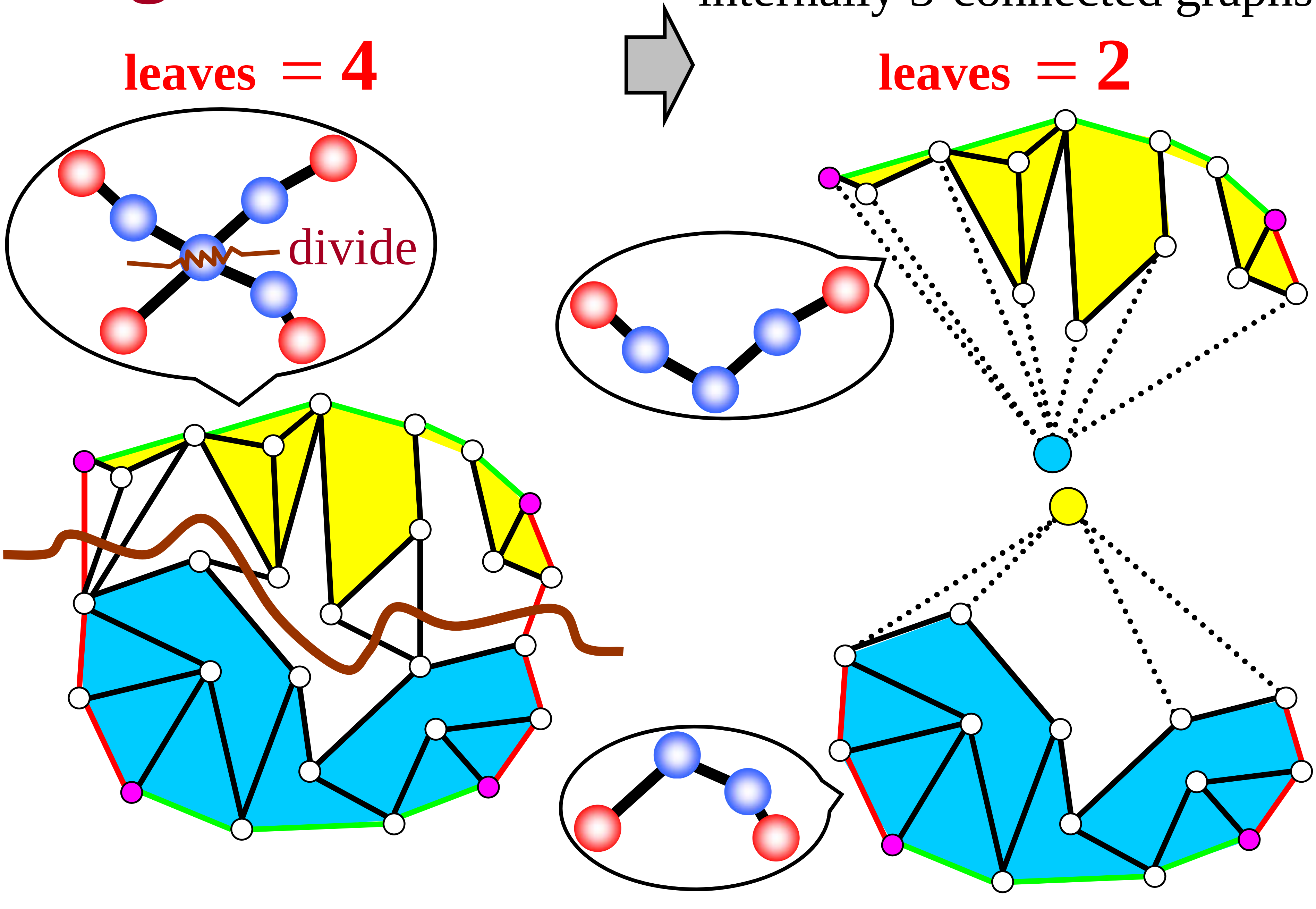
leaves = 4





Algorithm

internally 3-connected graphs

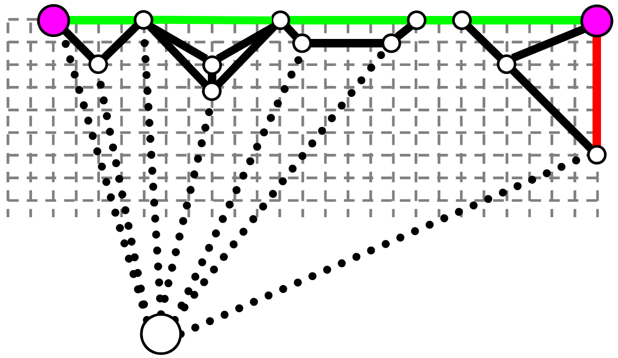


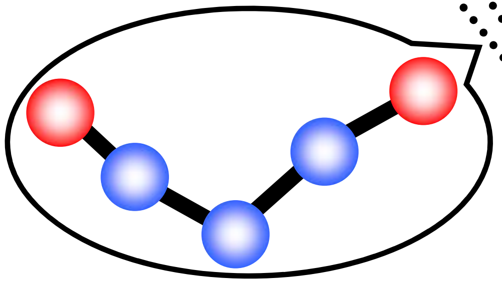
Algorithm

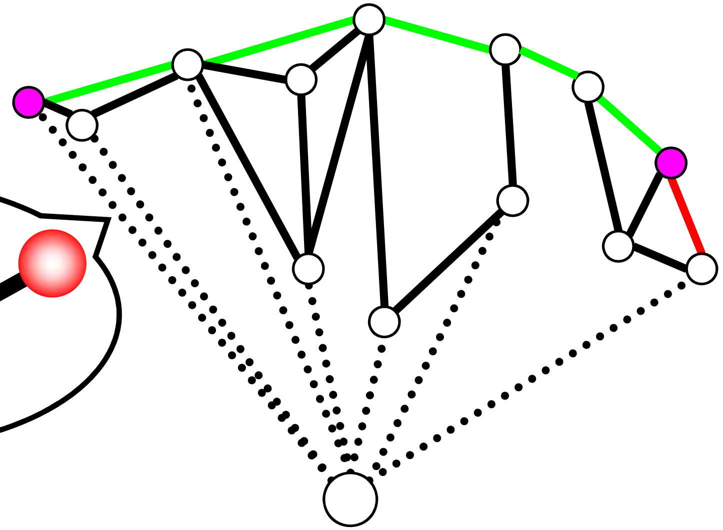
internally 3-connected graphs

inner convex grid drawings

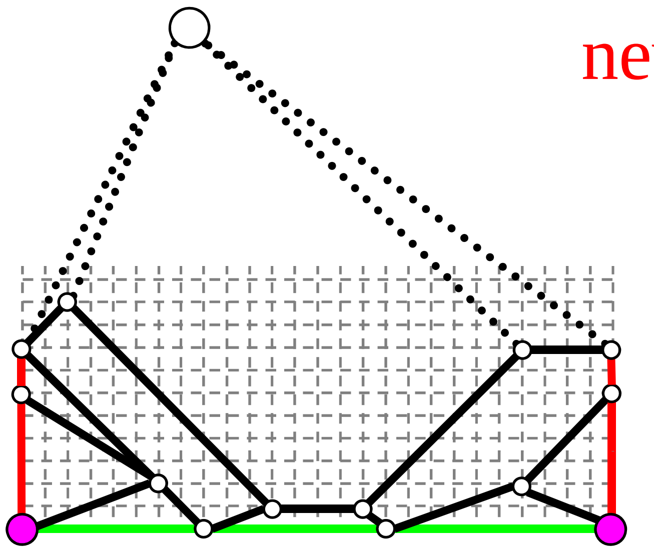
leaves = 2

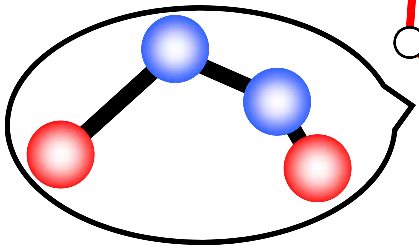


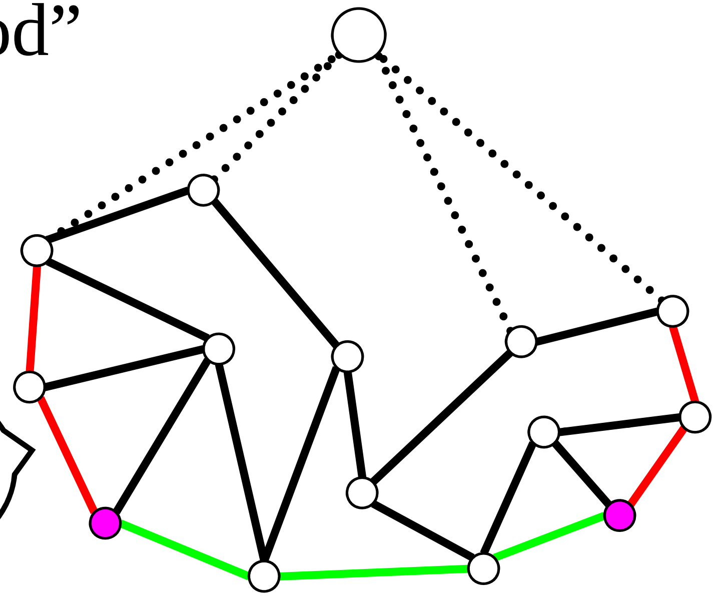




new "shift method"







Algorithm

internally 3-connected graph

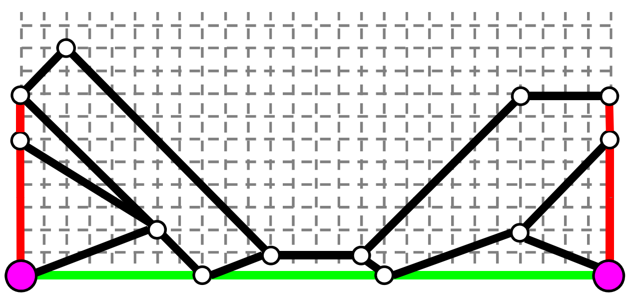
leaves $= \mathbf{2}$

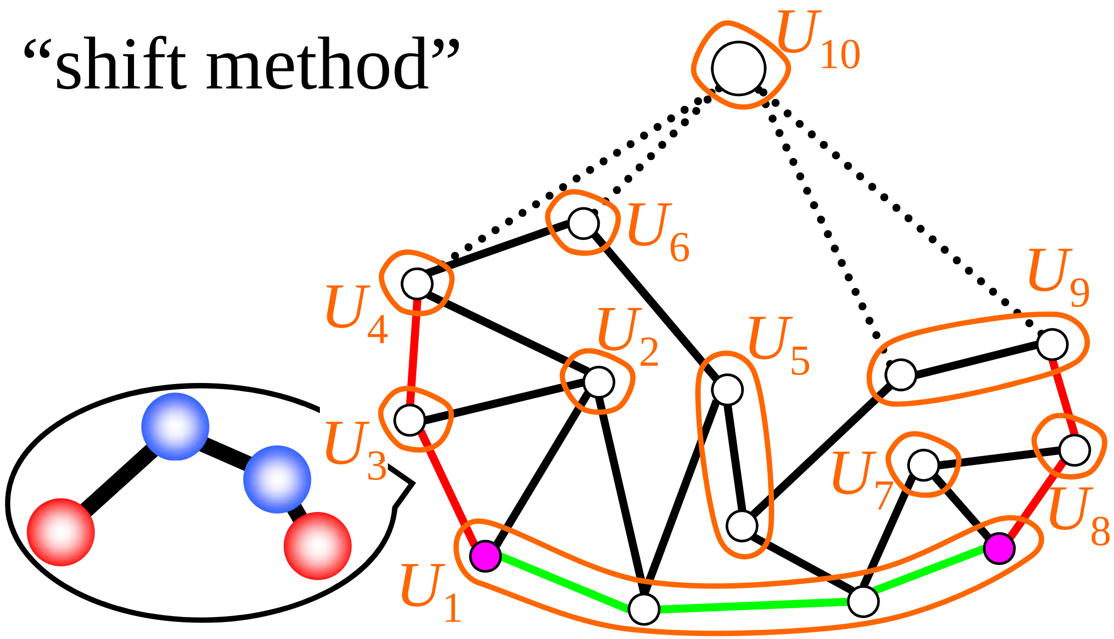
inner convex grid drawing

canonical decomposition

$$\Pi = (U_1 , U_2 , \cdots , U_{m-1} , U_m)$$

new “shift method”

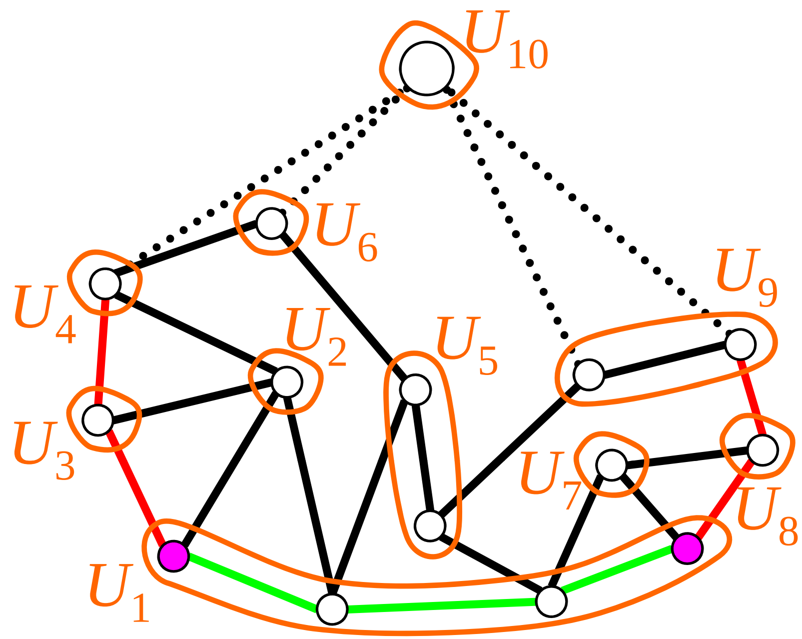




Canonical decomposition

canonical decomposition

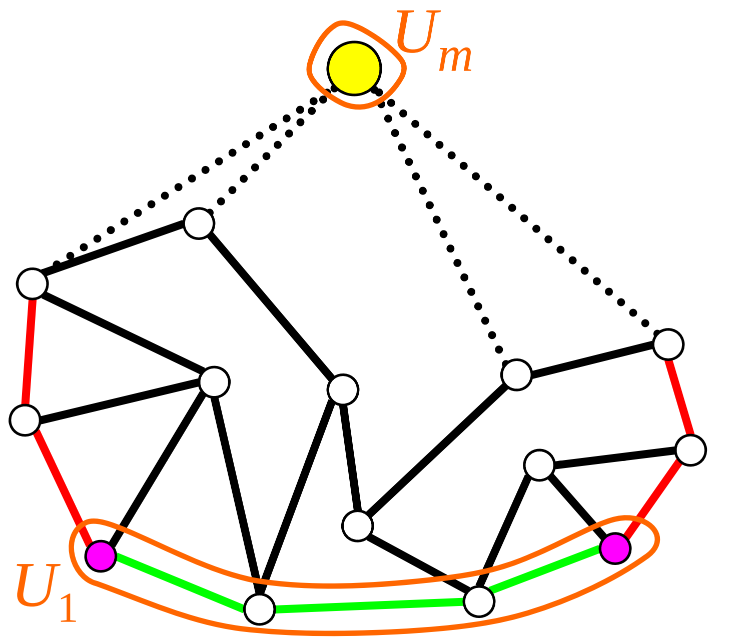
$$\Pi = (U_1 , U_2 , \cdots , U_{m-1} , U_m)$$



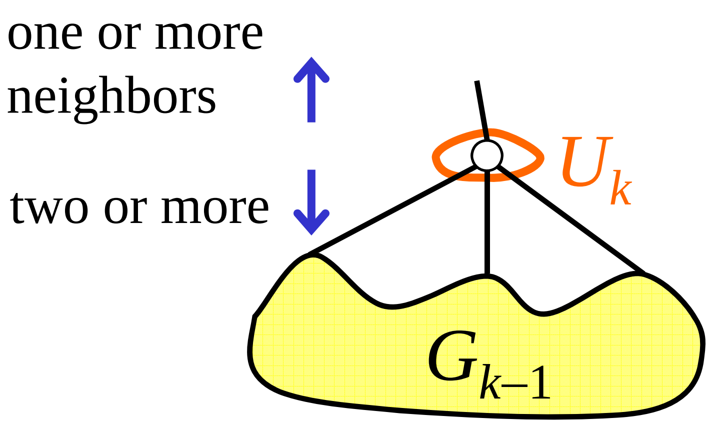
Canonical decomposition

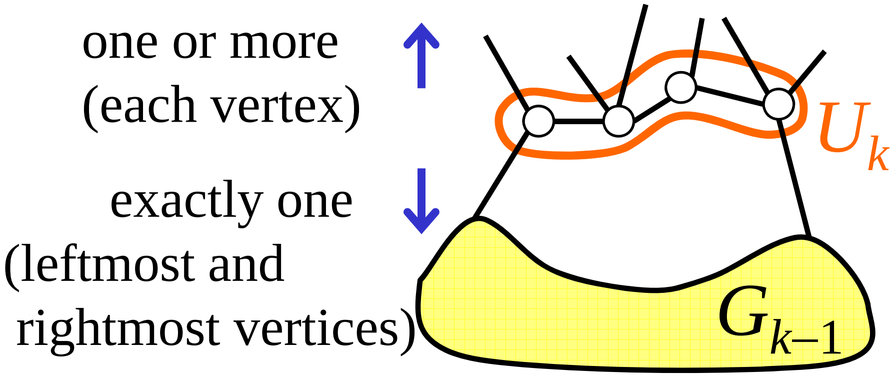
canonical decomposition

$$\Pi = (\boxed{U_1}, U_2, \cdots, U_{m-1}, \boxed{U_m})$$



Canonical decomposition

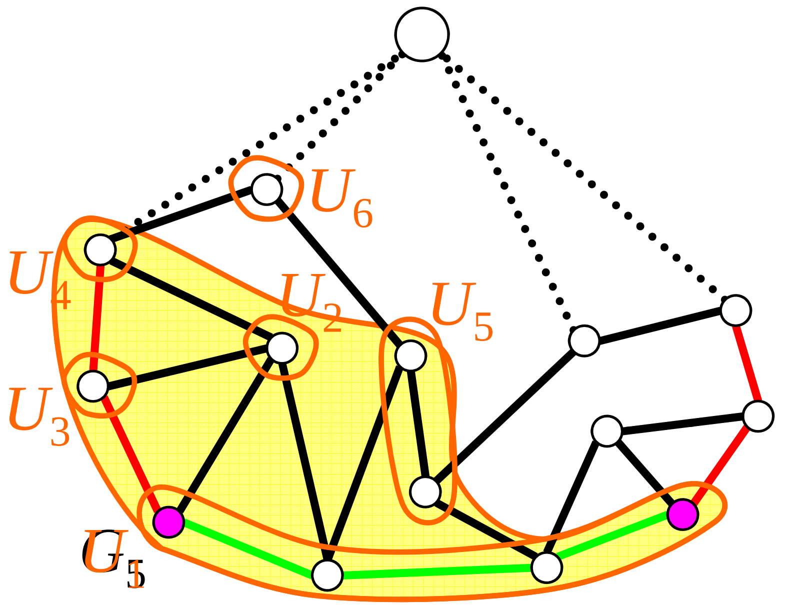




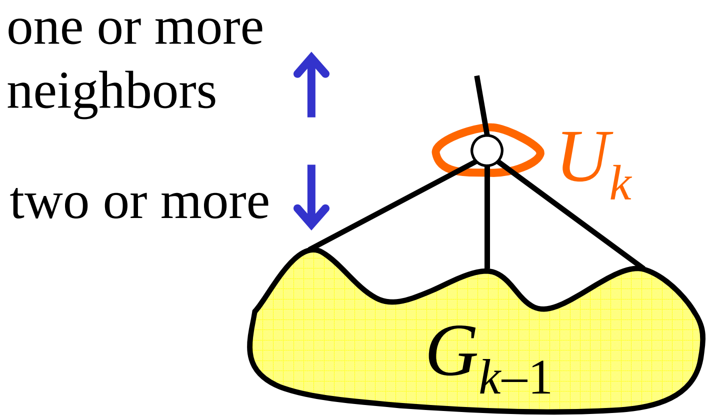
G_k is induced by
$U_1 \cup U_2 \cup \cdots \cup U_k$

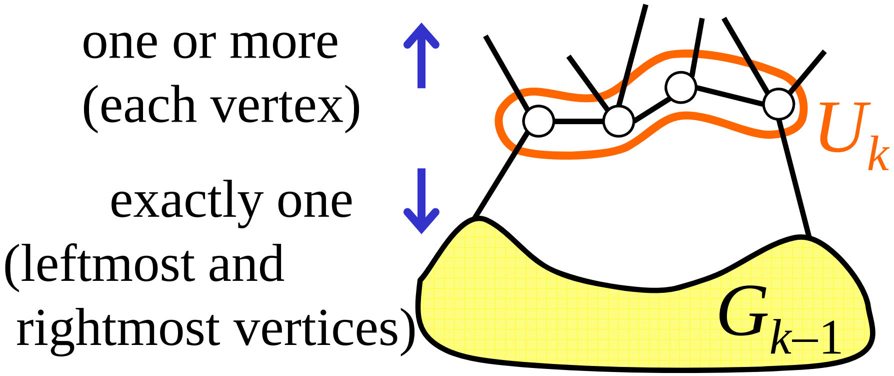
canonical decomposition

$$\Pi = (U_1 , \boxed{U_2 , \cdots , U_{m-1}} , U_m)$$



Canonical decomposition

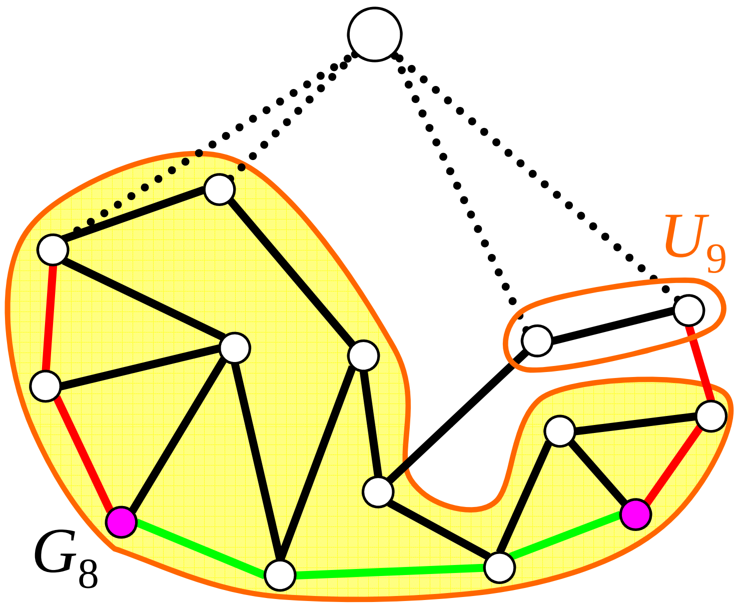




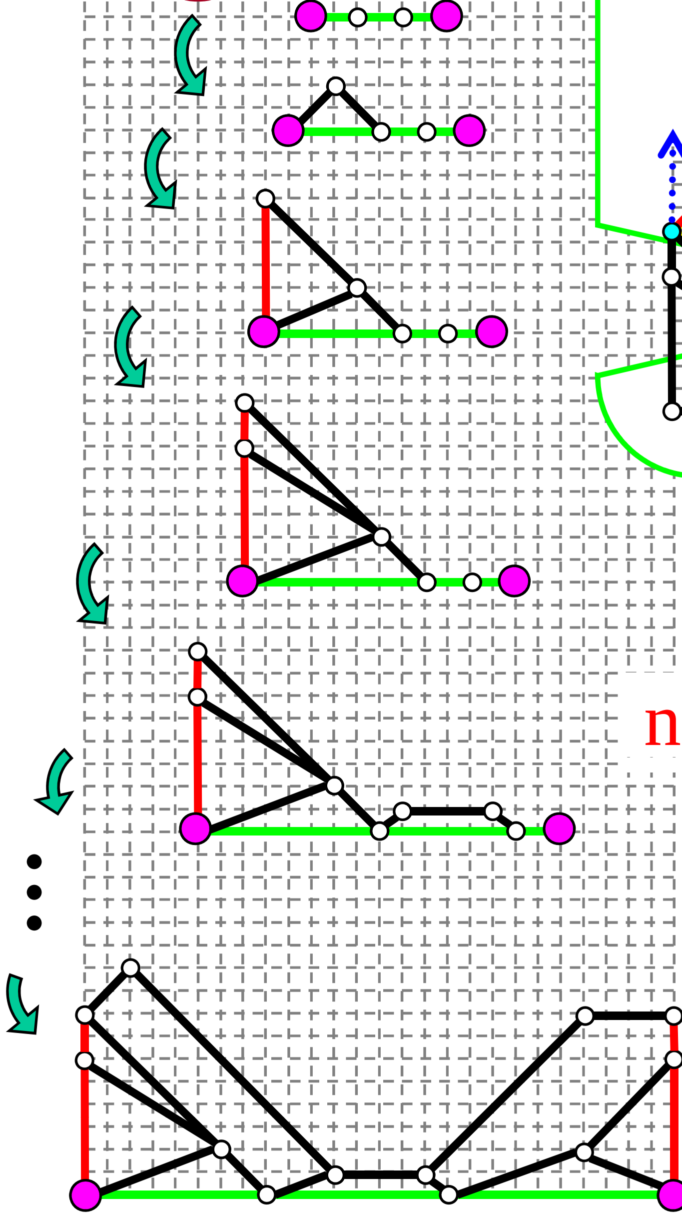
G_k is induced by $U_1 \cup U_2 \cup \cdots \cup U_k$

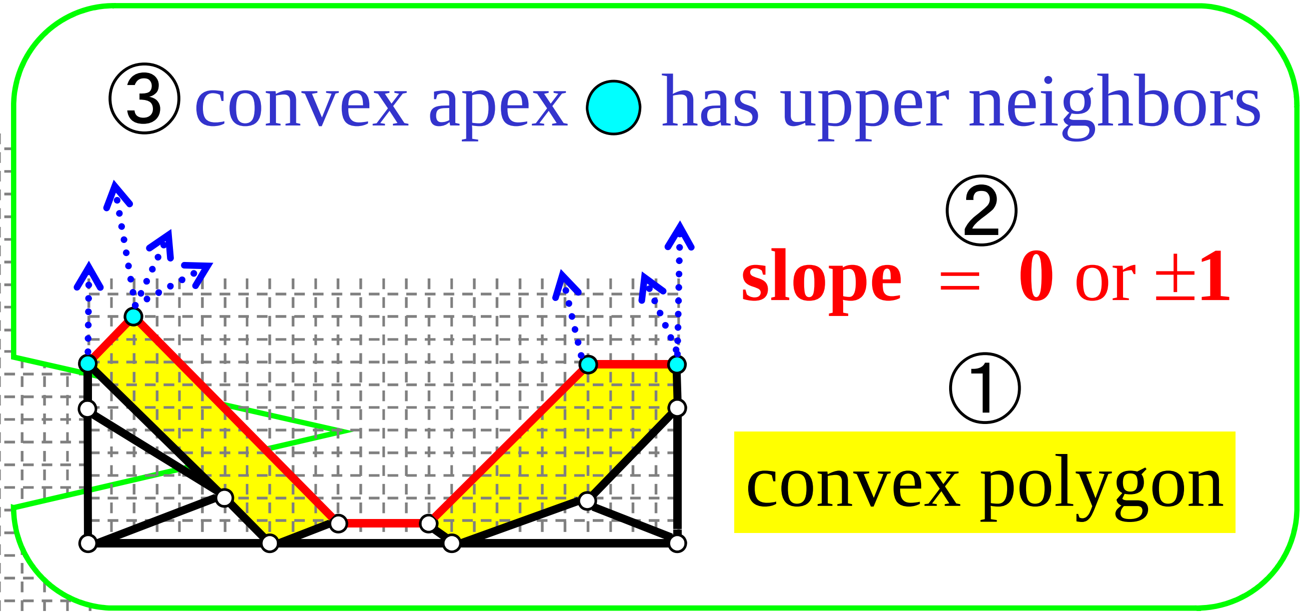
canonical decomposition

$$\Pi = (U_1 , U_2 , \cdots , U_{m-1} , U_m)$$

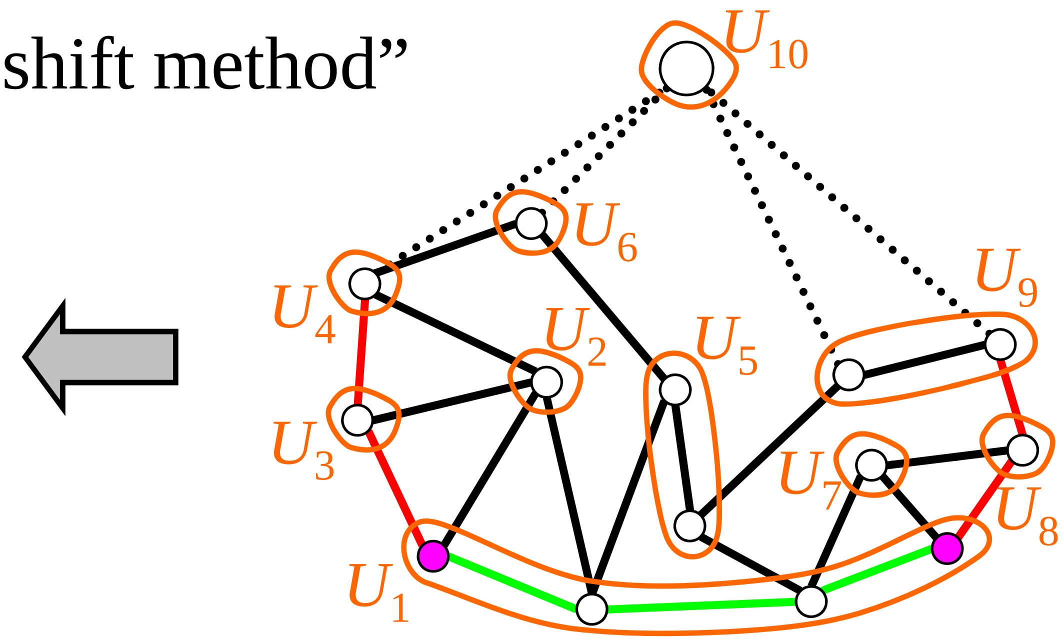


Algorithm

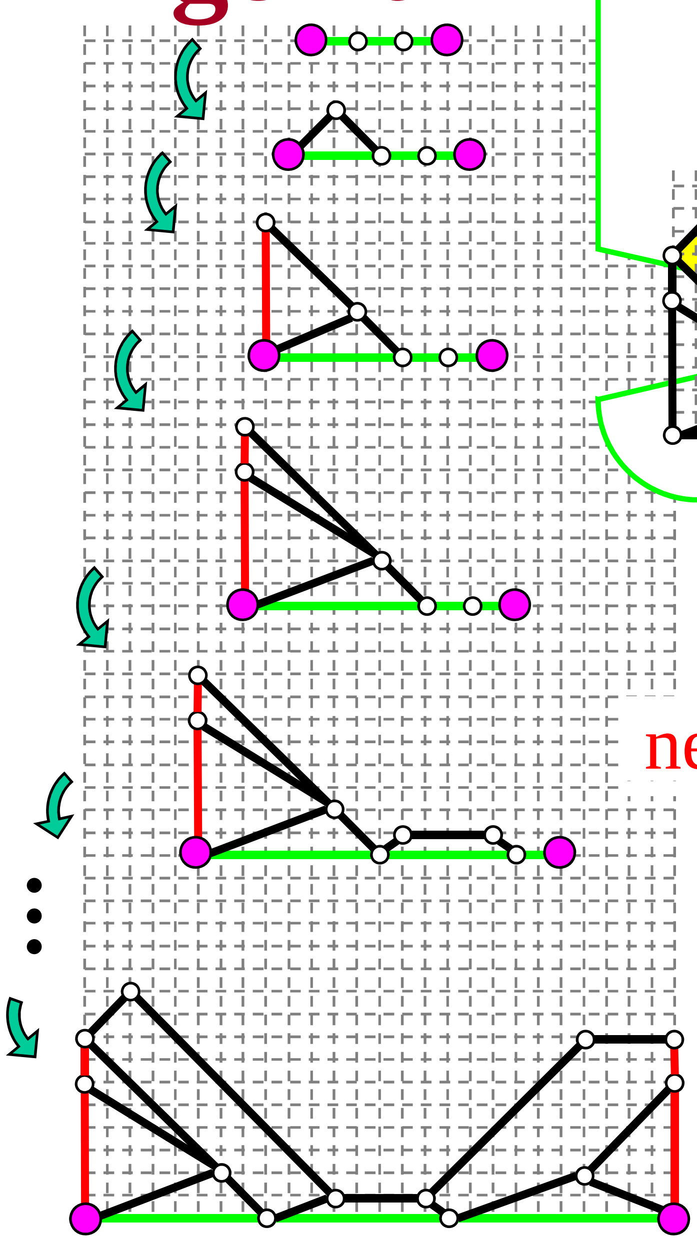


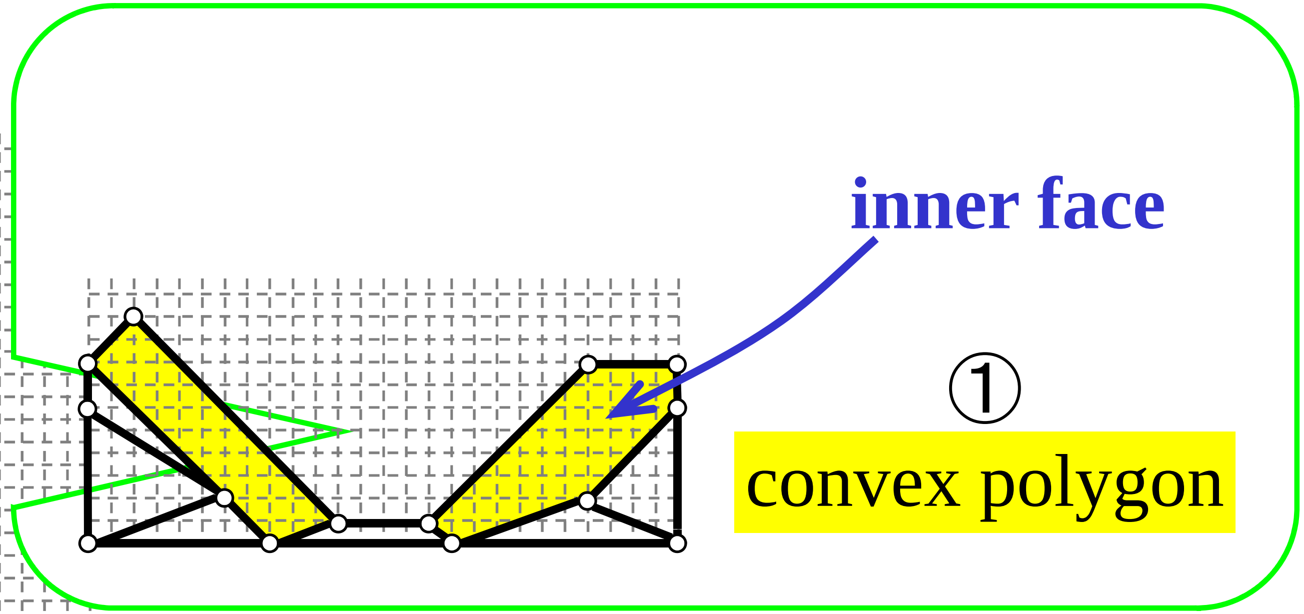


new "shift method"



Algorithm





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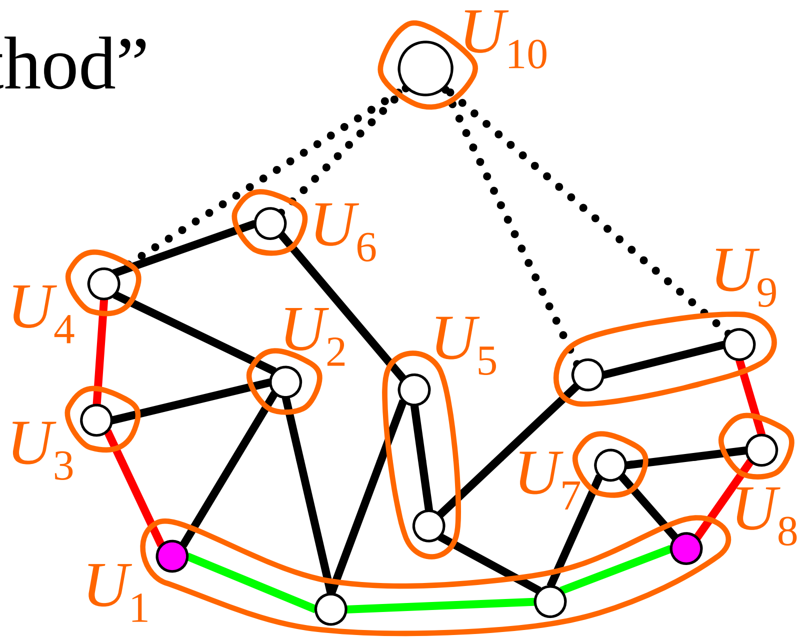
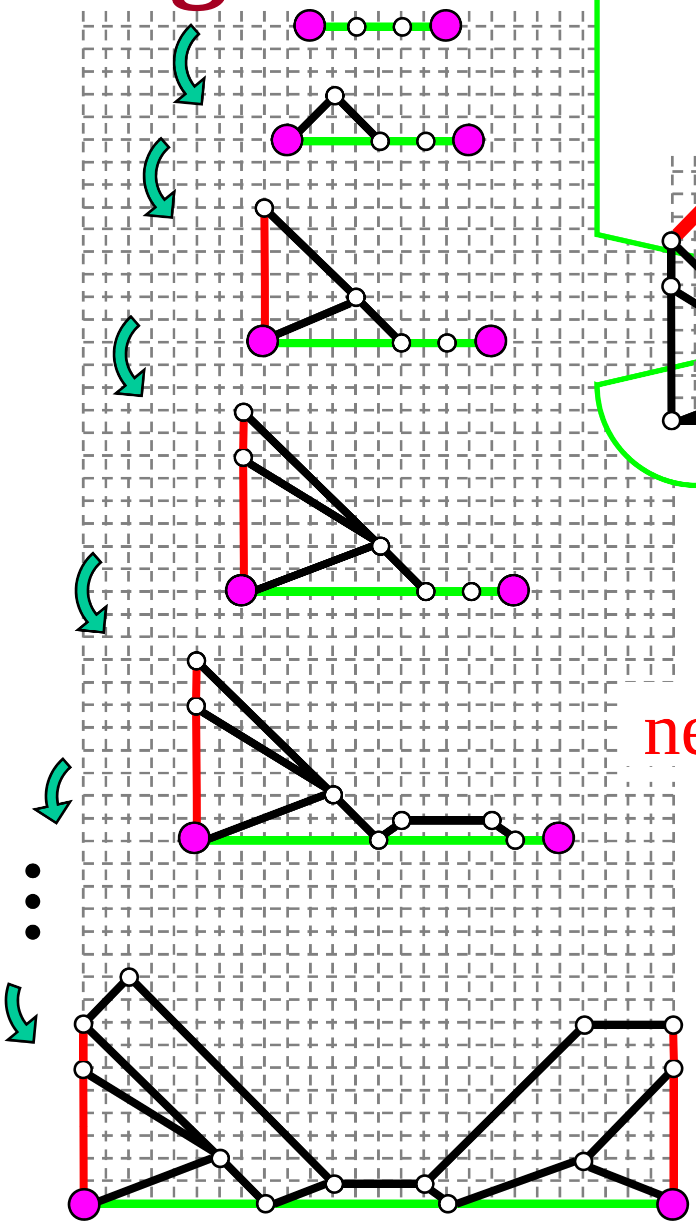
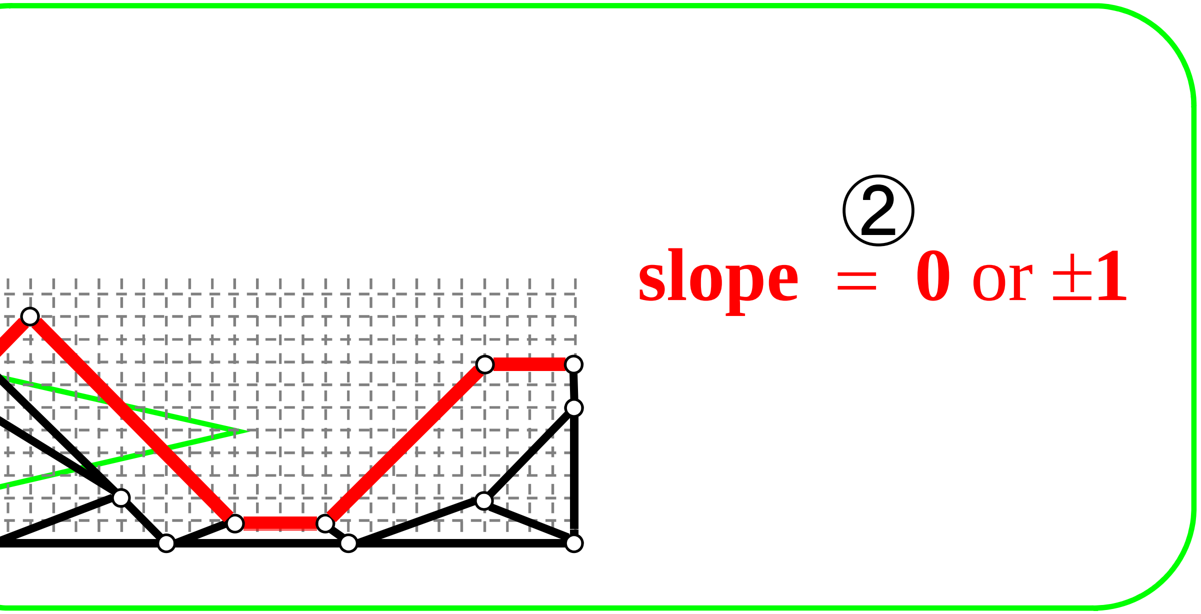
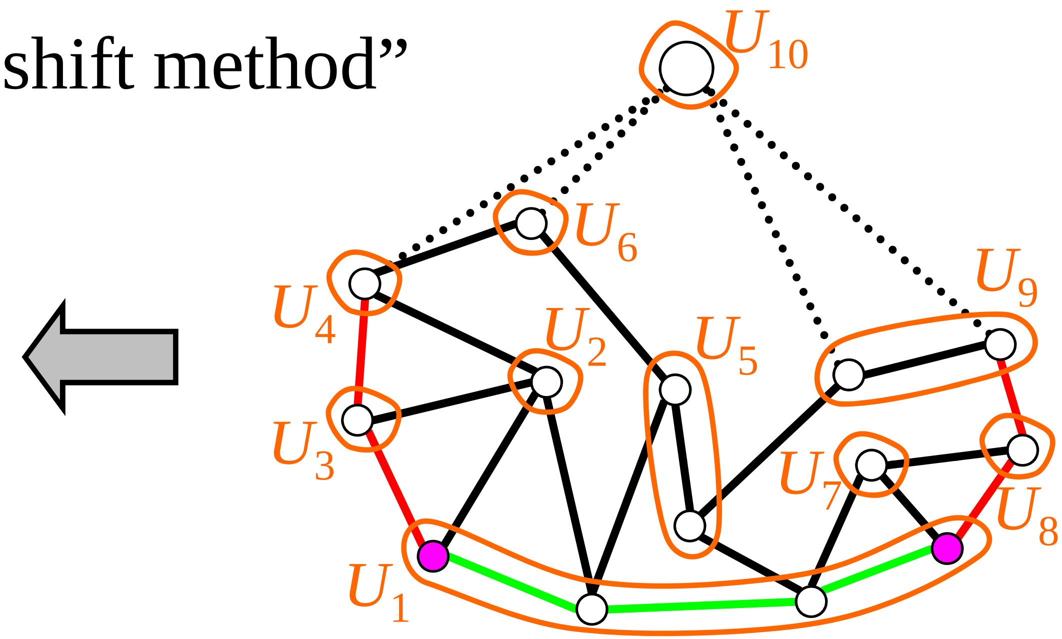


Algorithm

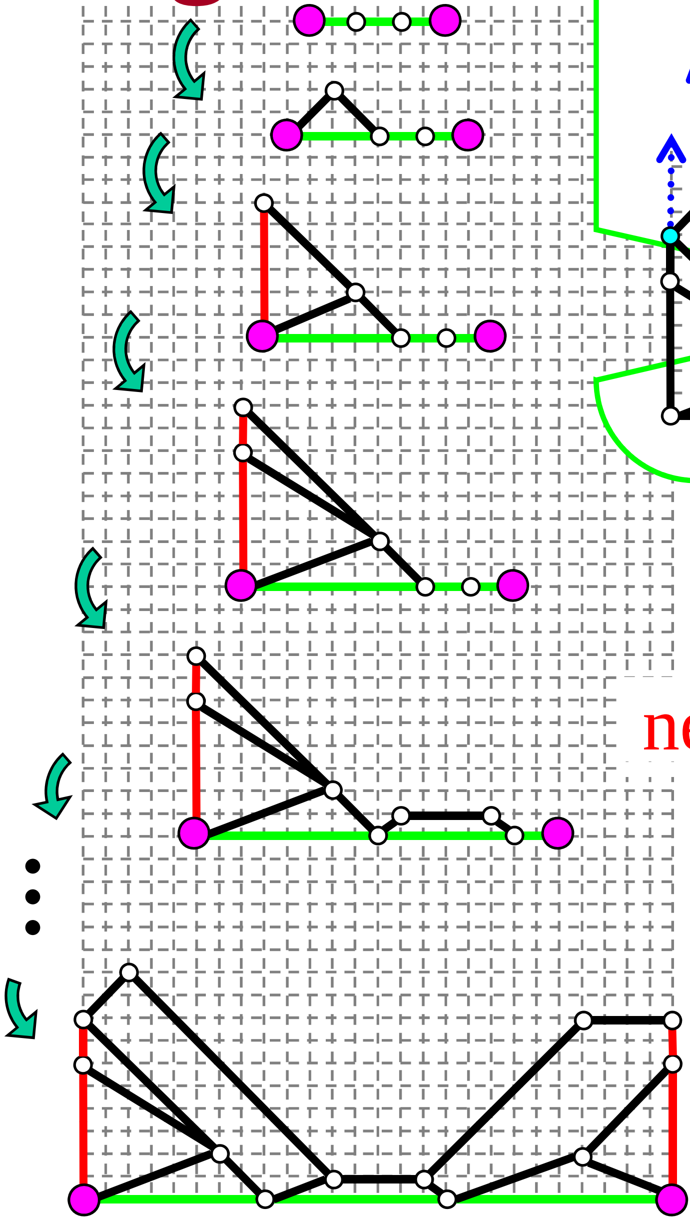


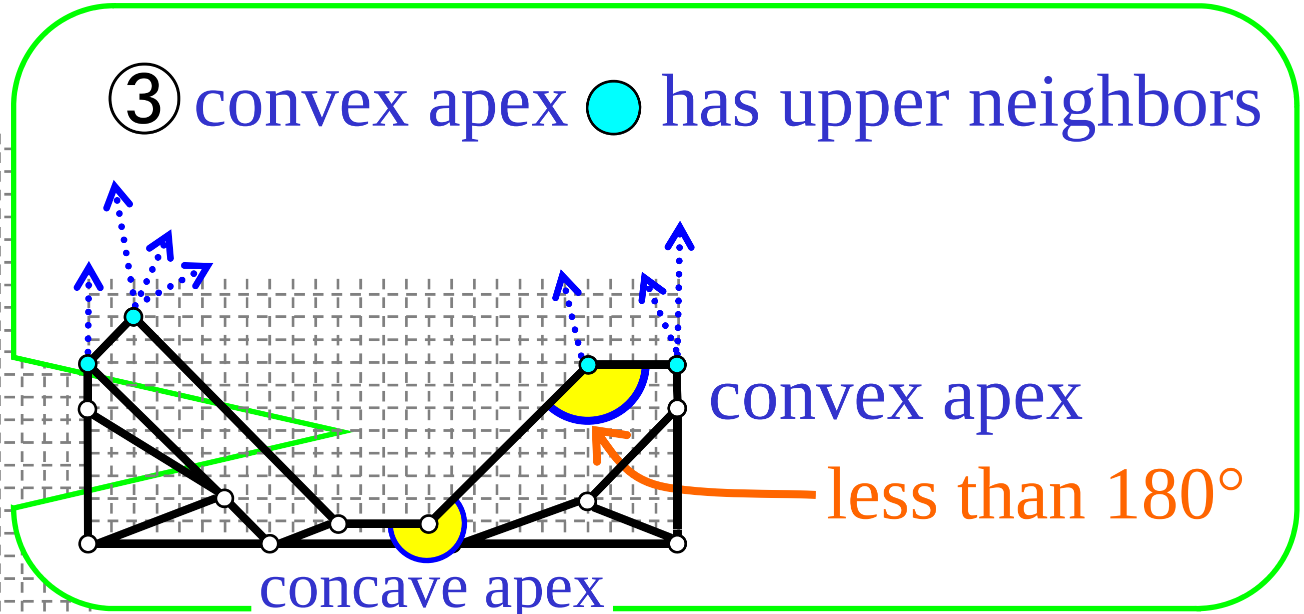


new “shift method”



Algorithm





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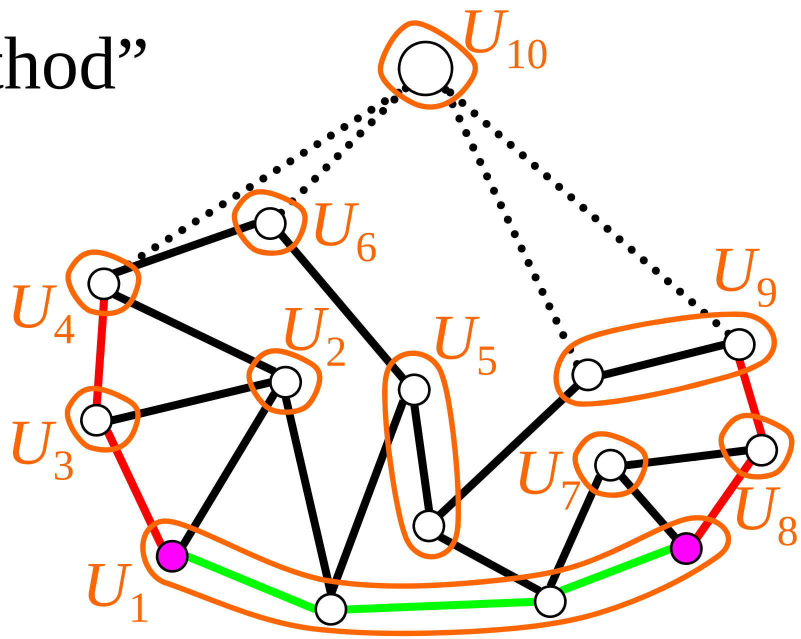
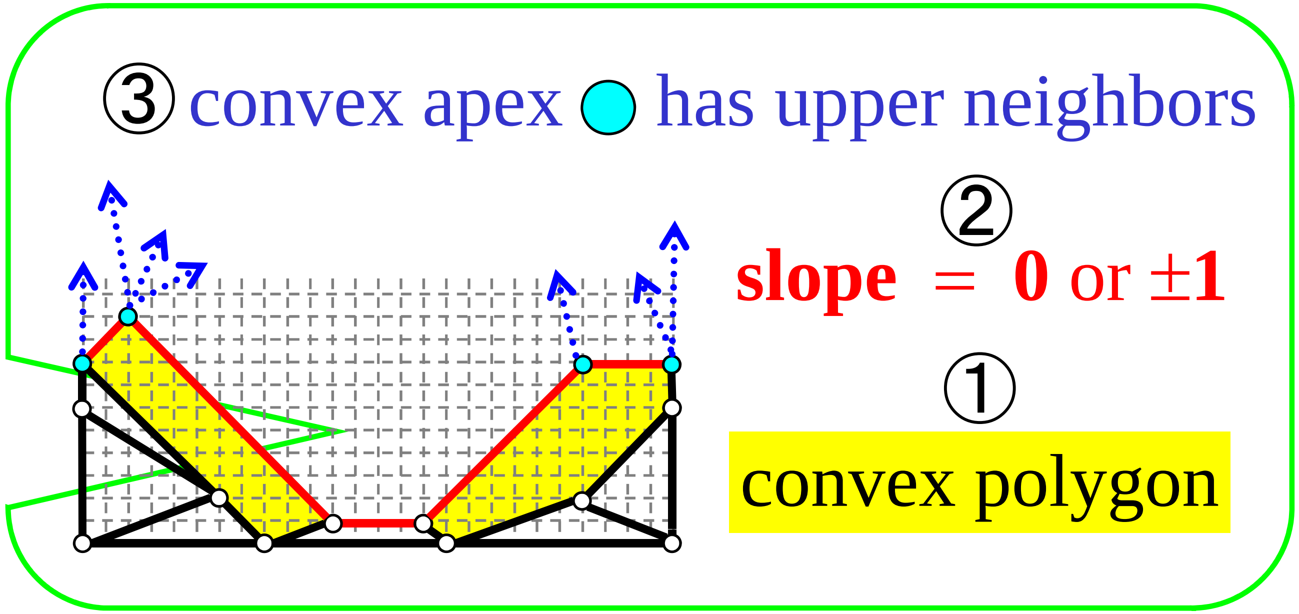


Algorithm



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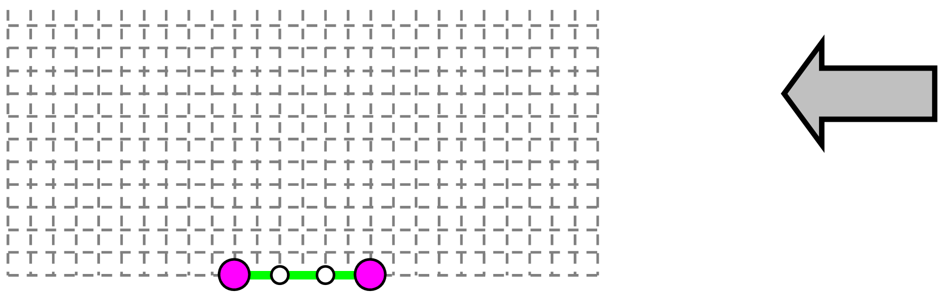
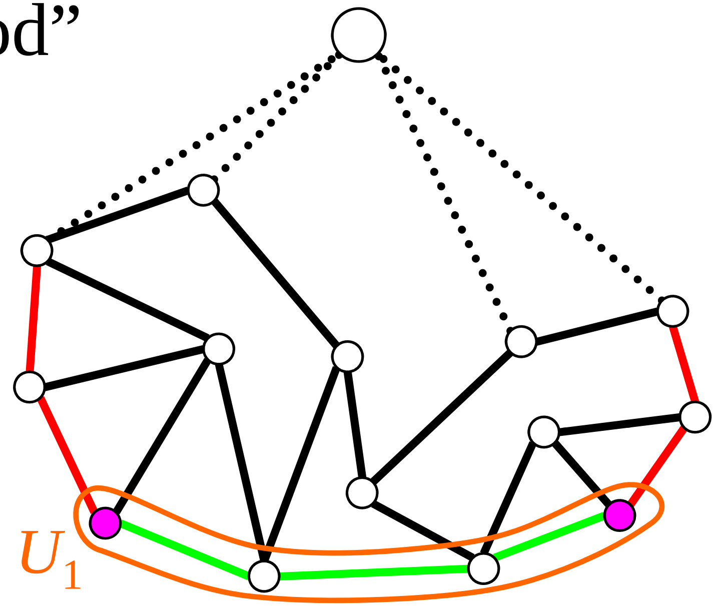
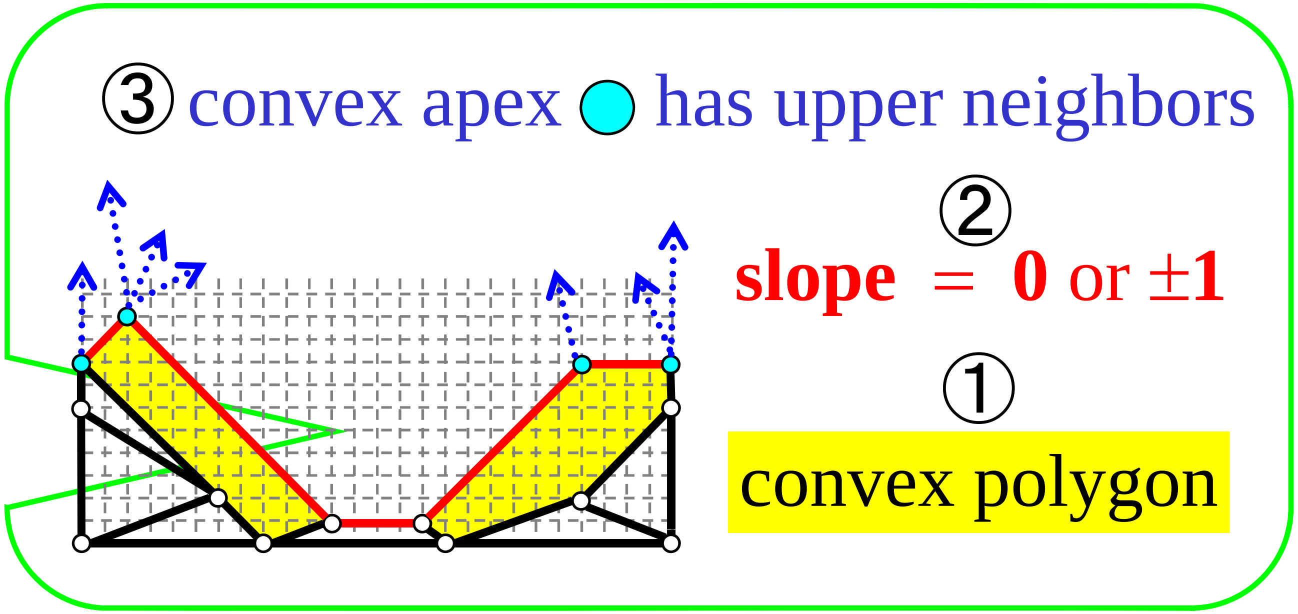
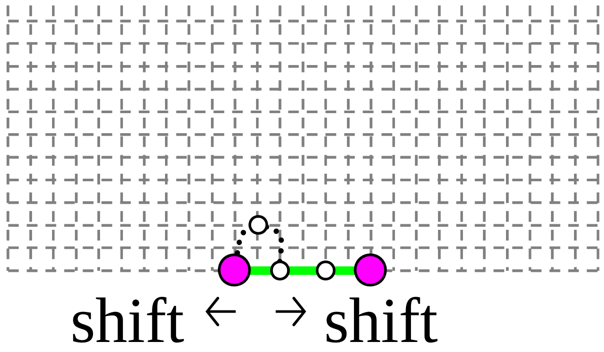


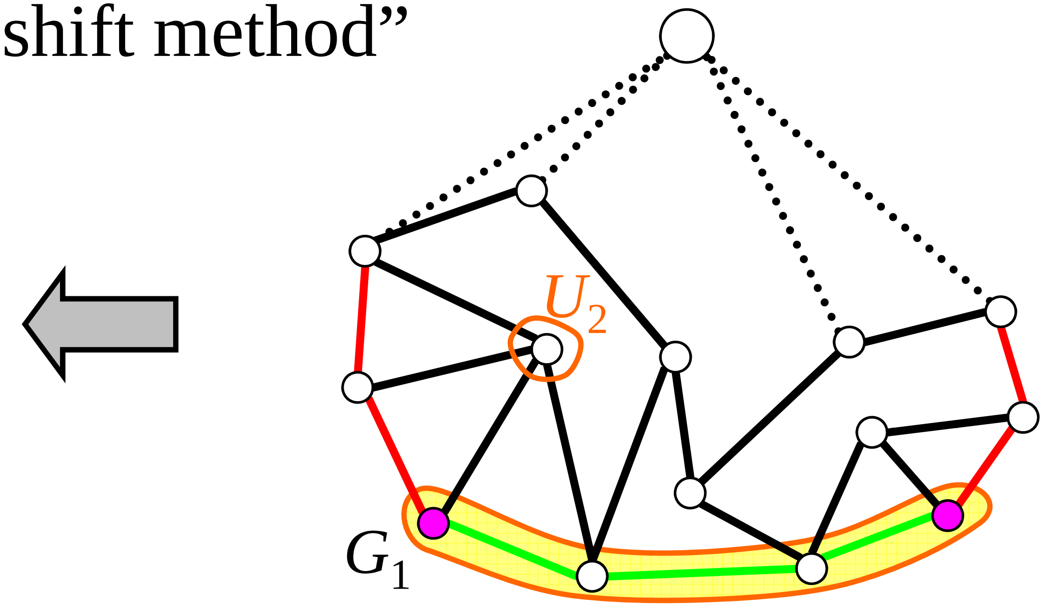


Algorithm



new "shift method"





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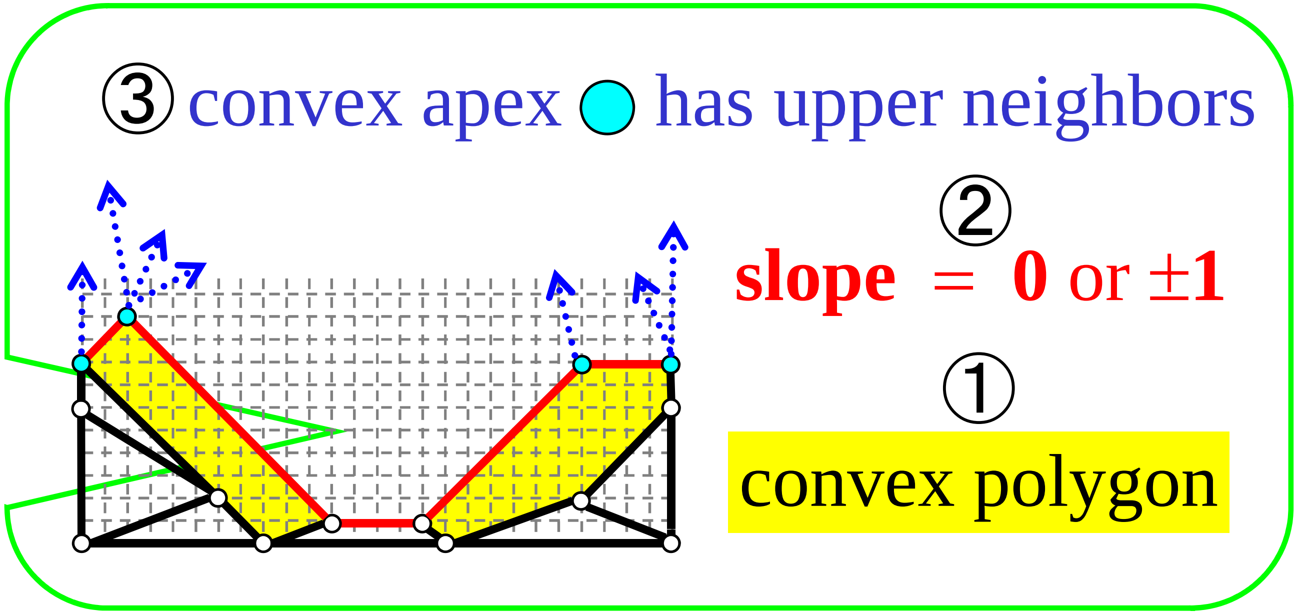

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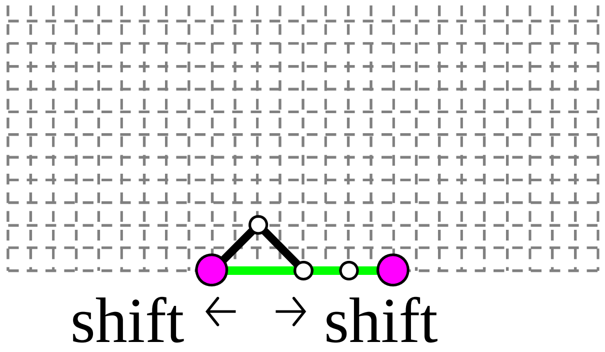
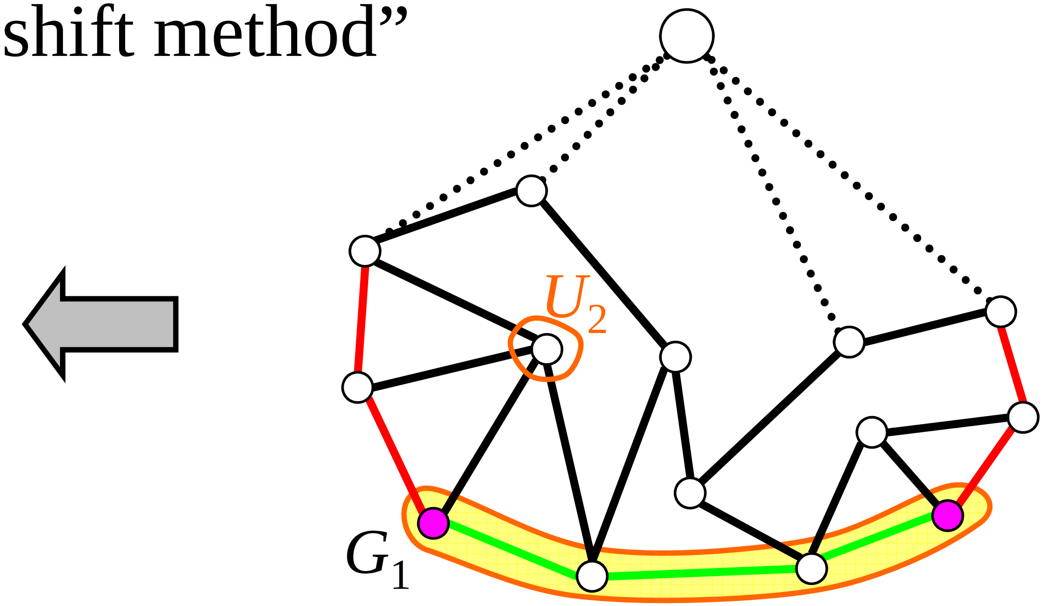
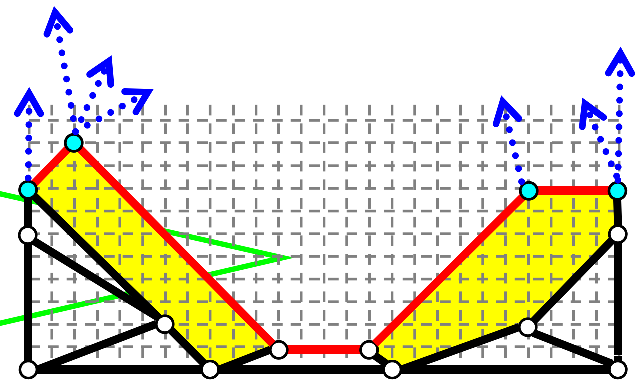




Algorithm

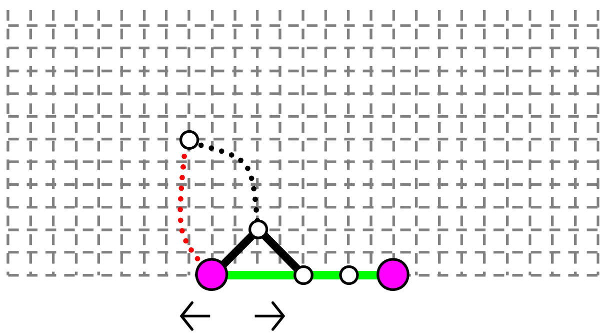
③ convex apex ● has upper neighbors

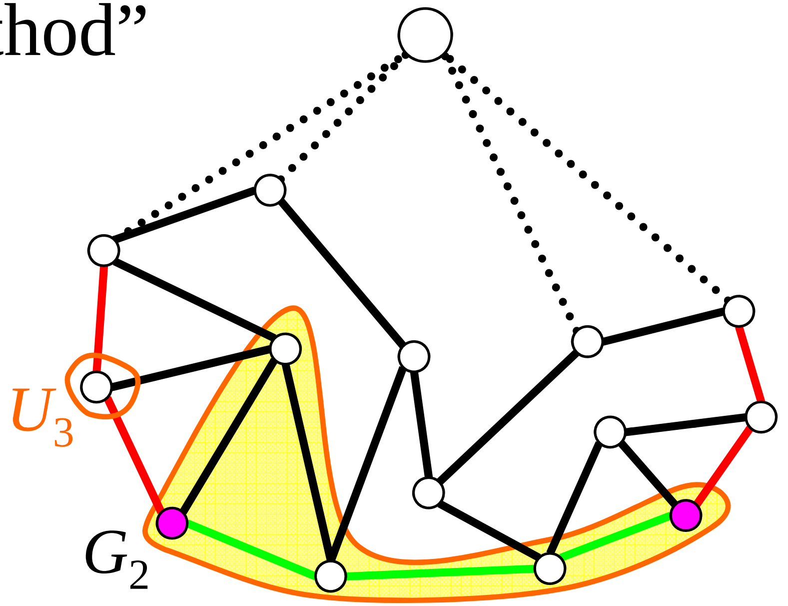


② **slope** = **0** or ±**1**

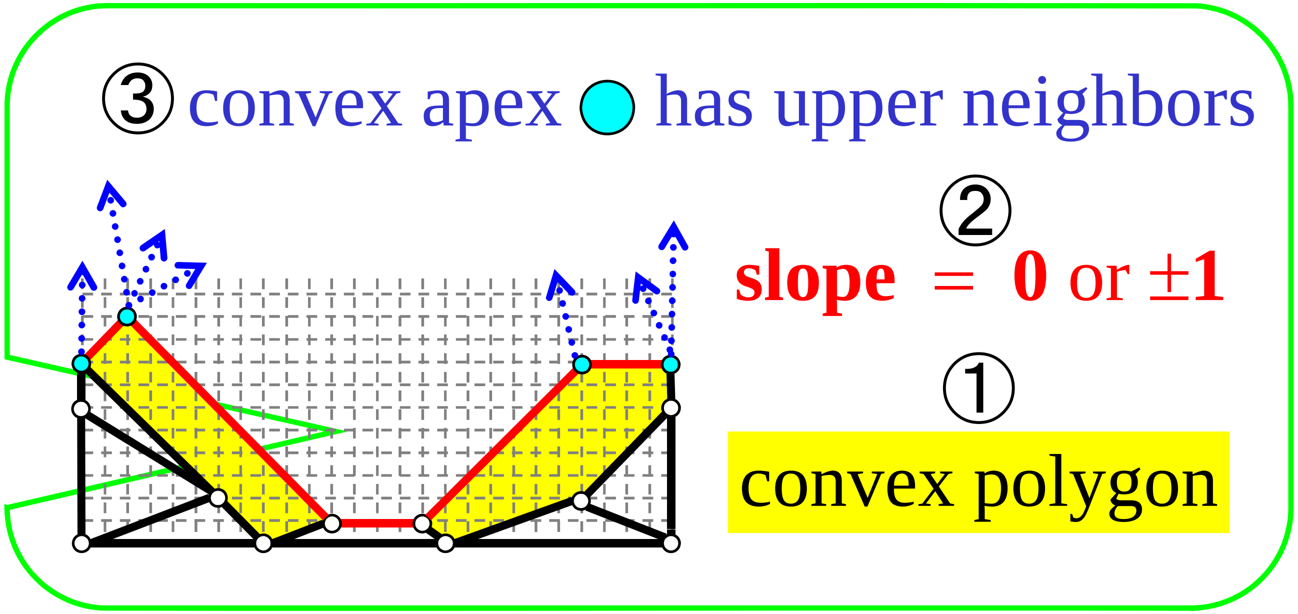
① convex polygon

new “shift method”

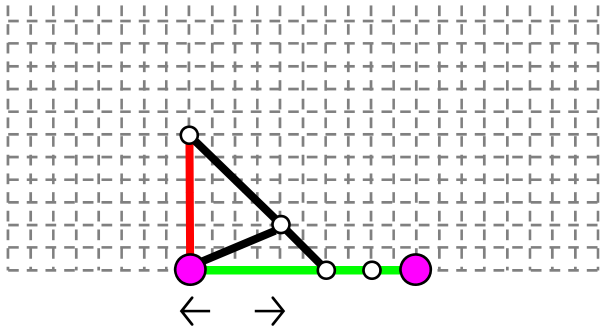


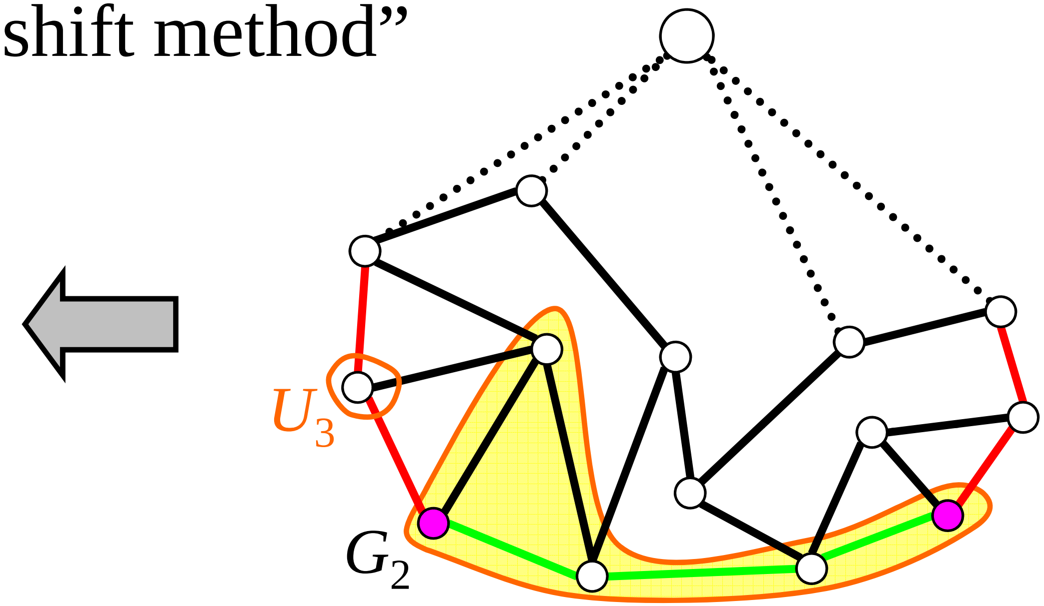


Algorithm



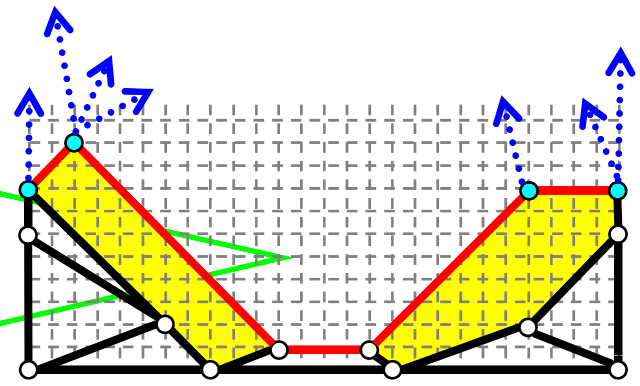
new "shift method"





Algorithm

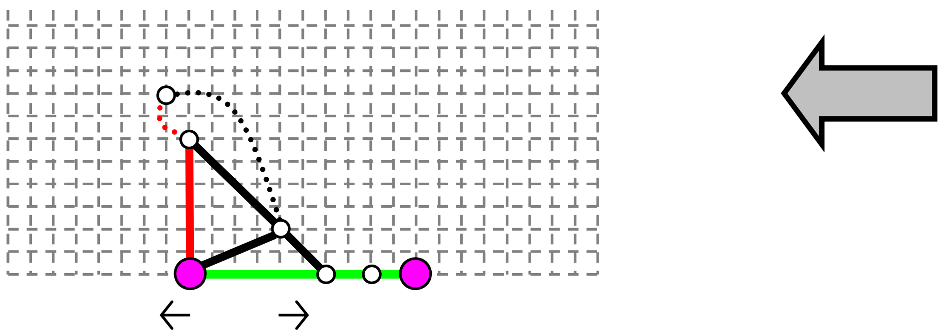
③ convex apex ● has upper neighbors

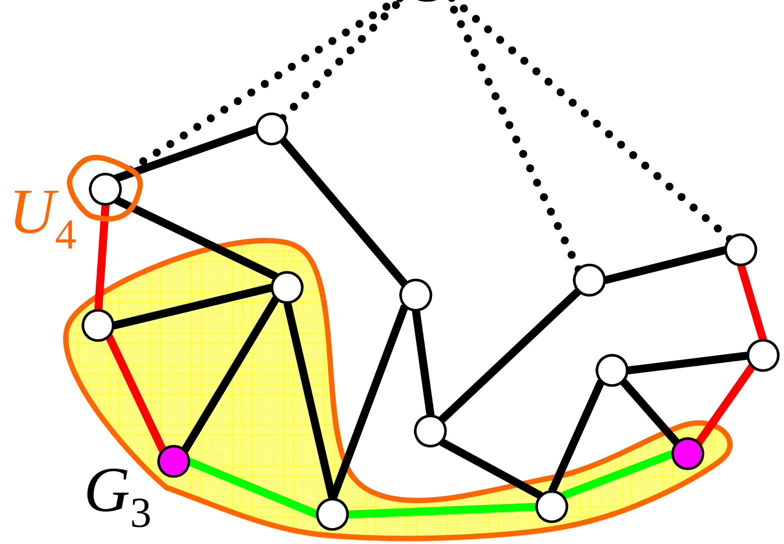


② **slope** = **0** or ±**1**

① convex polygon

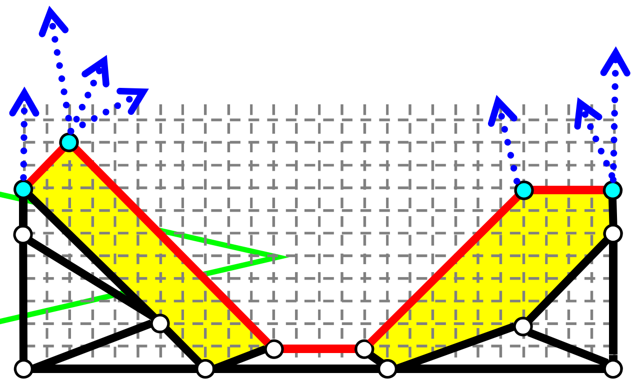
new “shift method”





Algorithm

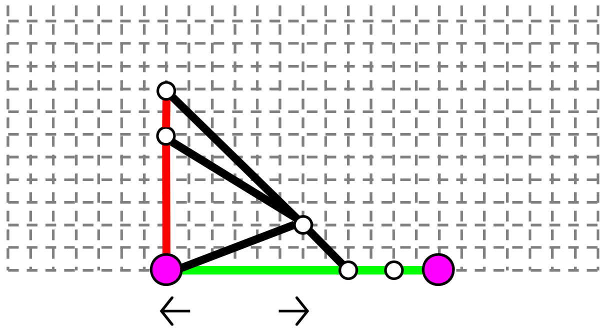
③ convex apex ● has upper neighbors

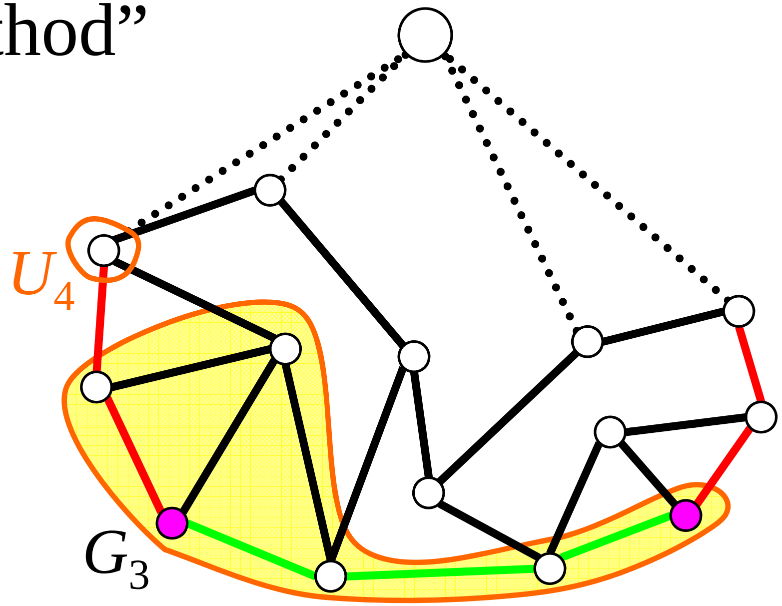


② slope = **0** or ±**1**

① convex polygon

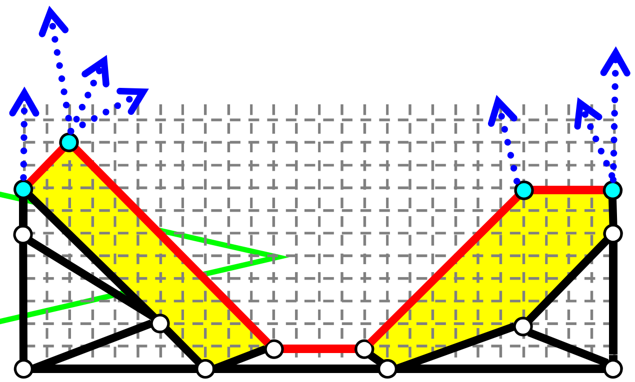
new “shift method”





Algorithm

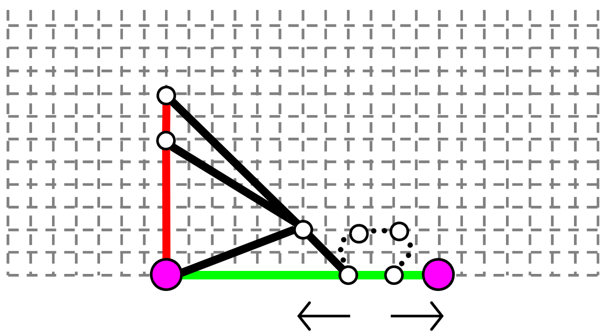
③ convex apex ● has upper neighbors

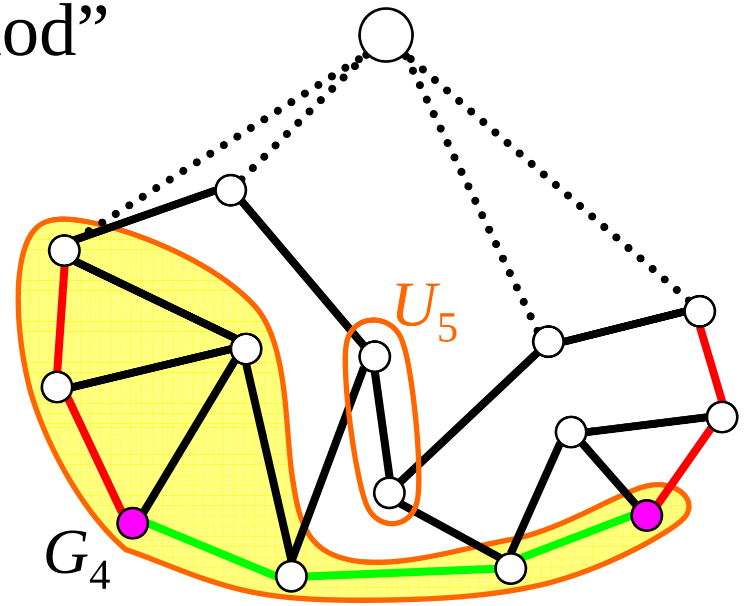


② **slope** = **0** or ±**1**

① convex polygon

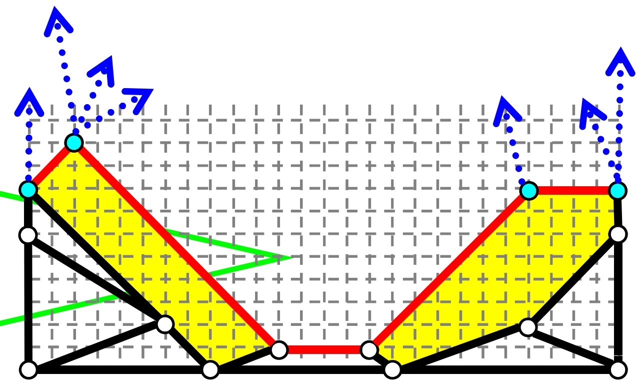
new "shift method"





Algorithm

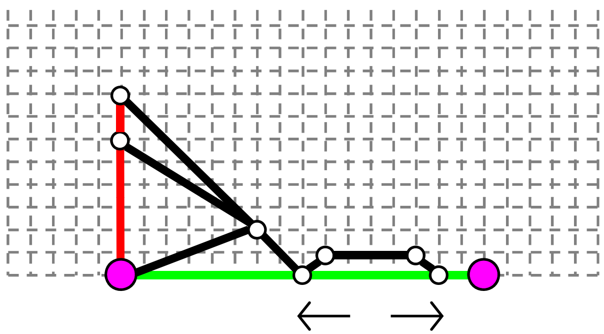
③ convex apex ● has upper neighbors

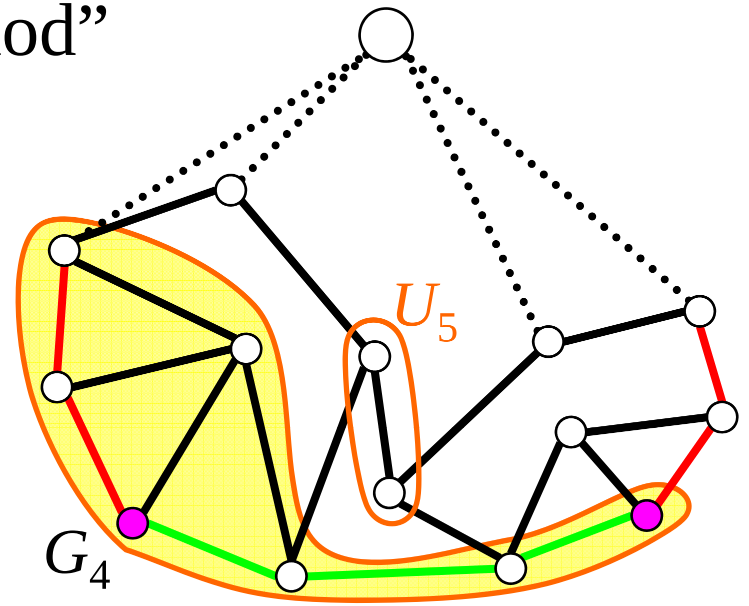


② **slope** = **0** or ±**1**

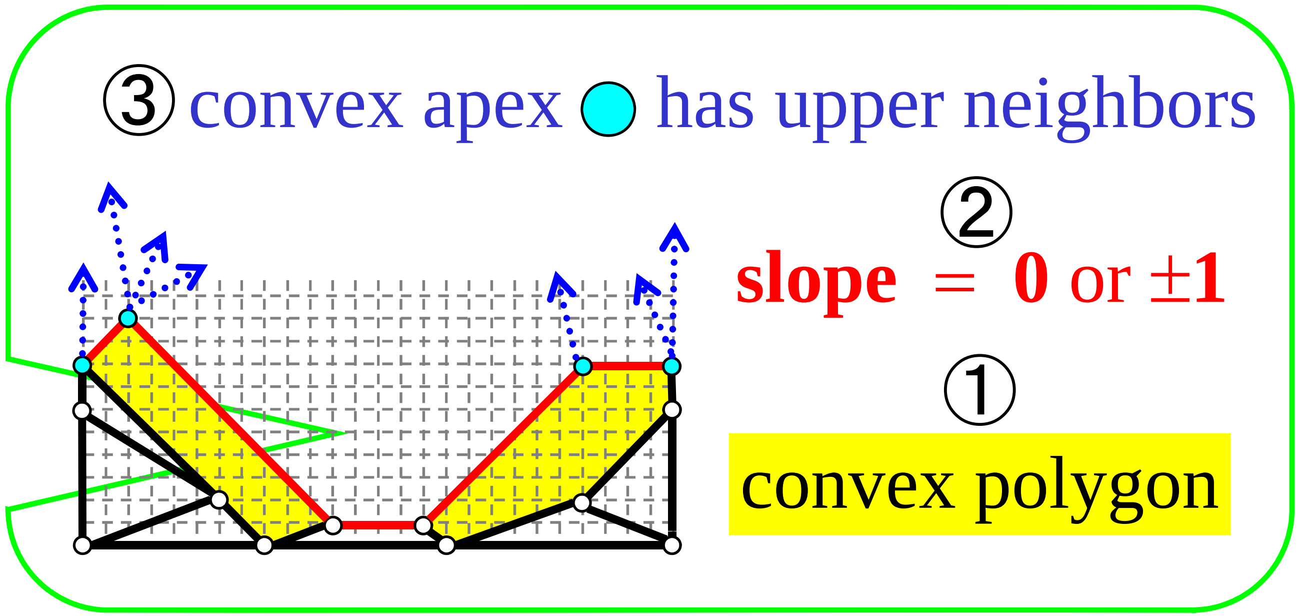
① convex polygon

new “shift method”

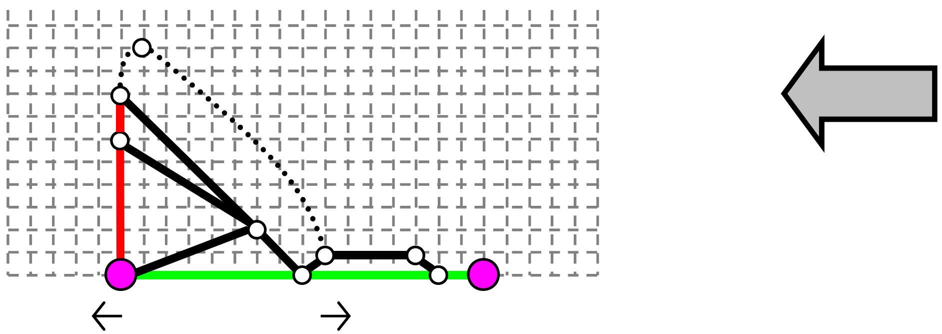


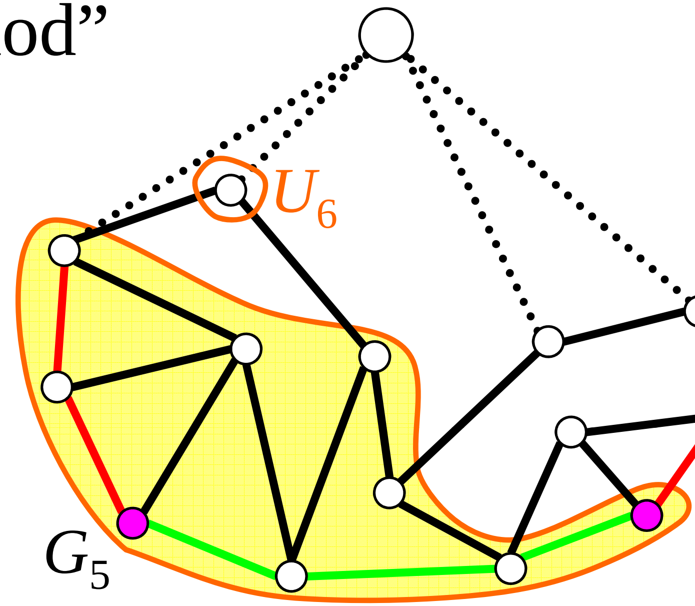


Algorithm



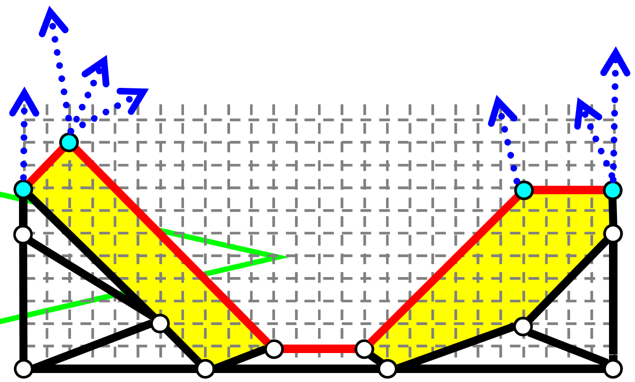
new "shift method"





Algorithm

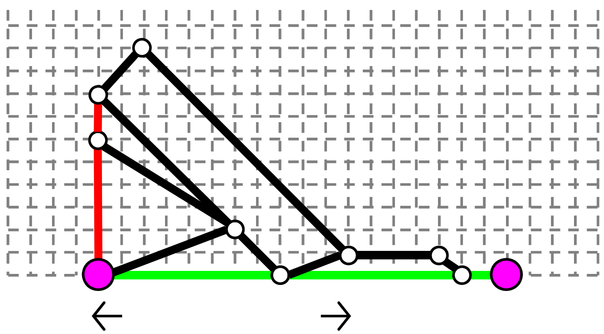
③ convex apex ● has upper neighbors

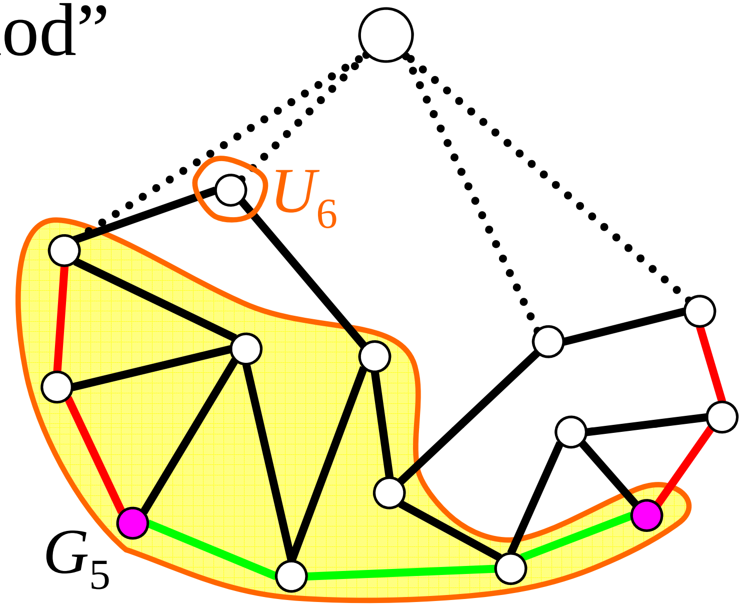


② **slope** = **0** or ±**1**

① convex polygon

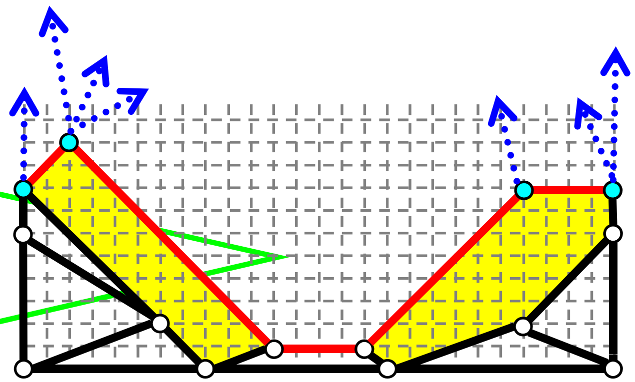
new “shift method”





Algorithm

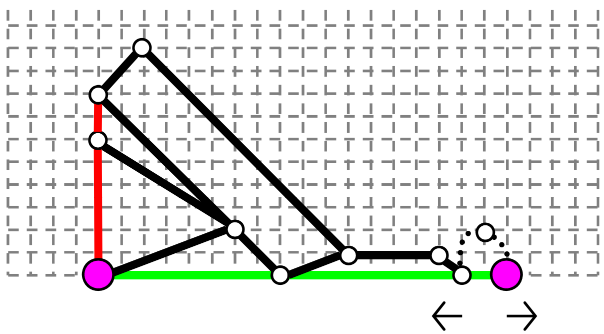
③ convex apex ● has upper neighbors

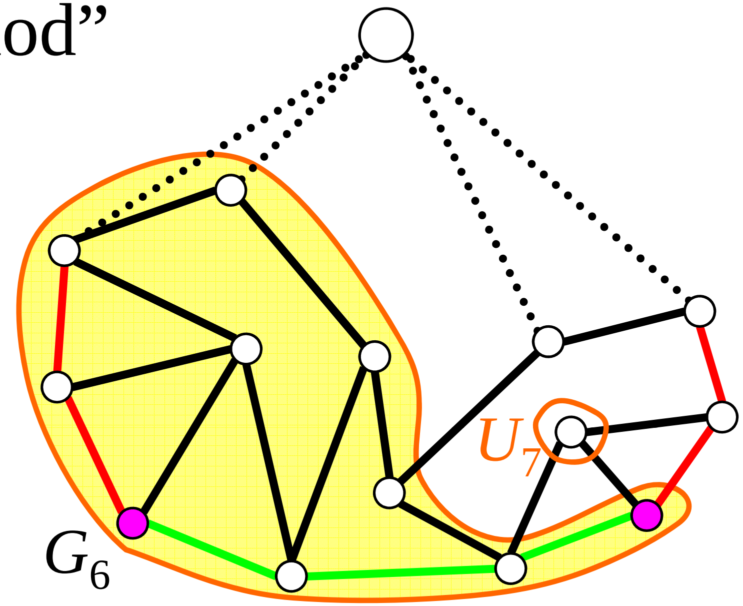


② slope = **0** or ±**1**

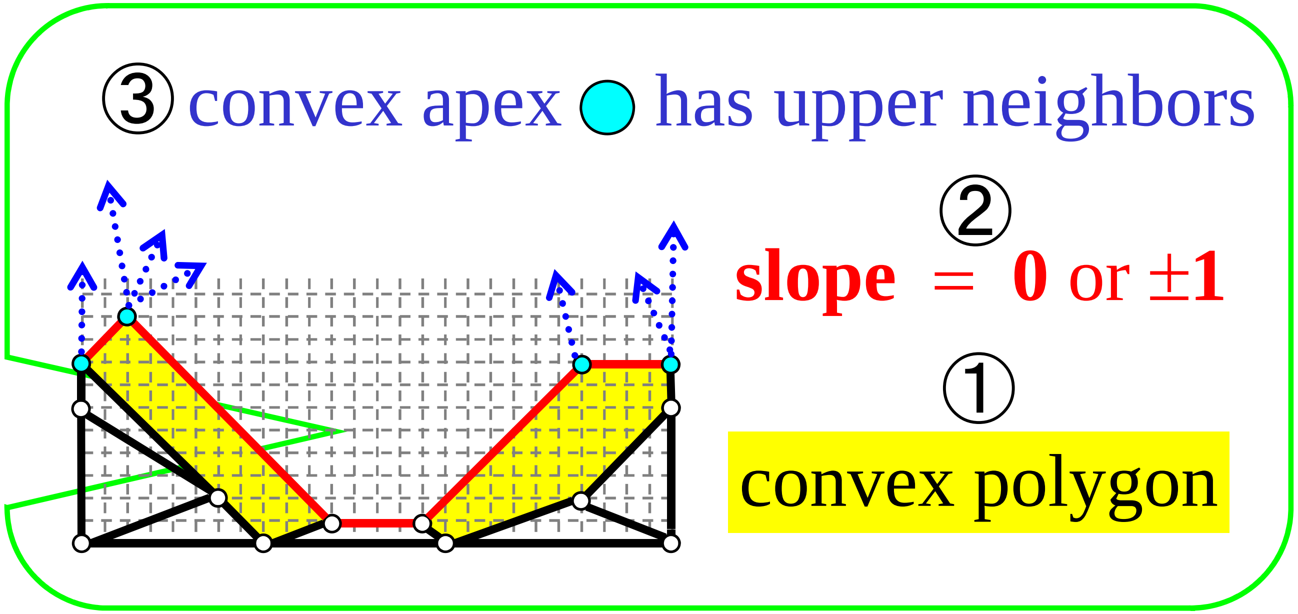
① convex polygon

new "shift method"

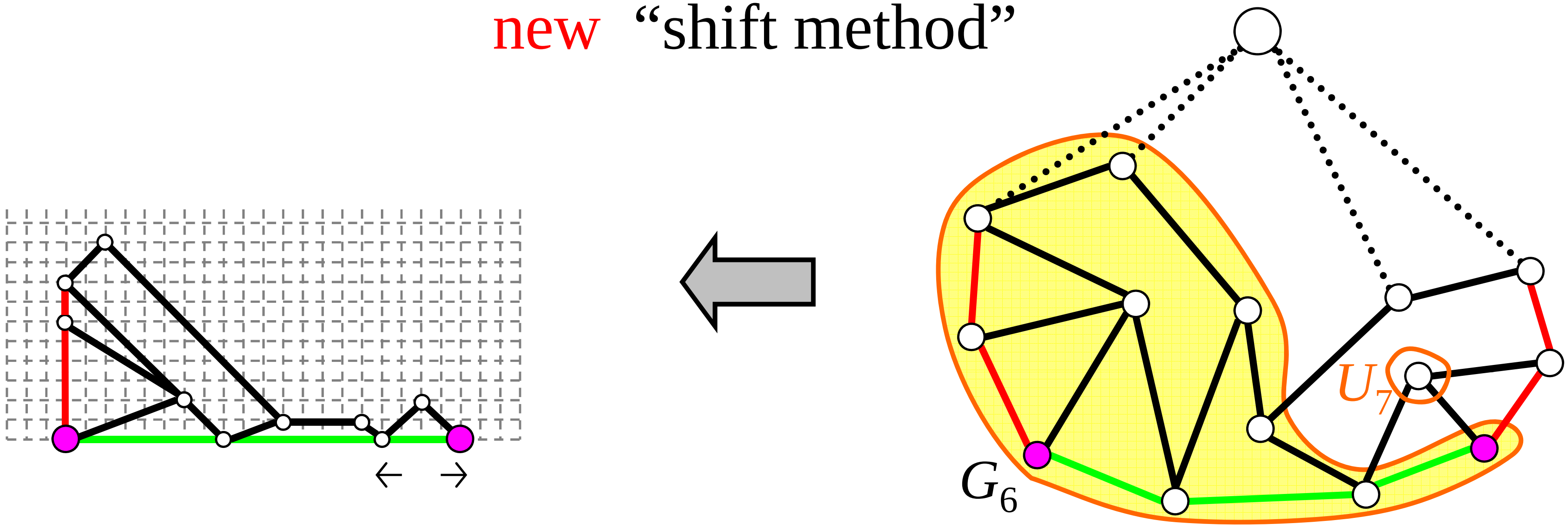




Algorithm

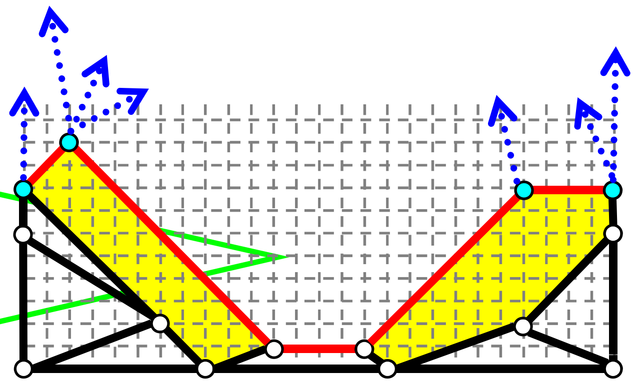


new "shift method"



Algorithm

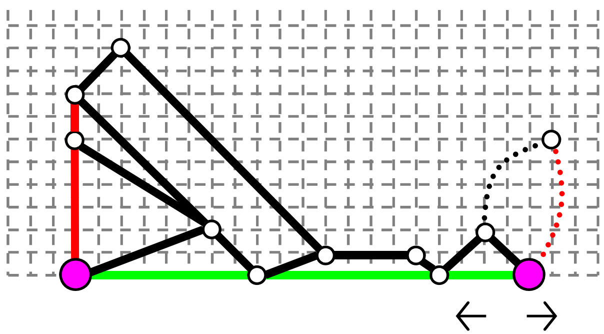
③ convex apex ● has upper neighbors

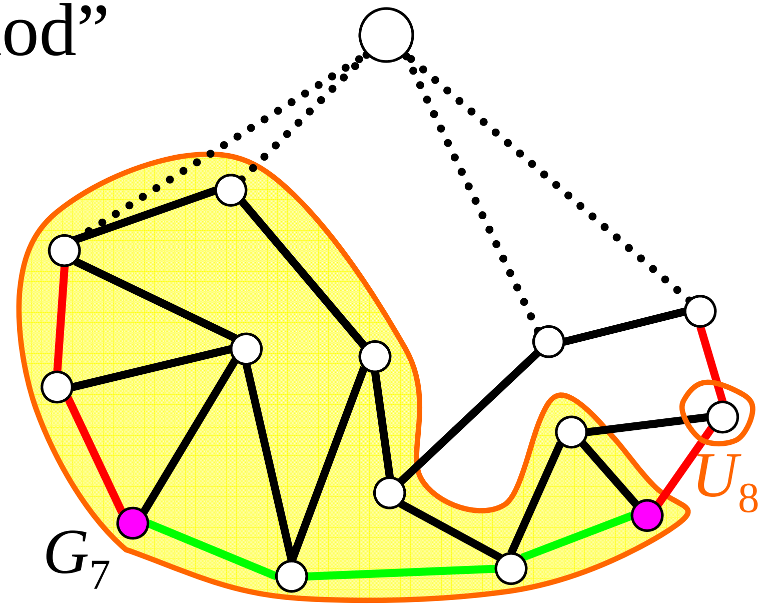


② **slope** = **0** or ±**1**

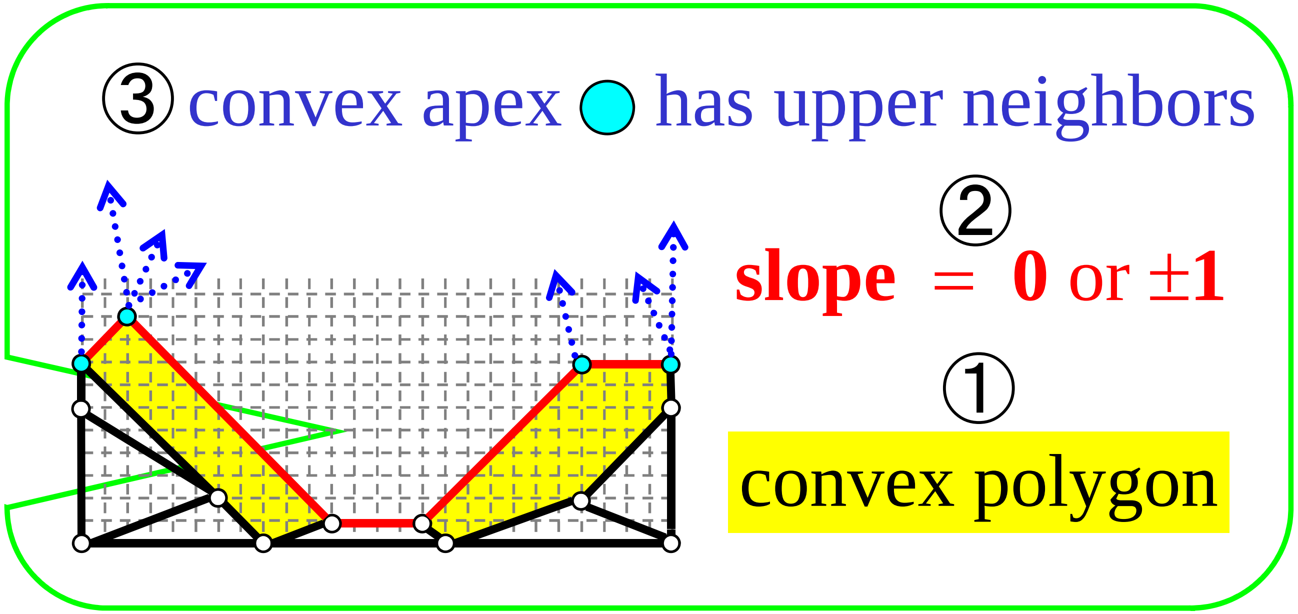
① convex polygon

new “shift method”

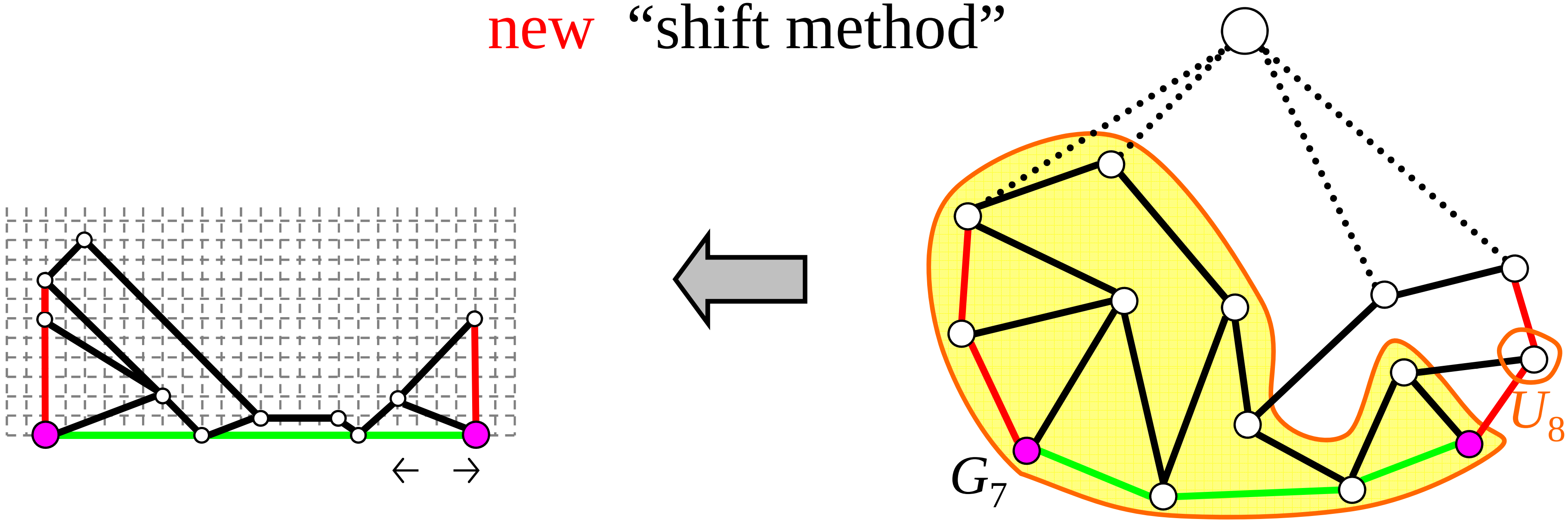




Algorithm

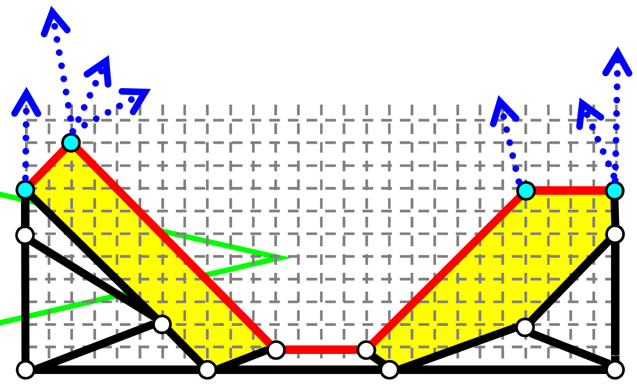


new "shift method"



Algorithm

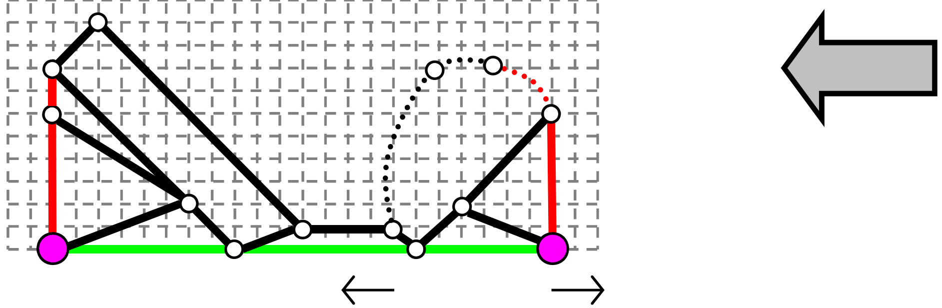
③ convex apex ● has upper neighbors

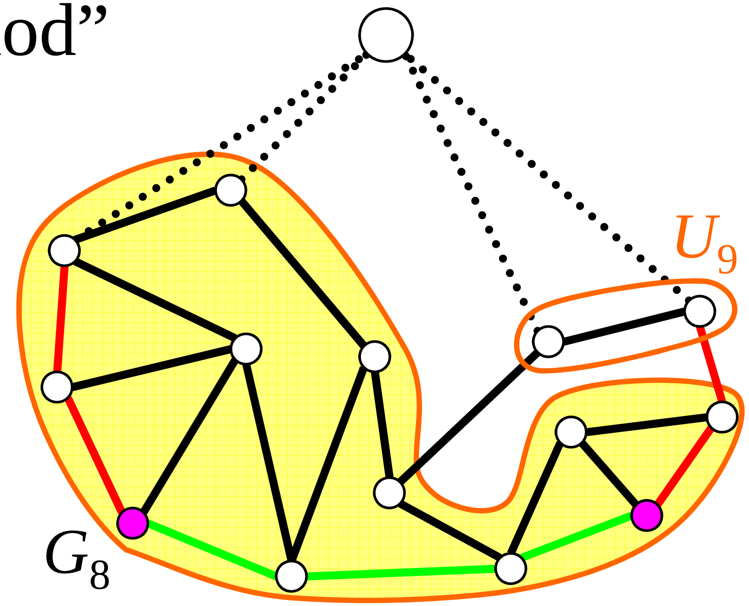


② **slope** = **0** or ±**1**

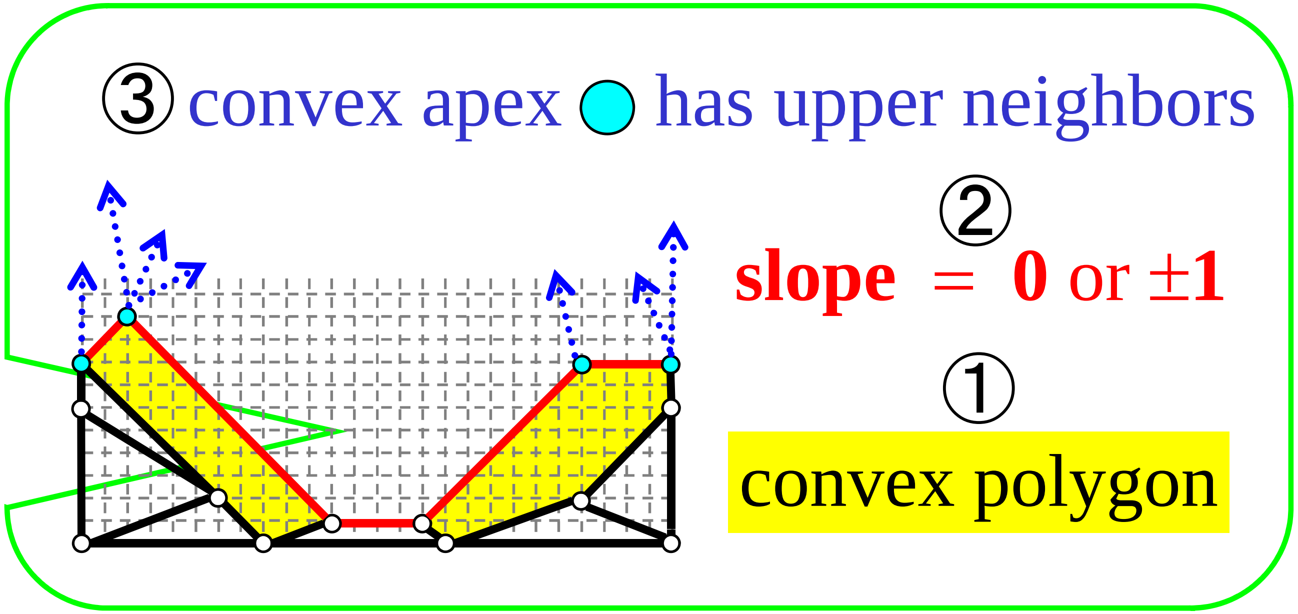
① convex polygon

new “shift method”





Algorithm



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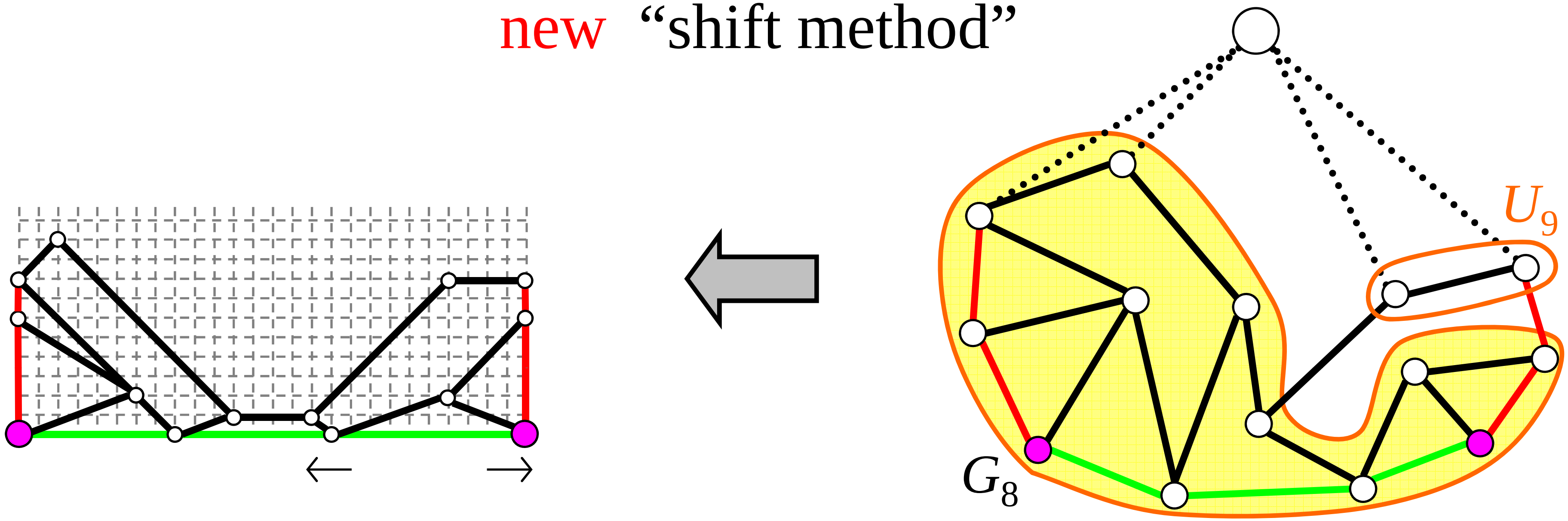
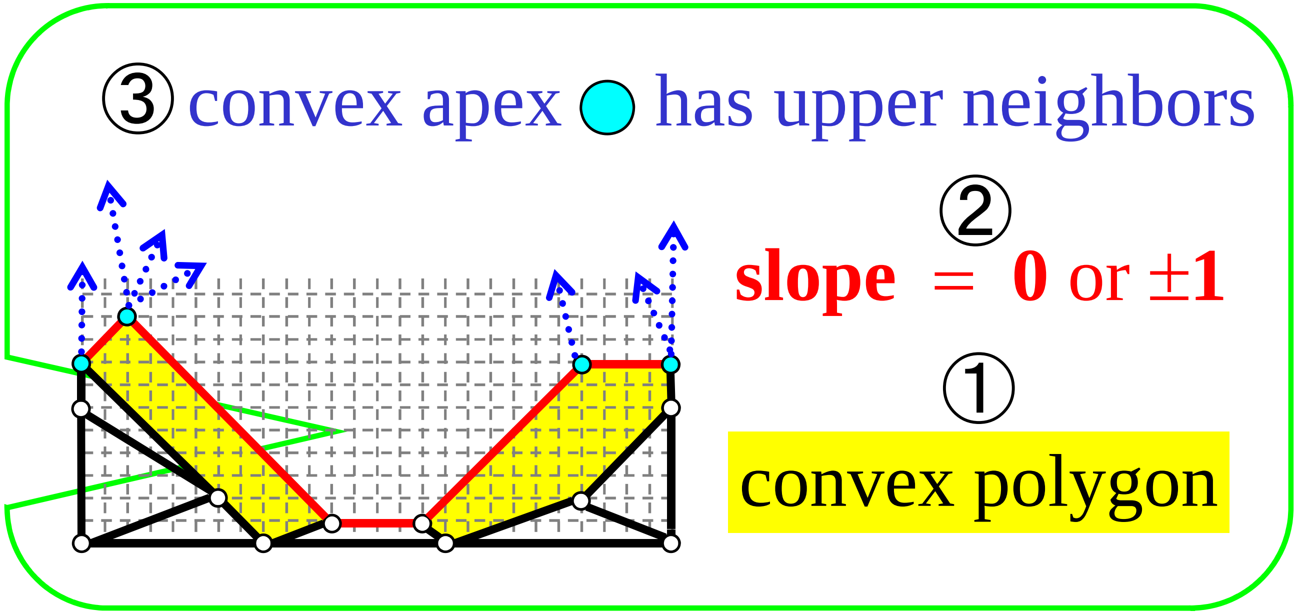


Algorithm



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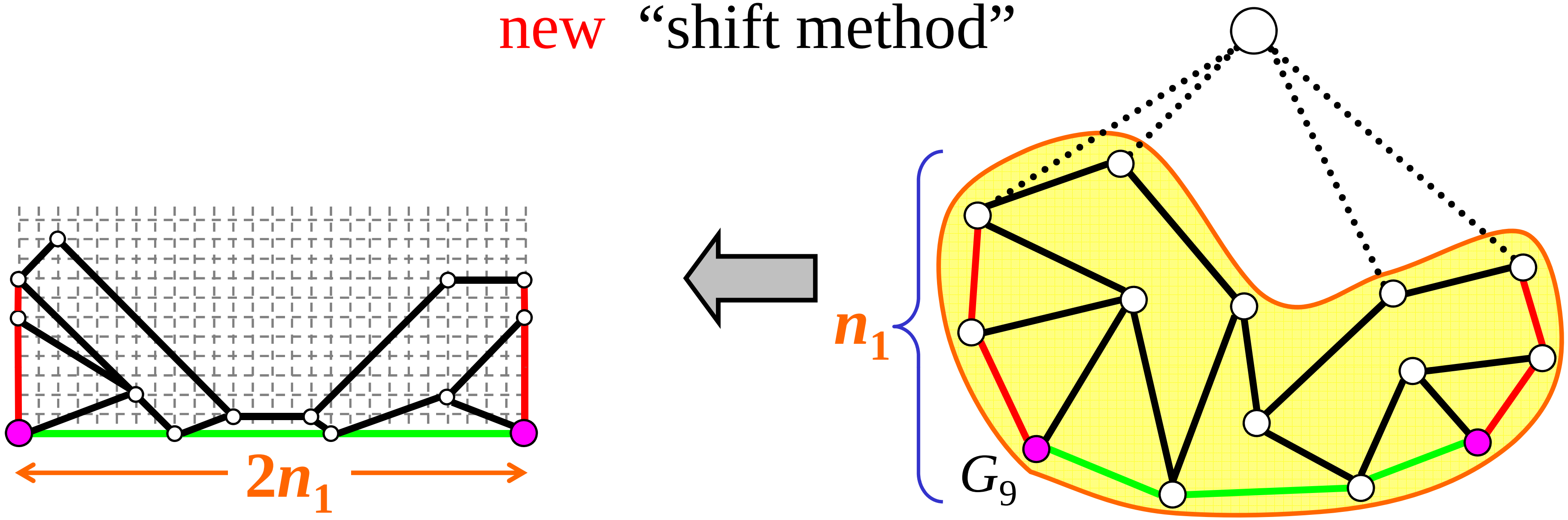
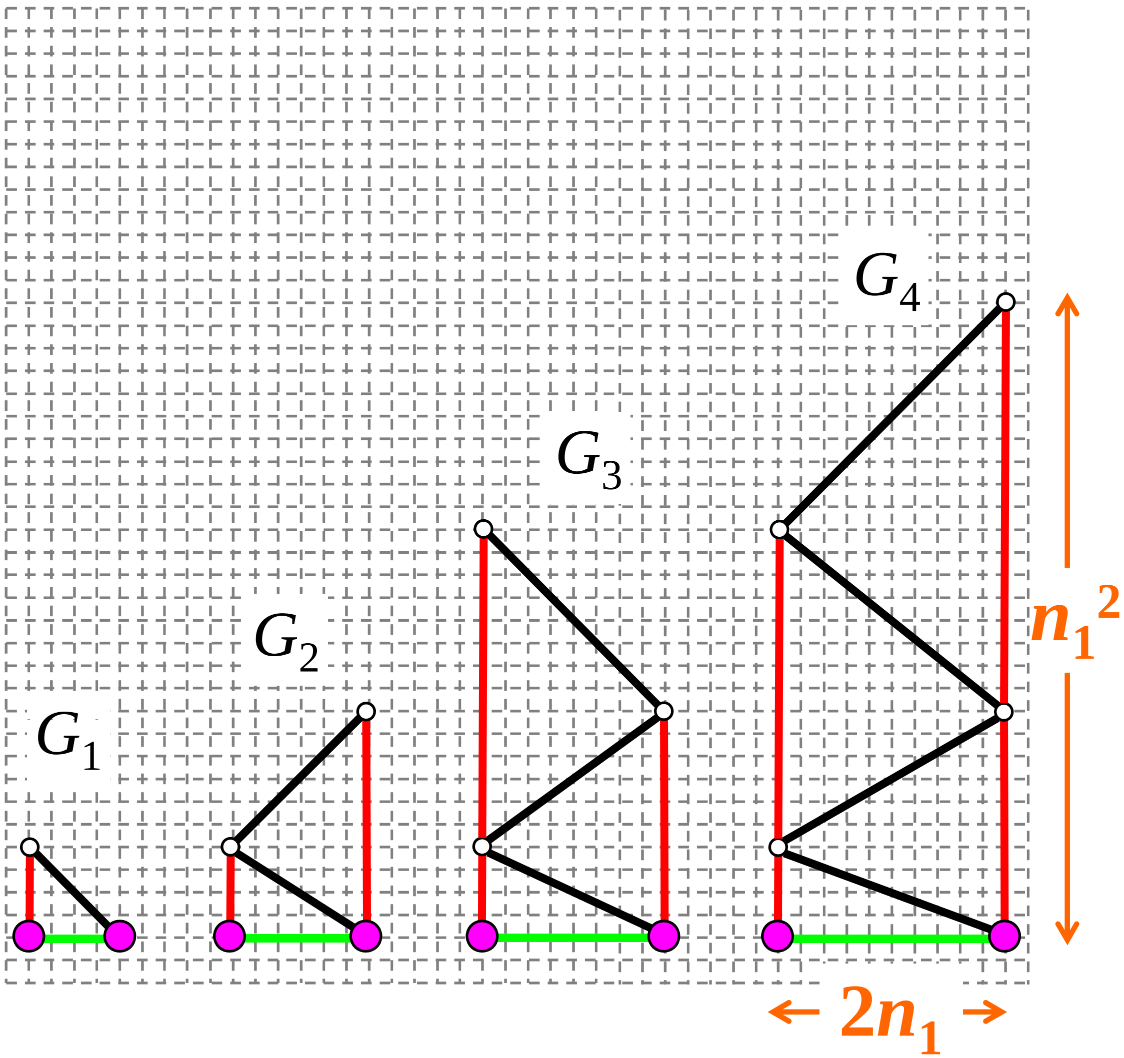
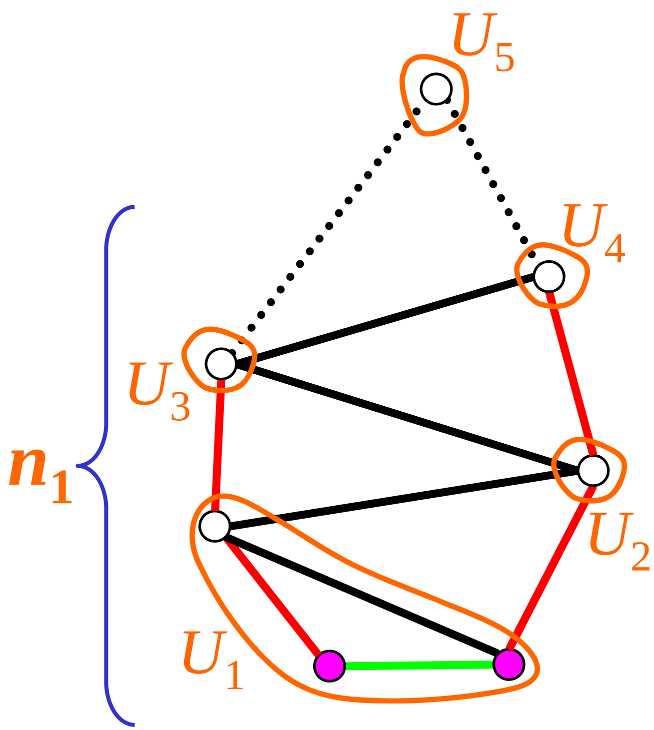


Grid size



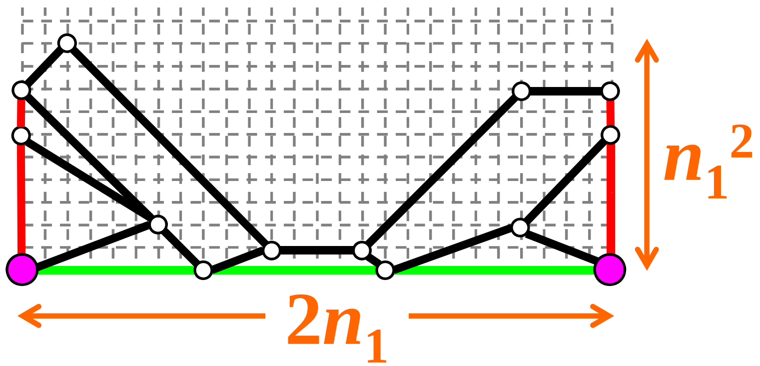


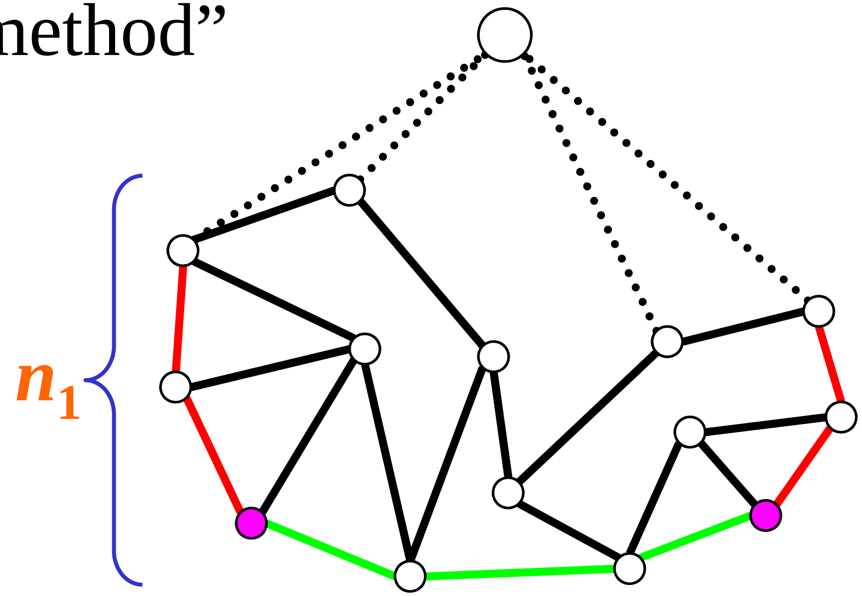
Algorithm

inner convex grid drawing ⇐ internally 3-connected graph

leaves = 2

new “shift method”





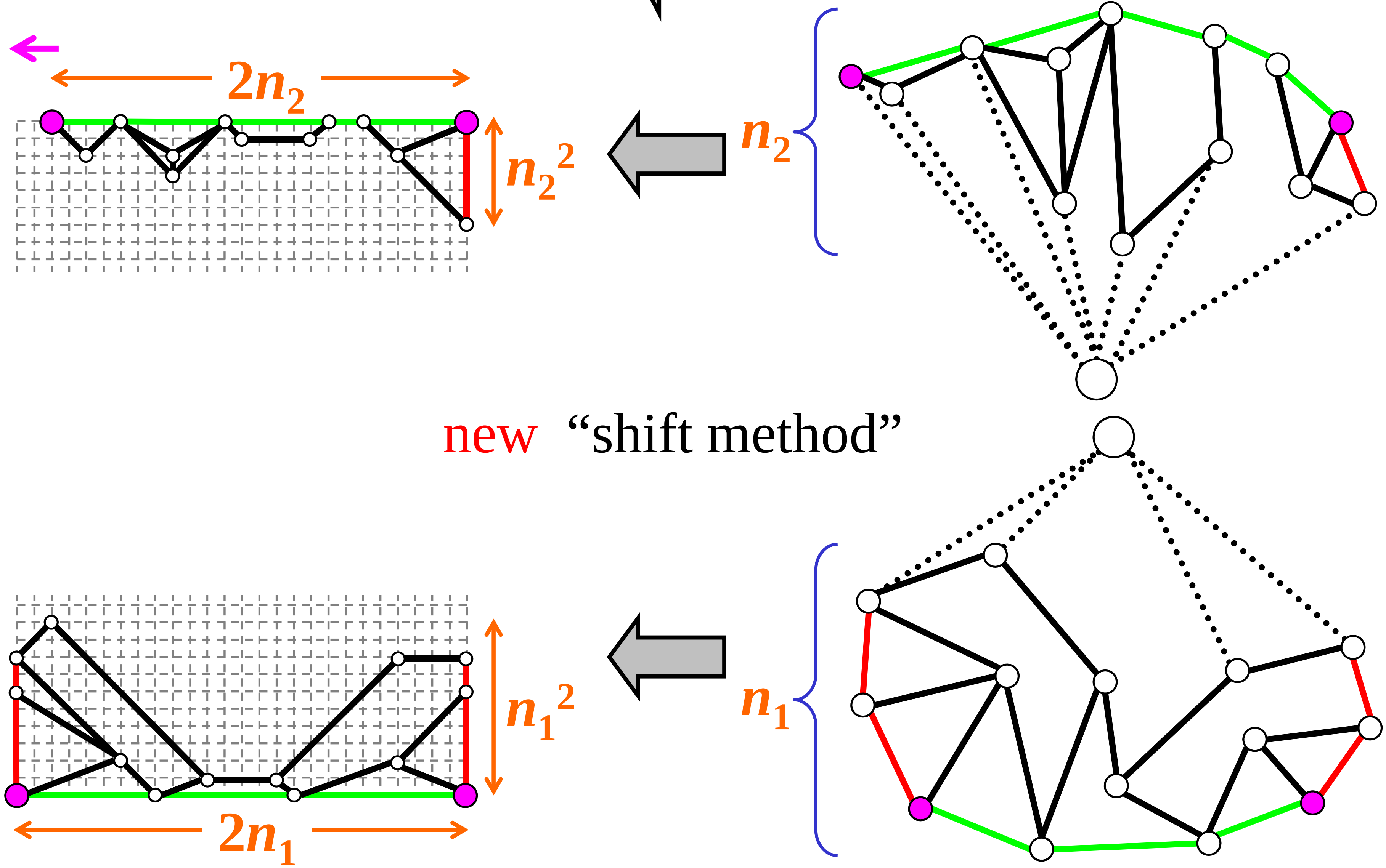
Algorithm

internally 3-connected graphs

leaves $= 2$

inner convex grid drawings

new "shift method"

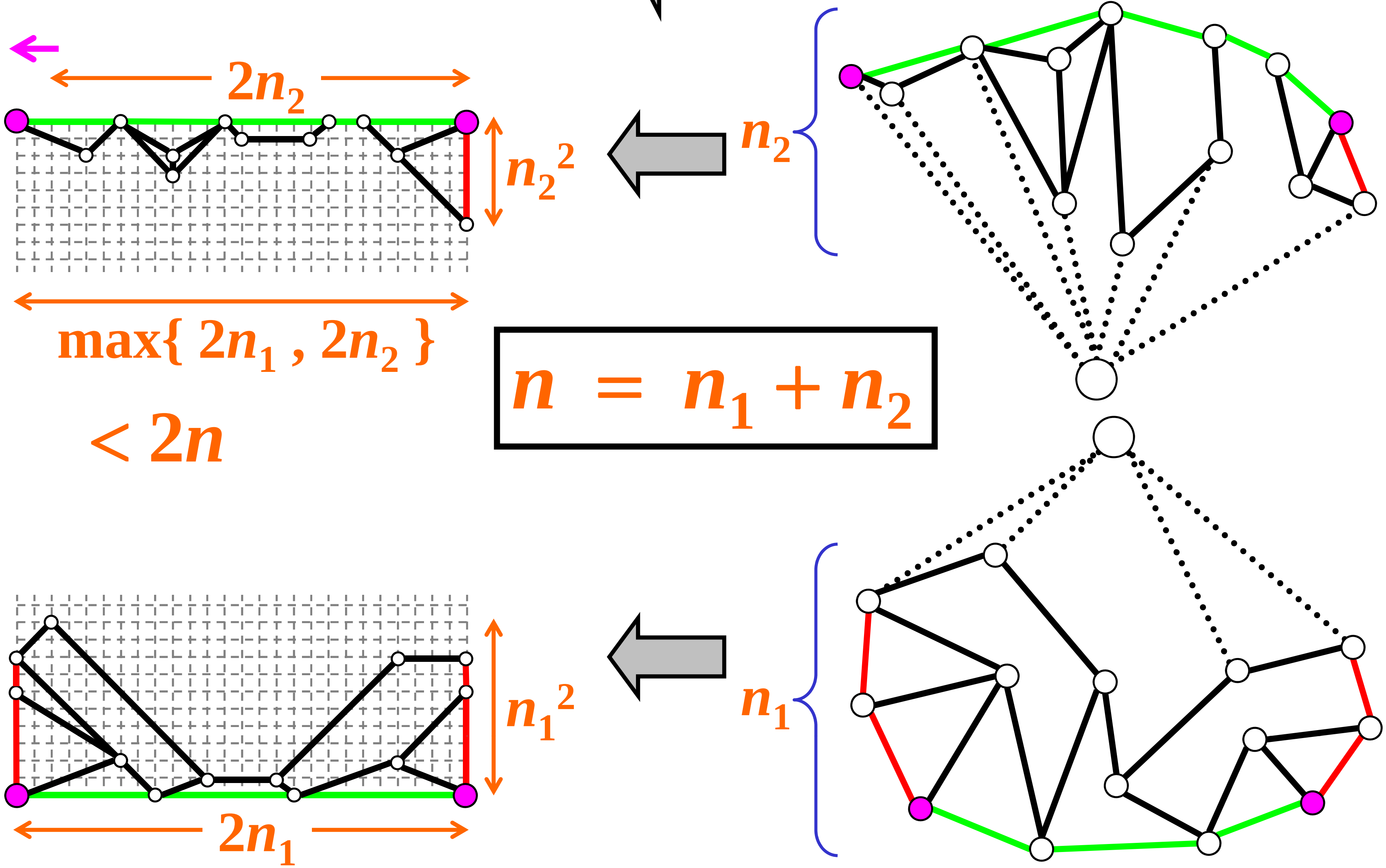


Algorithm

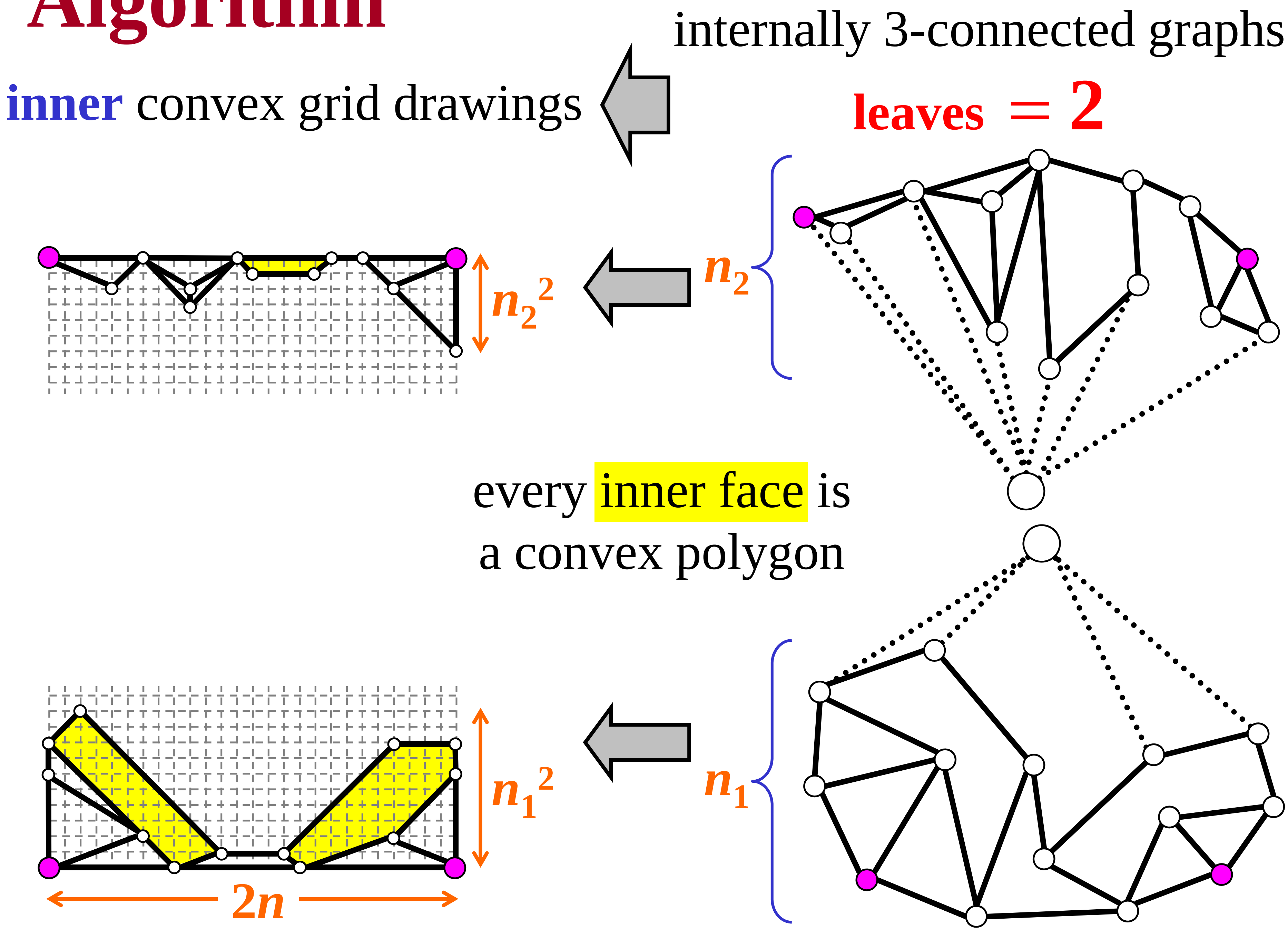
internally 3-connected graphs

inner convex grid drawings

leaves = 2



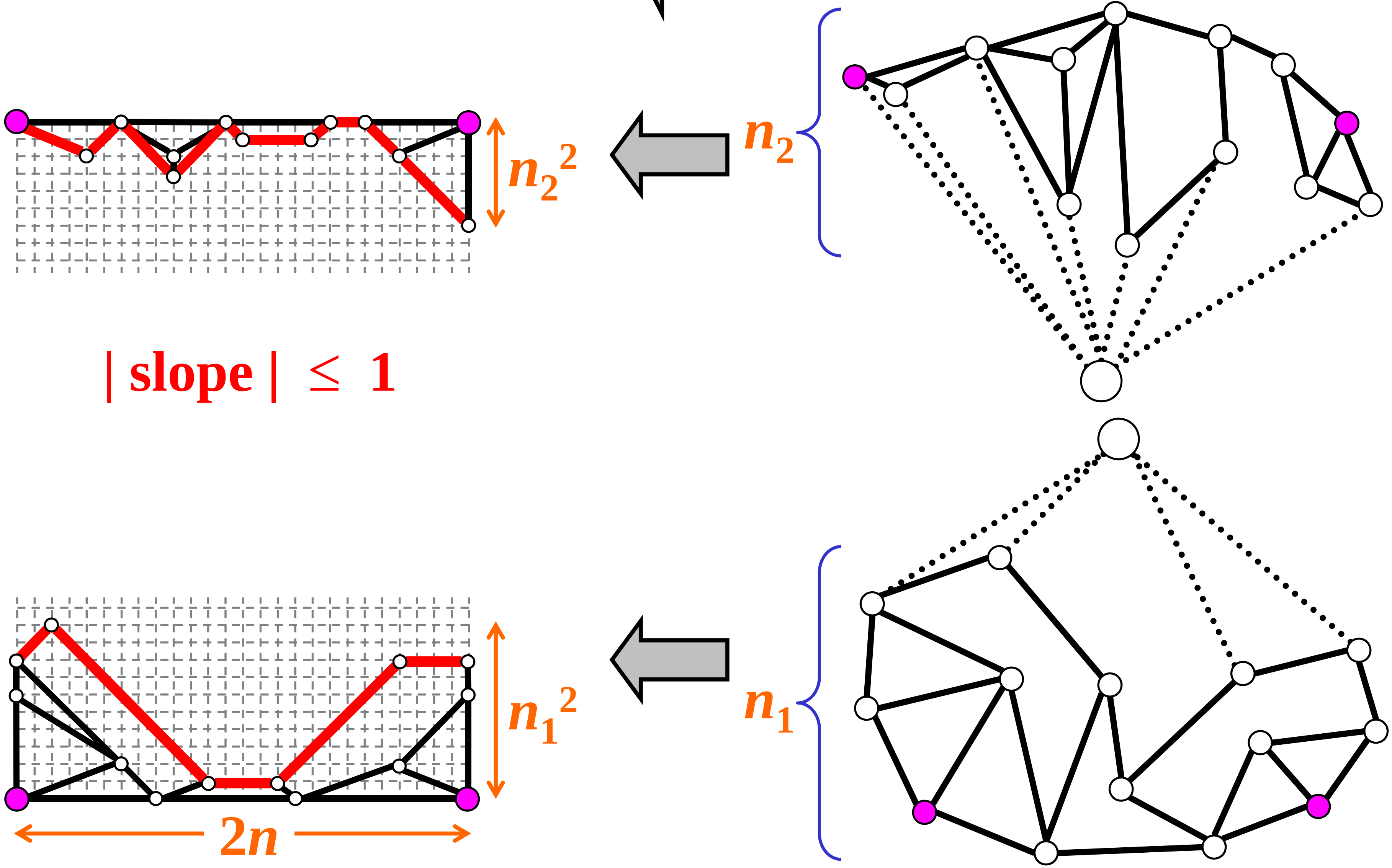
Algorithm


internally 3-connected graphs
inner convex grid drawings
leaves = 2
n_2
n_2^2
every inner face is a convex polygon
n_1
n_1^2
$2n$

Algorithm

inner convex grid drawings ⟵ internally 3-connected graphs

leaves = 2

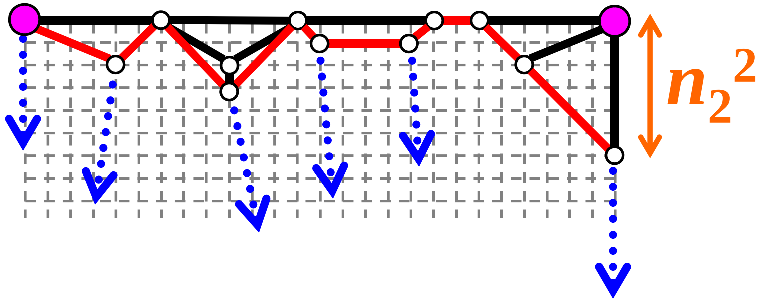


Algorithm

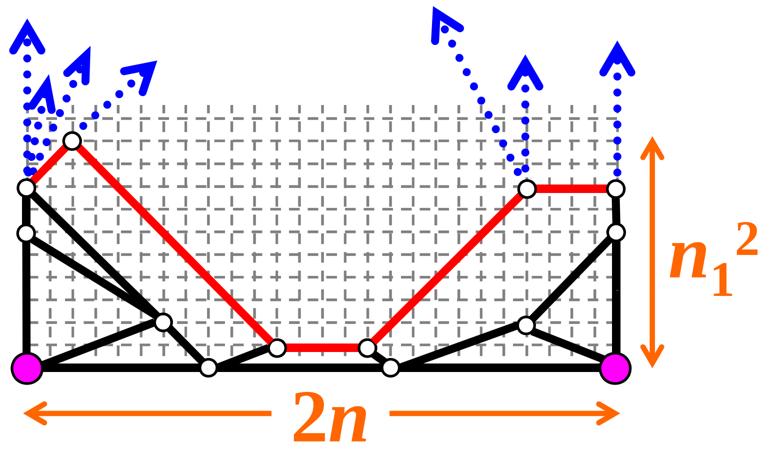
internally 3-connected graphs

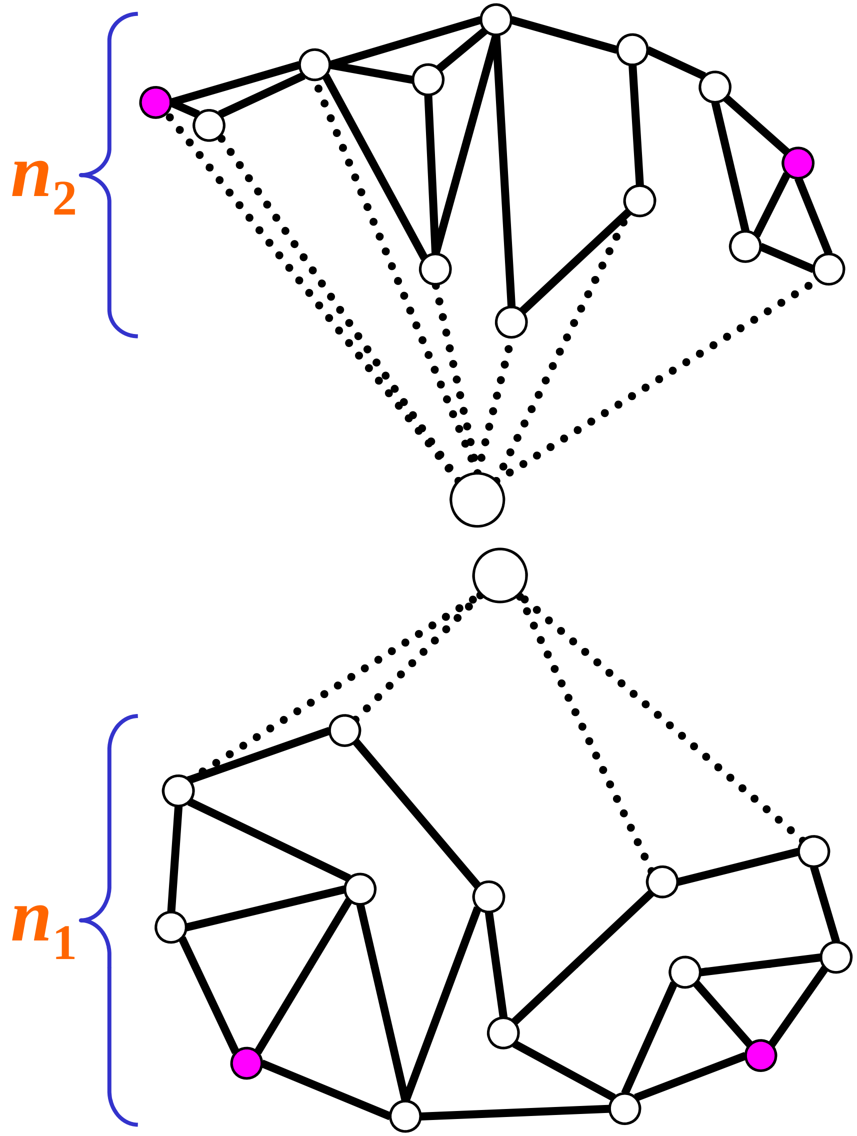
inner convex grid drawings

leaves $= 2$



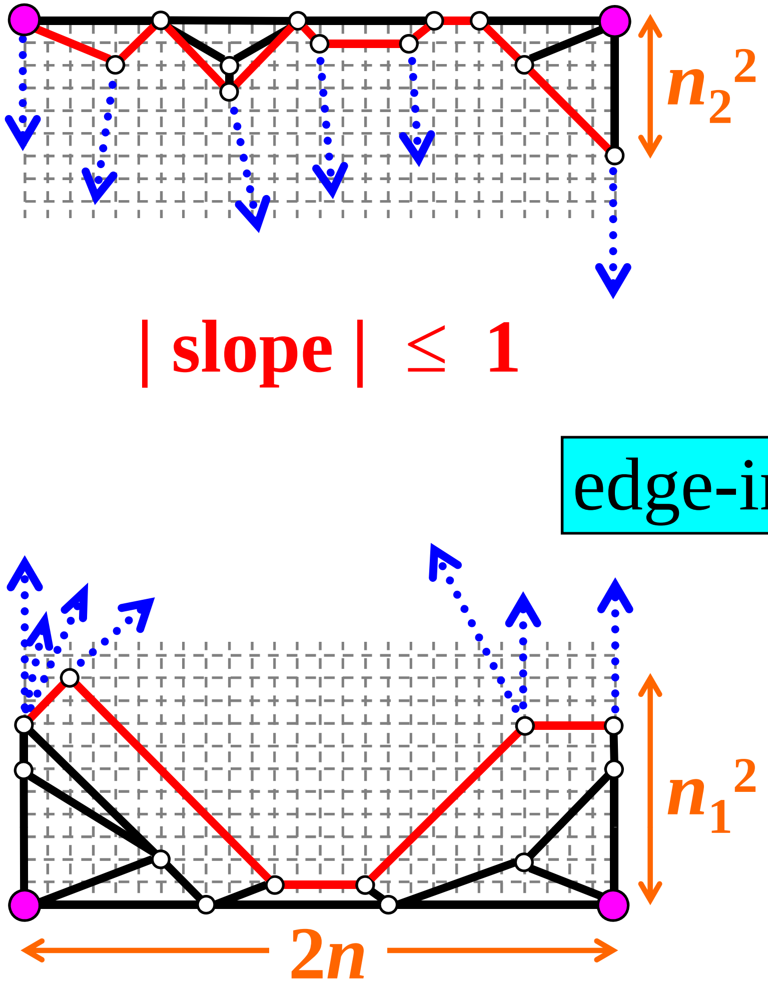
$|$ slope $|\ \leq\ 1$

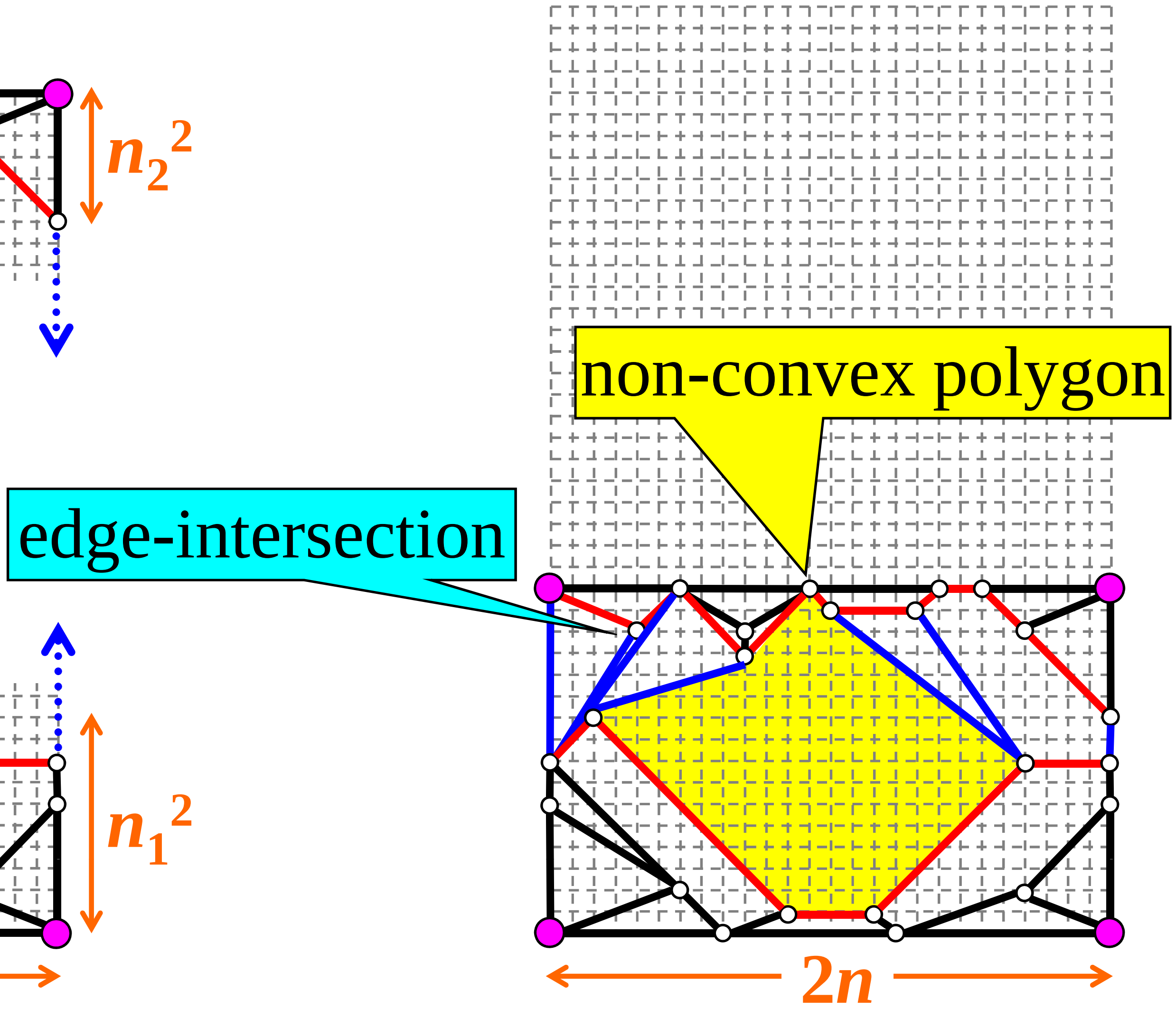




Algorithm

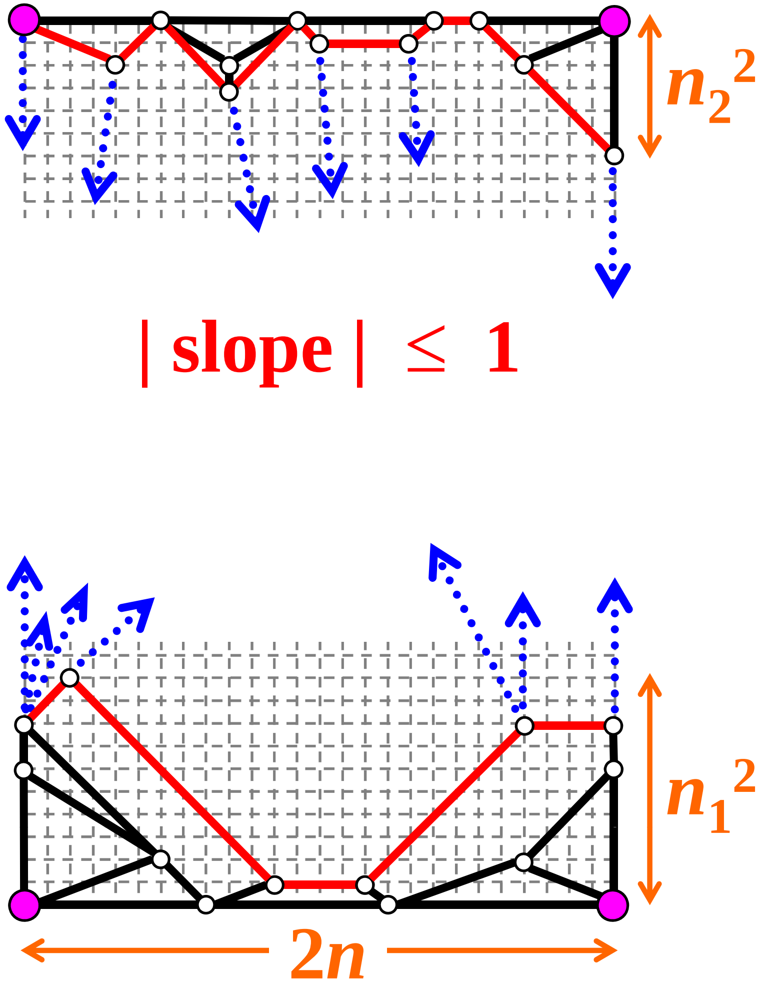
inner convex grid drawings ⇒ convex grid drawing

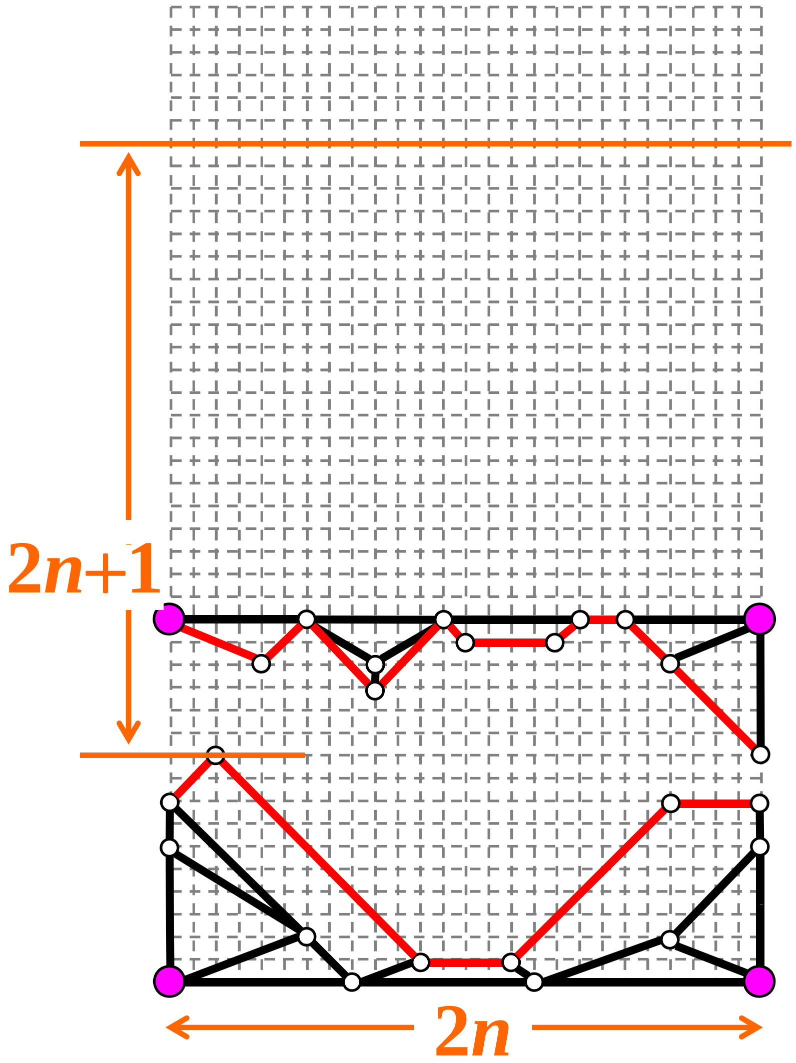




Algorithm

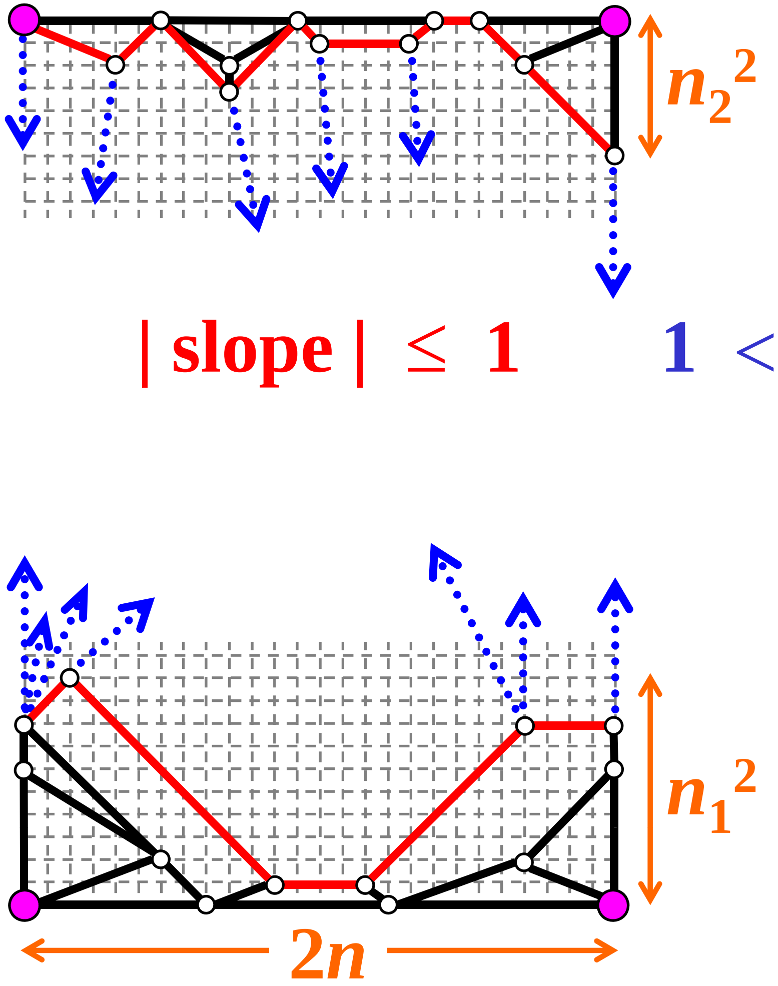
inner convex grid drawings ⇒ convex grid drawing





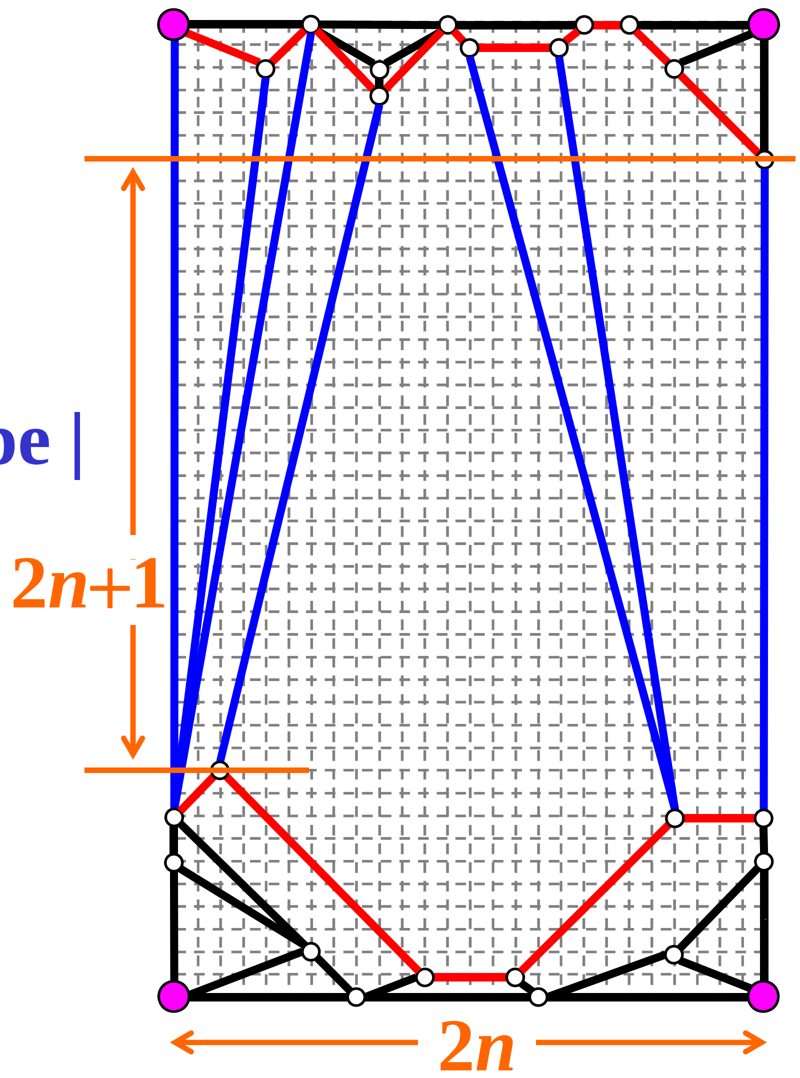
Algorithm

inner convex grid drawings ⇨ convex grid drawing

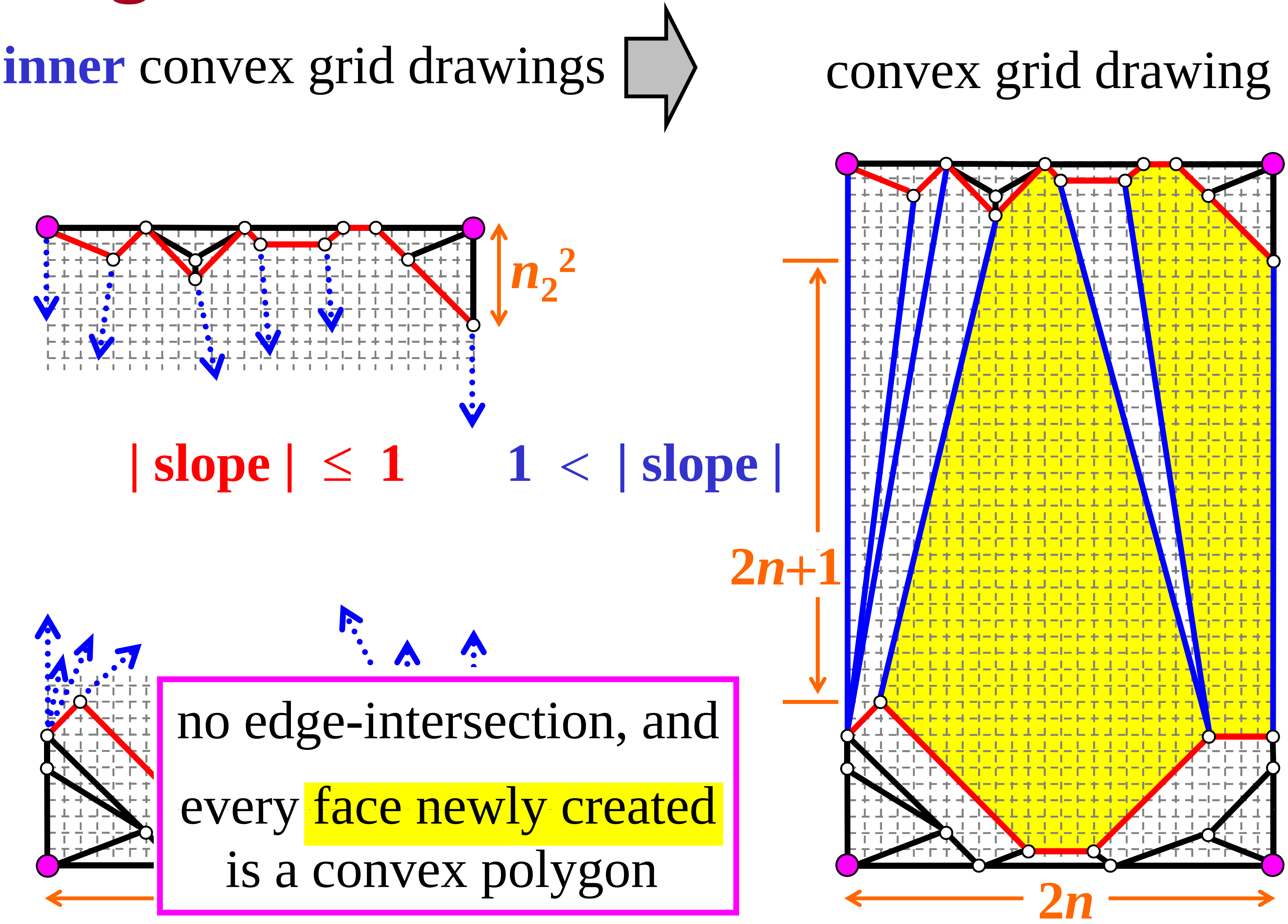


| slope | ≤ 1

1 < | slope |



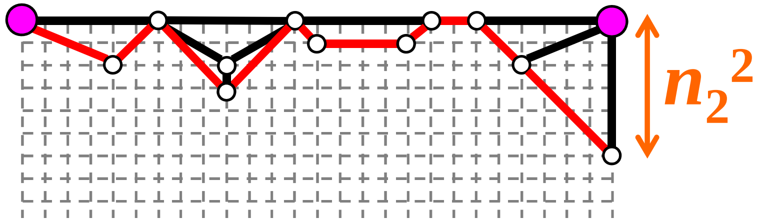
Algorithm



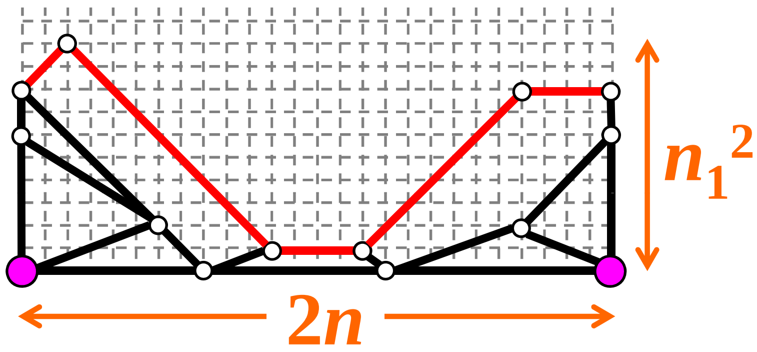
inner convex grid drawings
convex grid drawing
n_2^2
| slope | ≤ 1
1 < | slope |
2n+1
2n
no edge-intersection, and
every face newly created
is a convex polygon

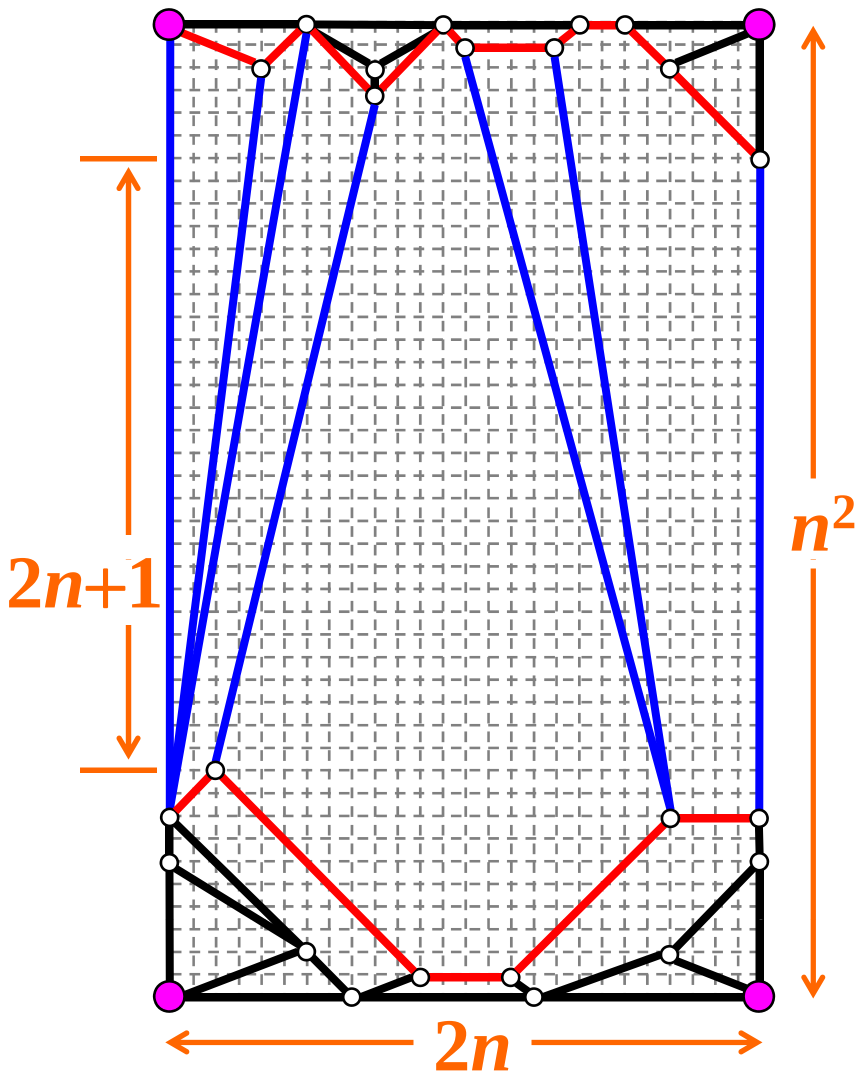
Algorithm

inner convex grid drawings → convex grid drawing

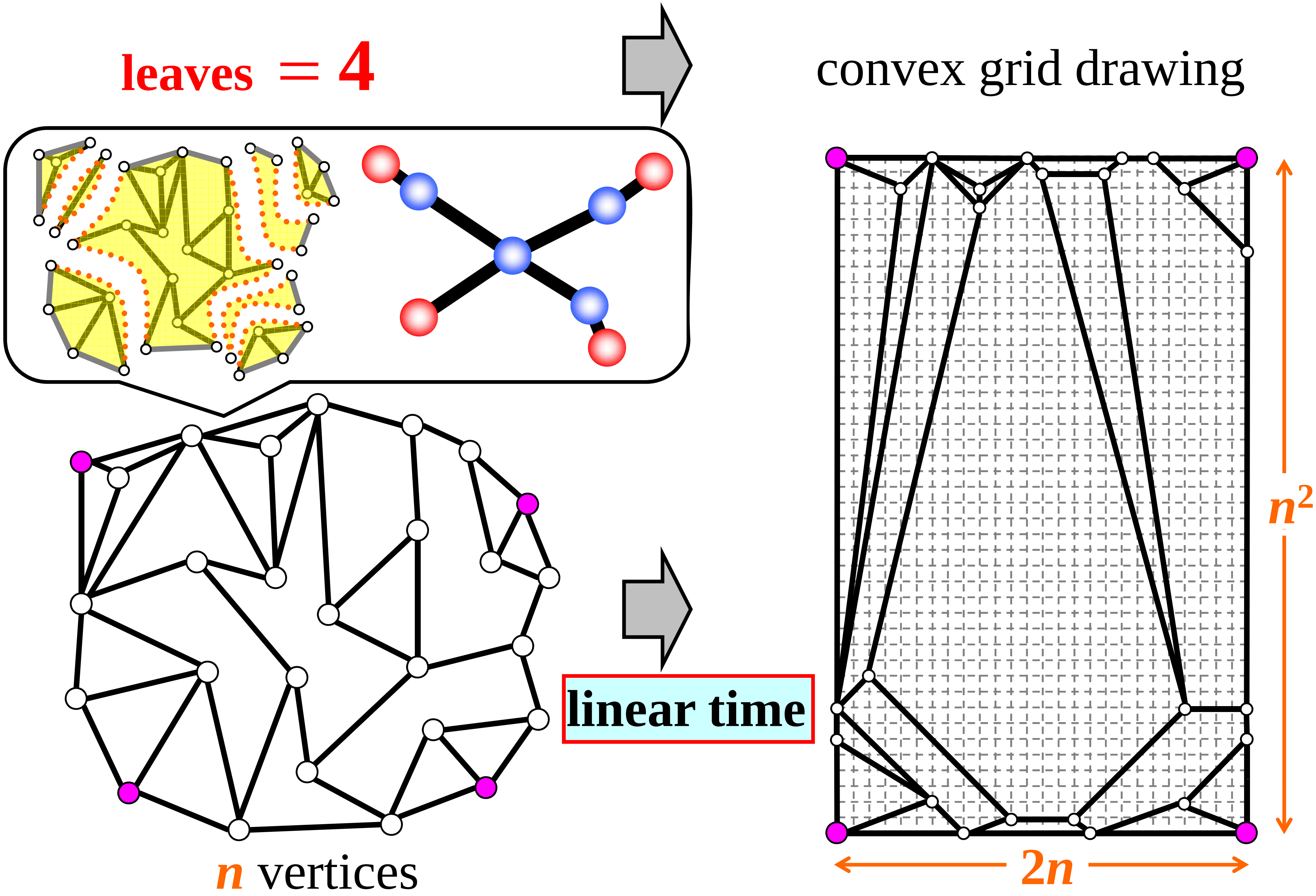


$$H = n_1^2 + 2n+1 + n_2^2$$
$$< n^2$$

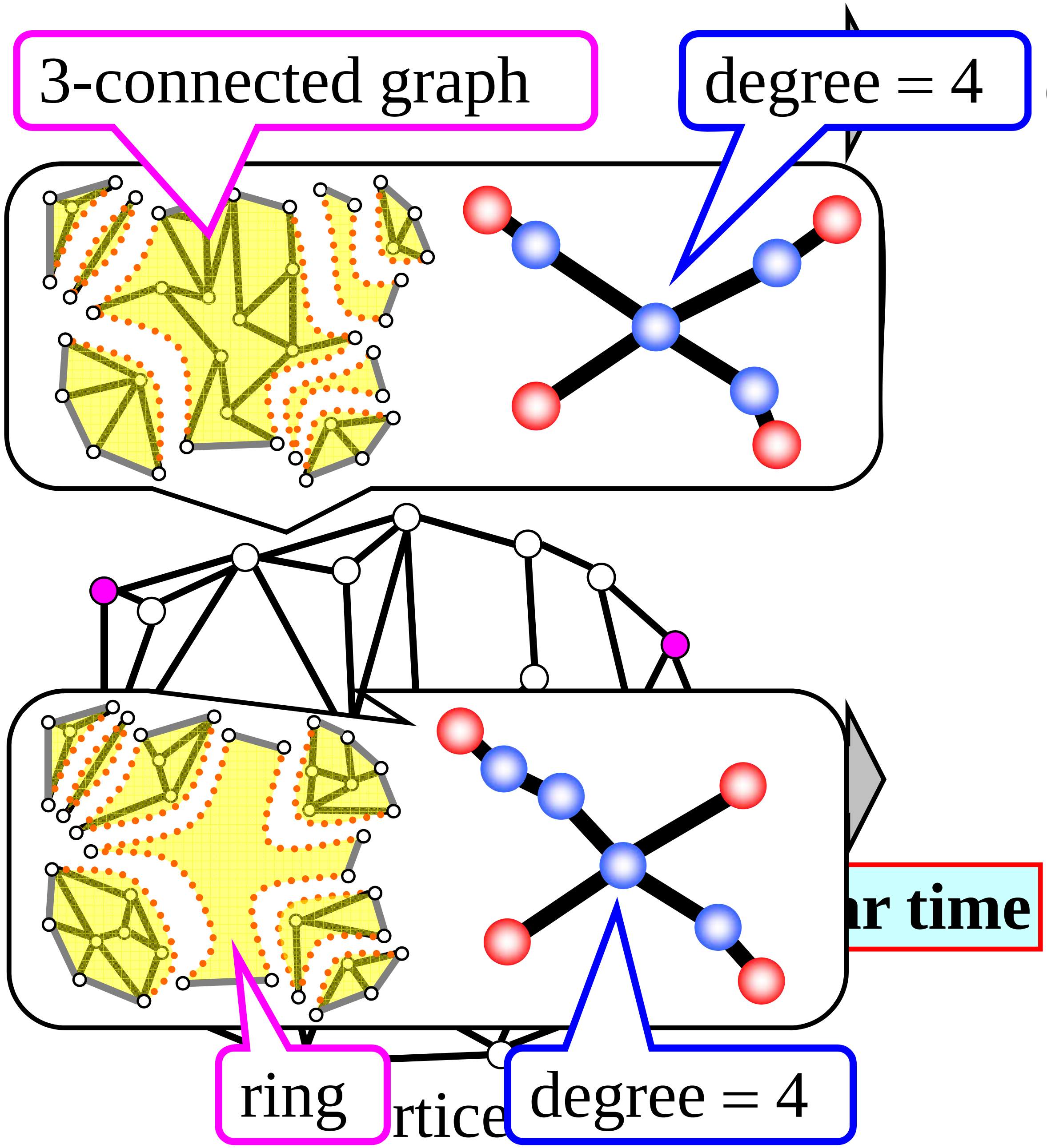




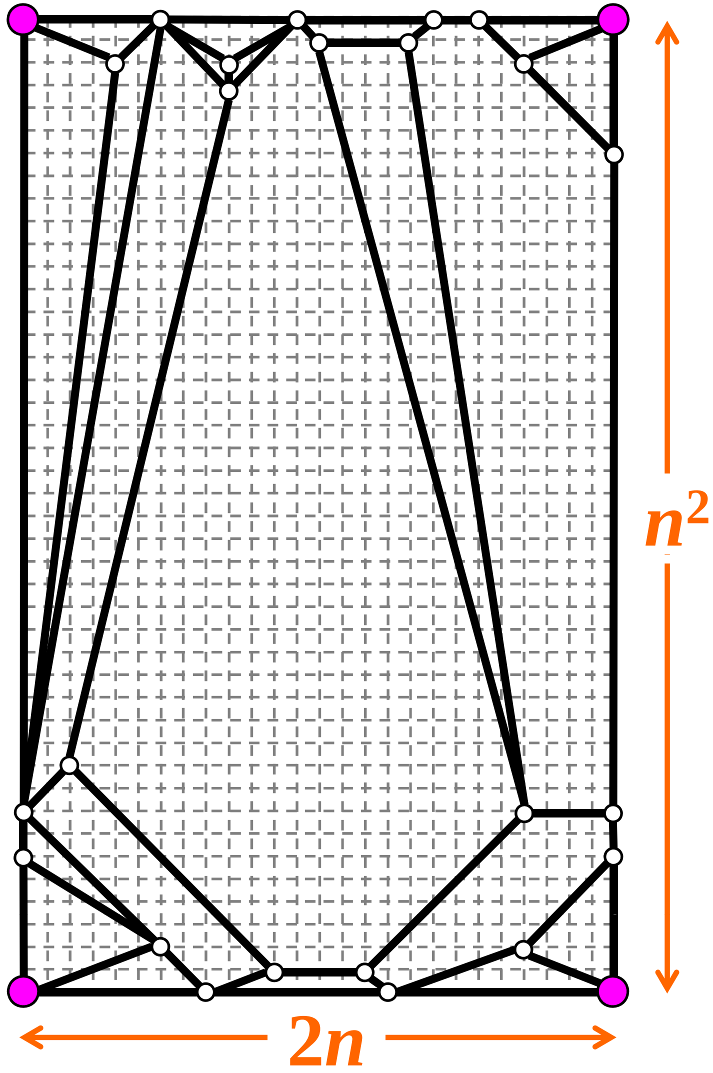
Conclusions



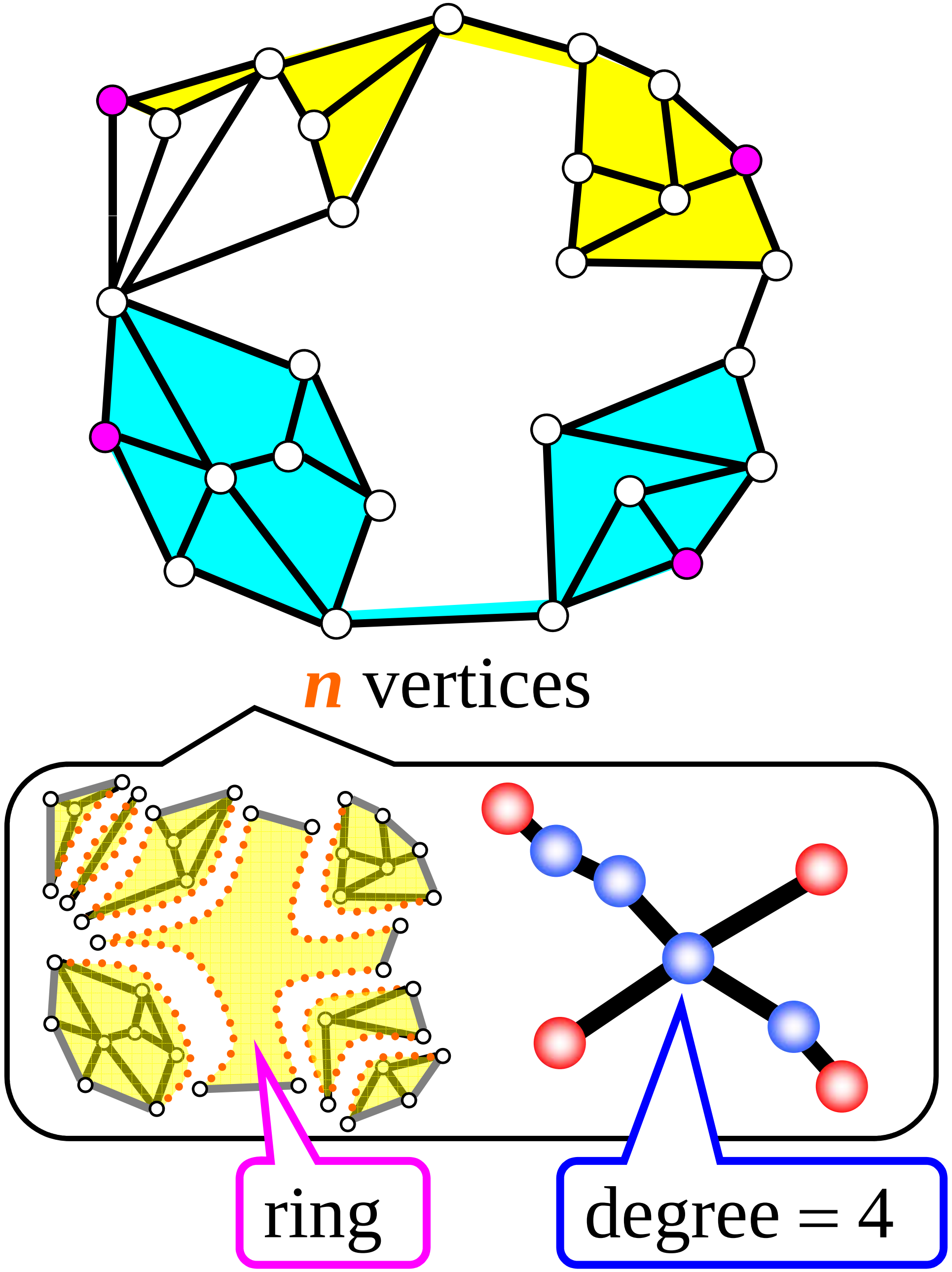
Conclusions

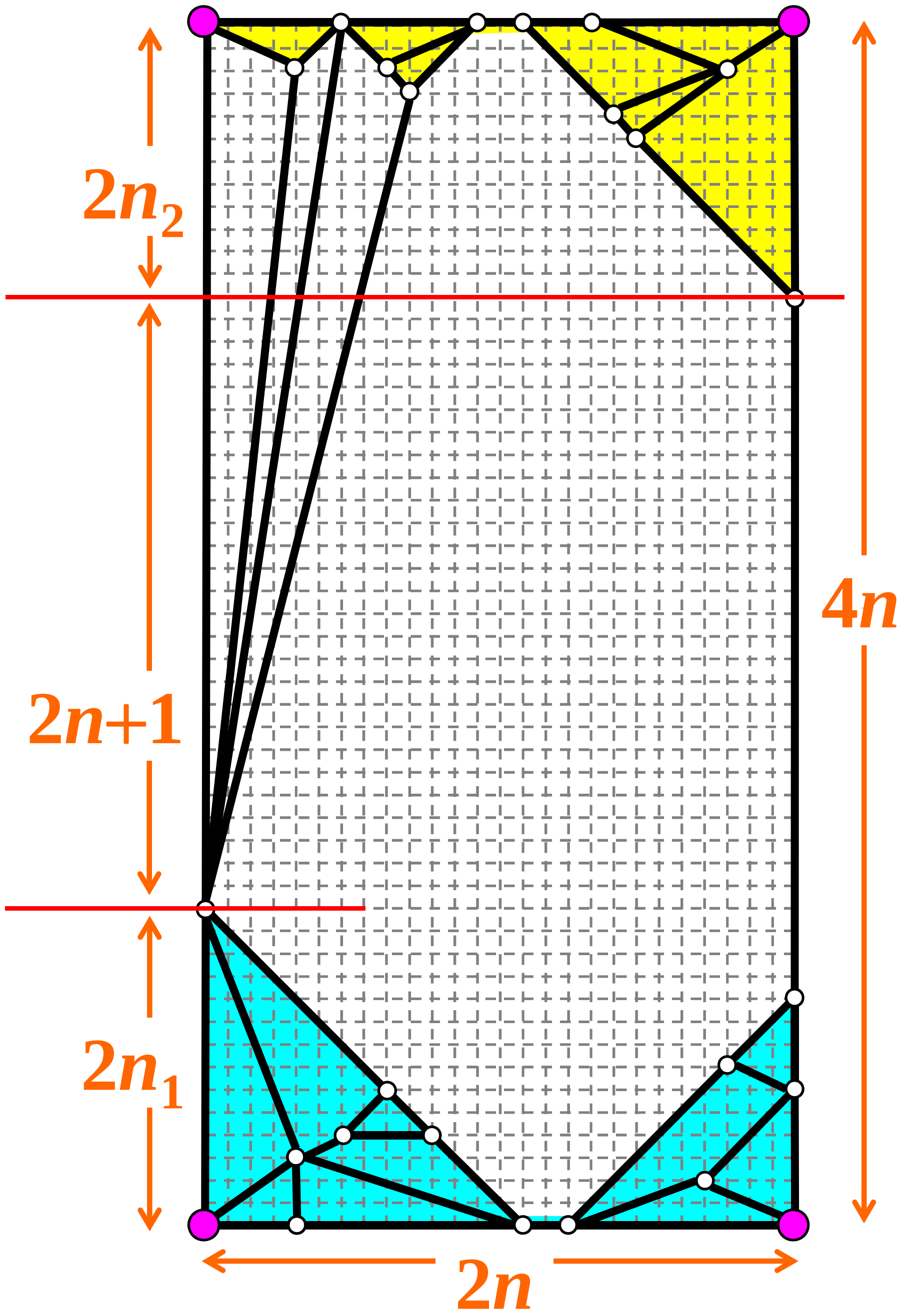


convex grid drawing

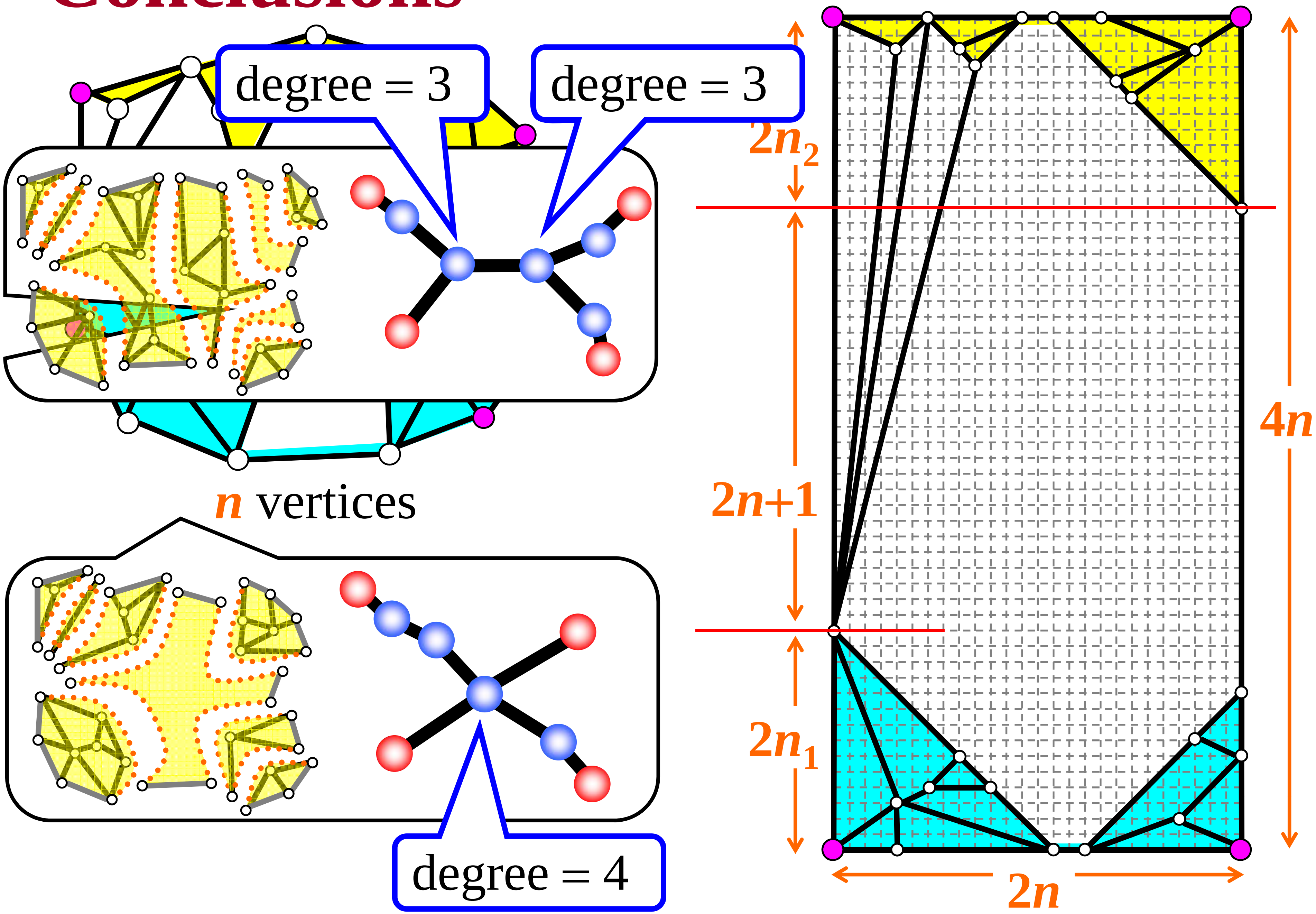


Conclusions

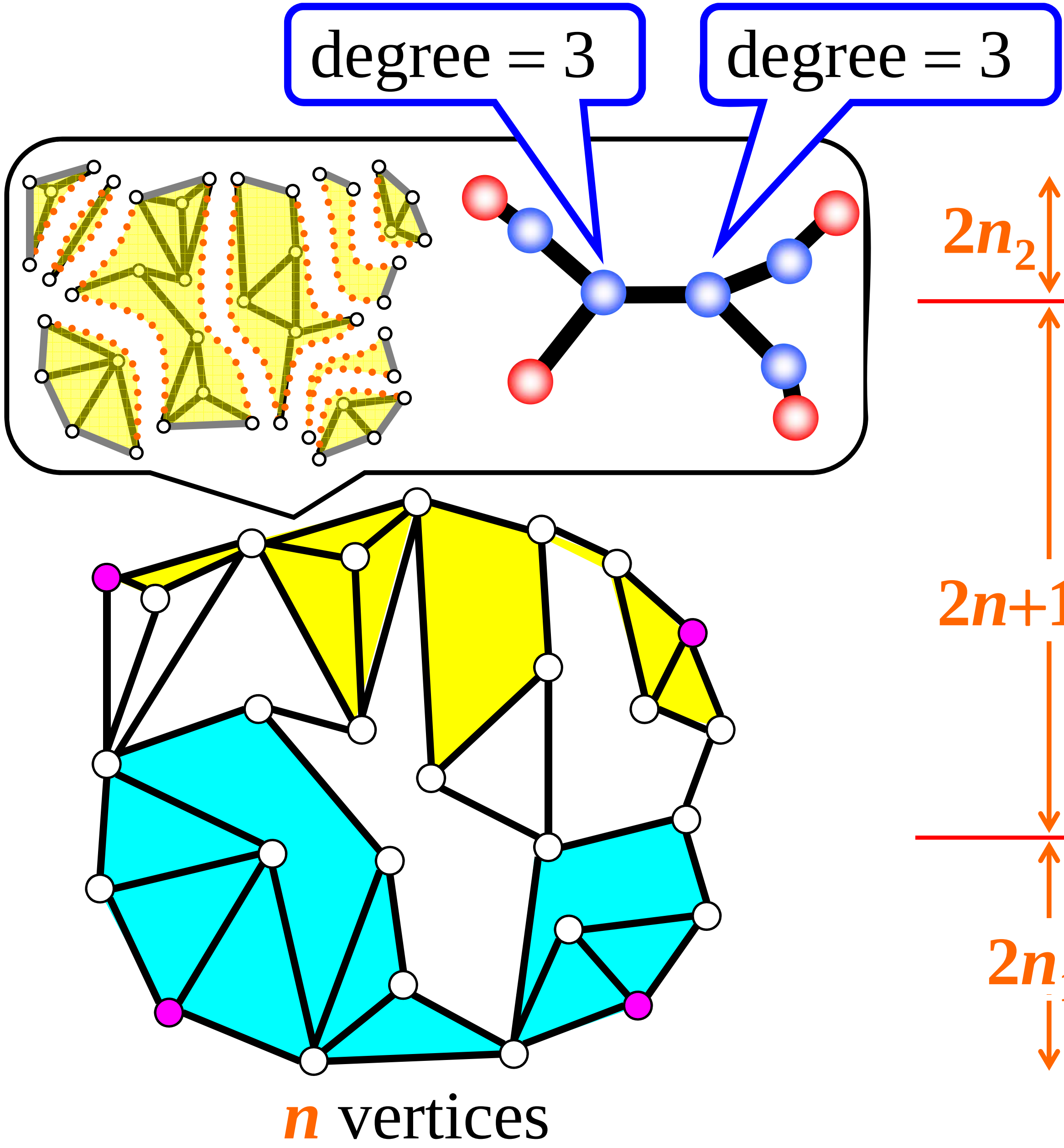


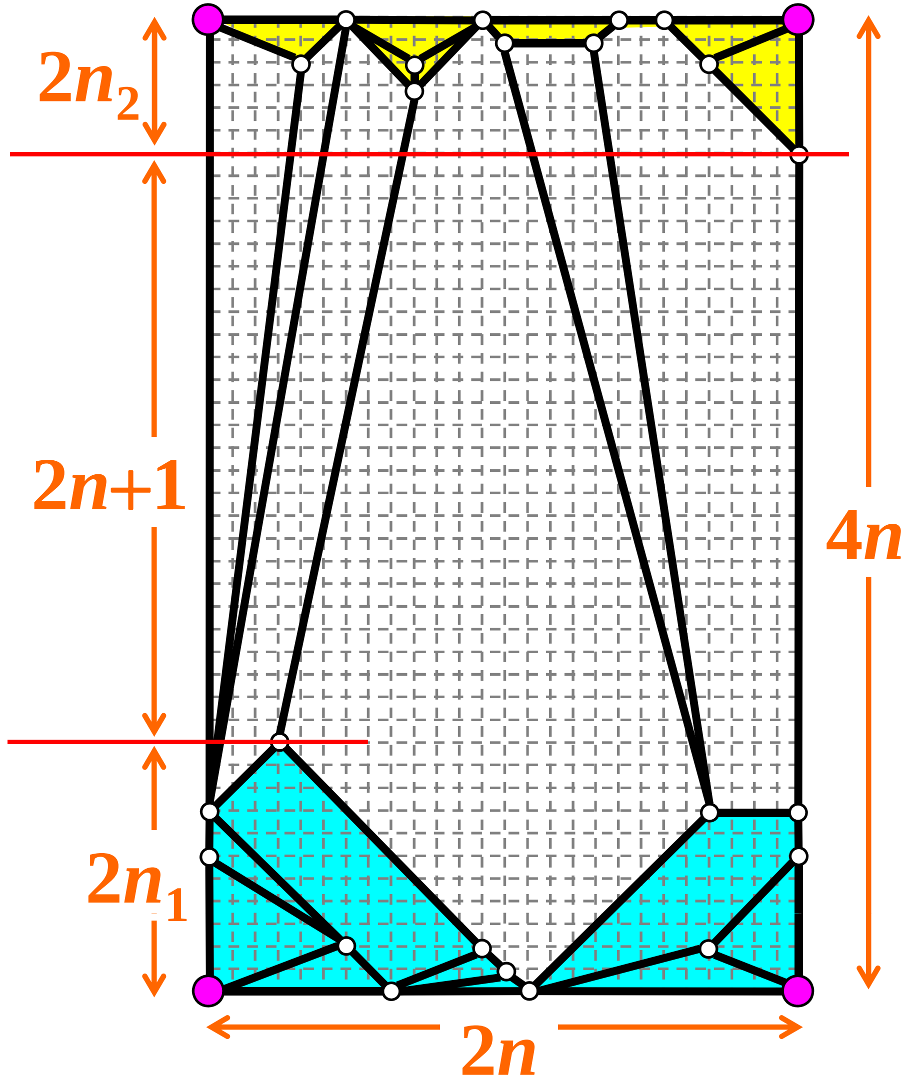


Conclusions

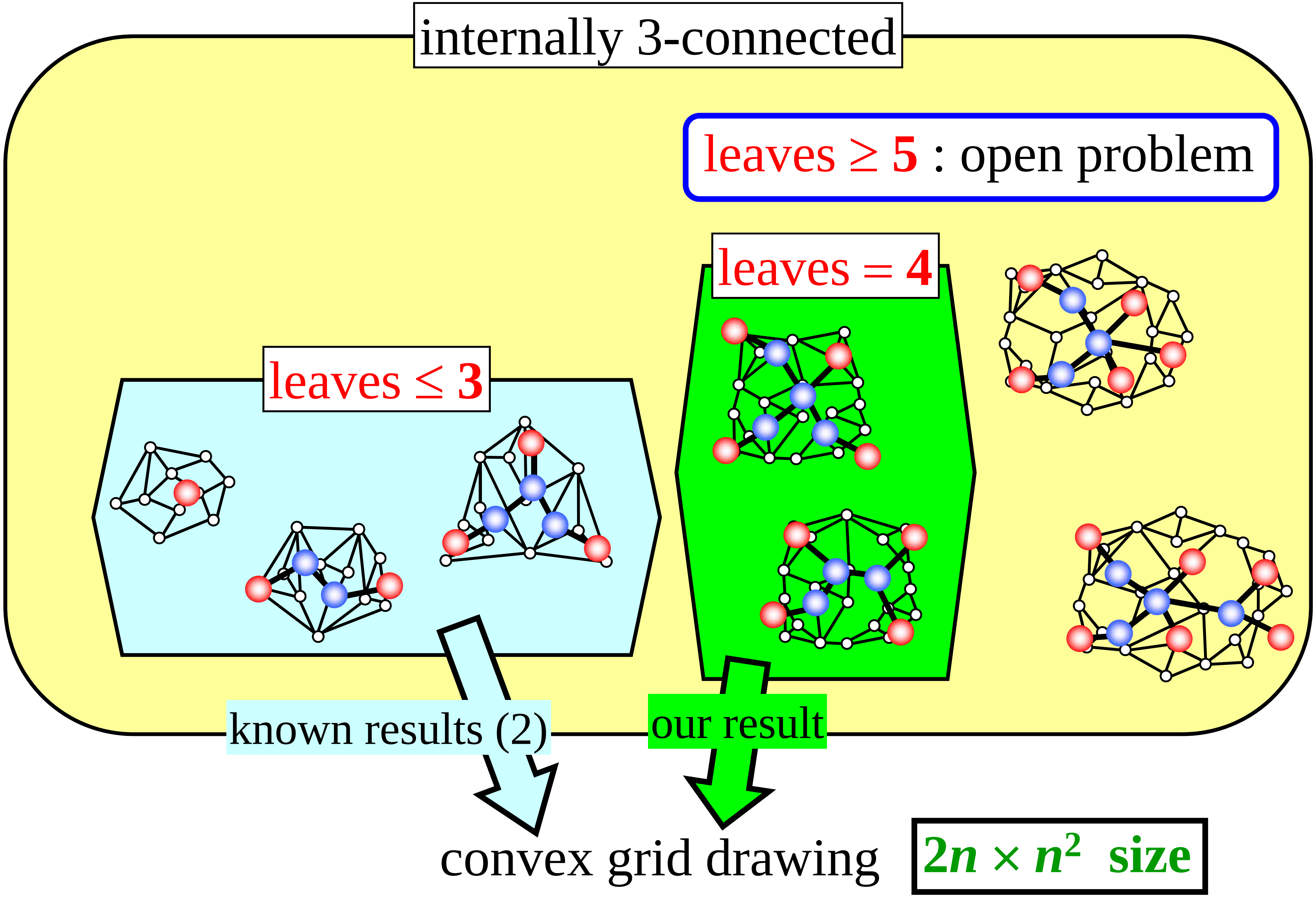


Conclusions

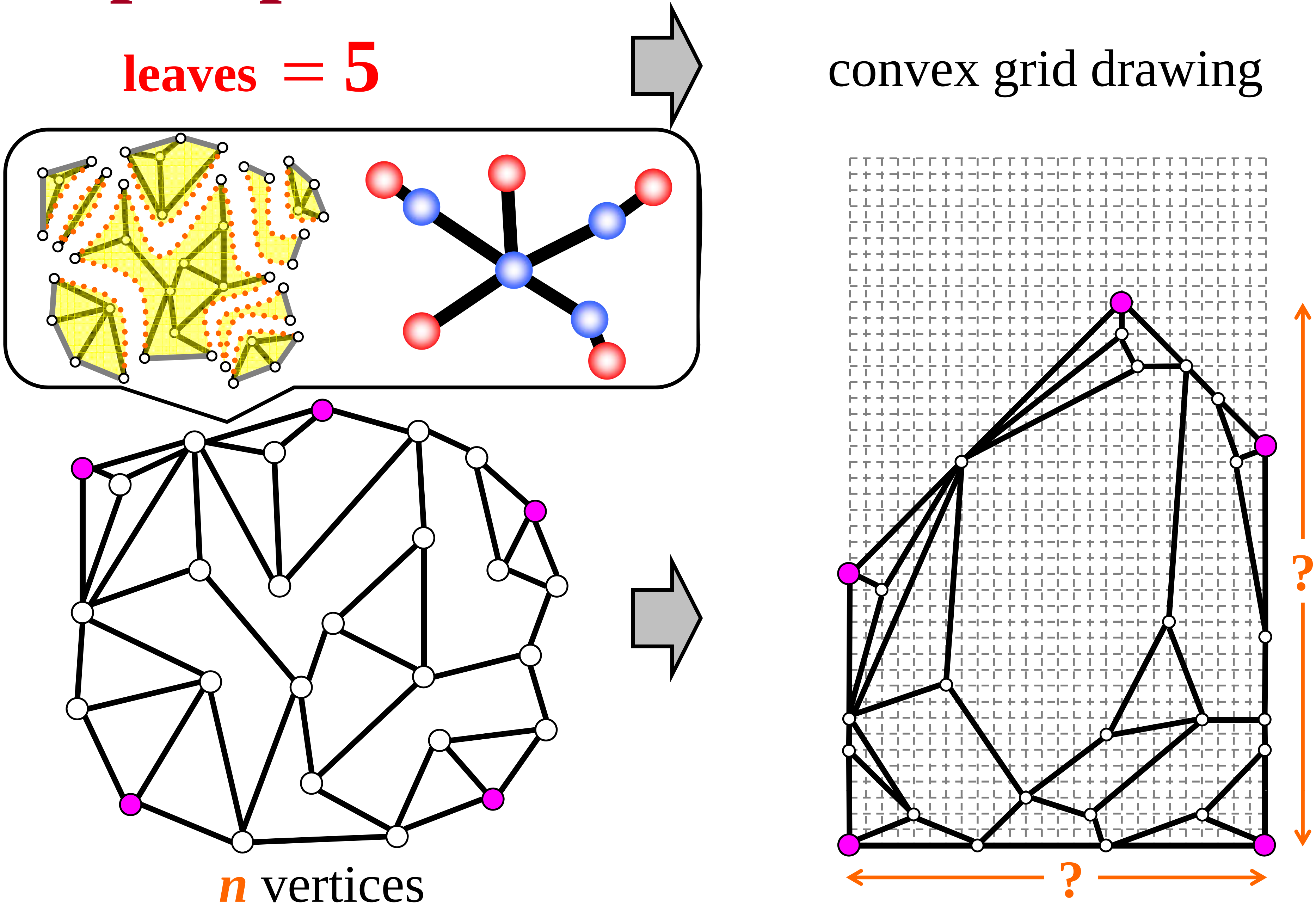




Open problem



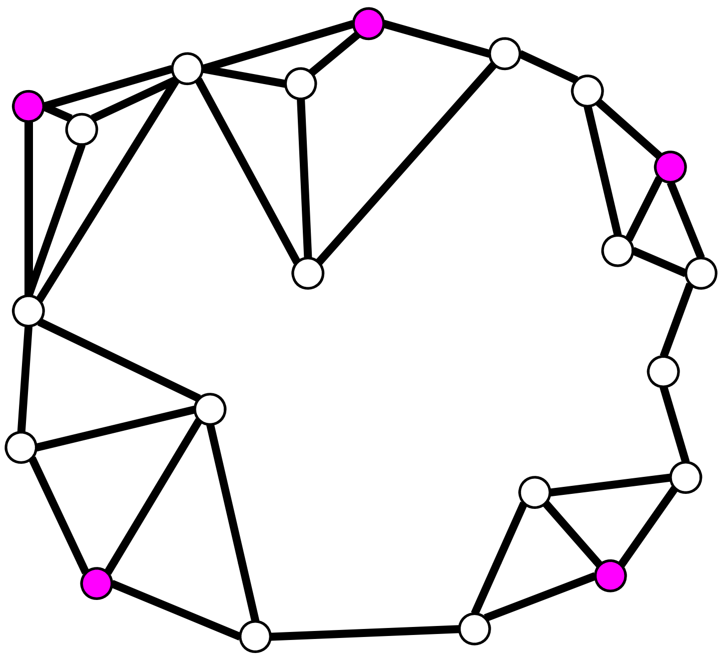
Open problem



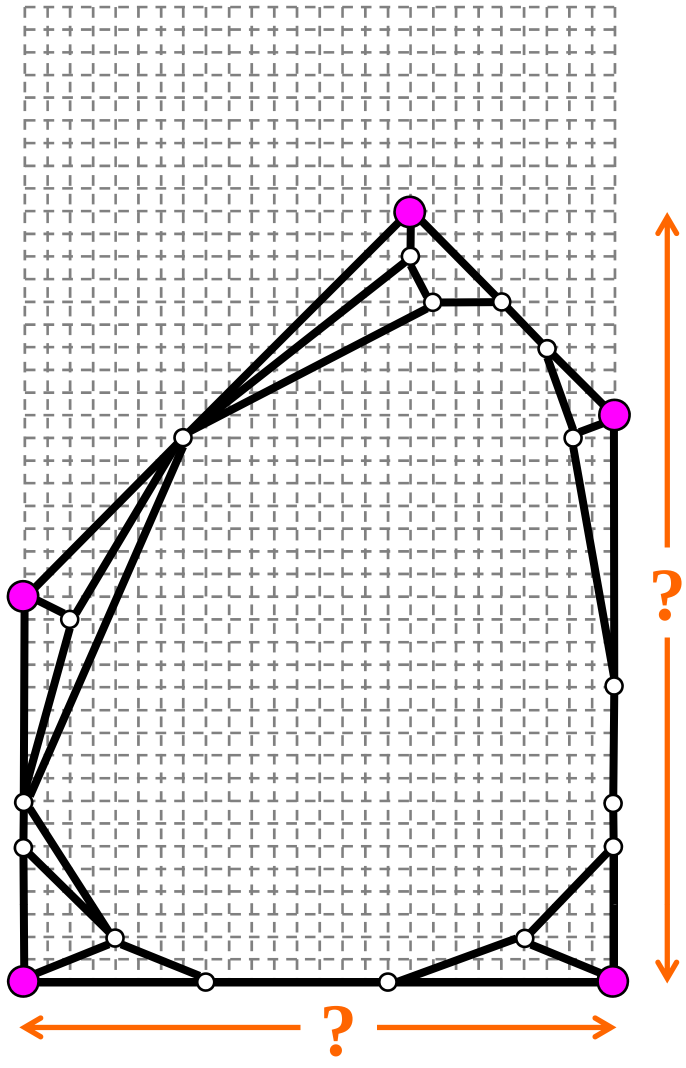
Open problem

leaves $= 5$

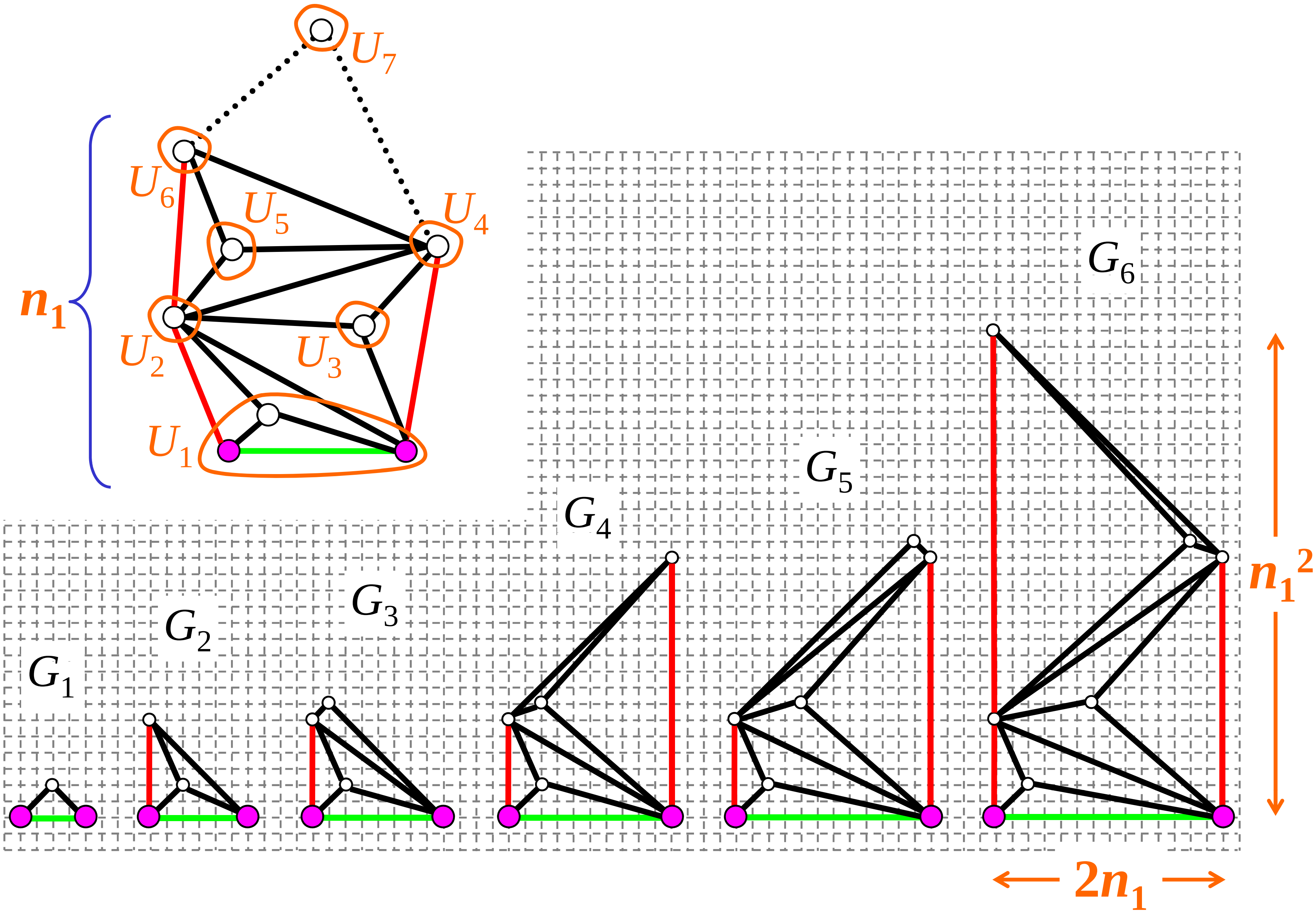
convex grid drawing



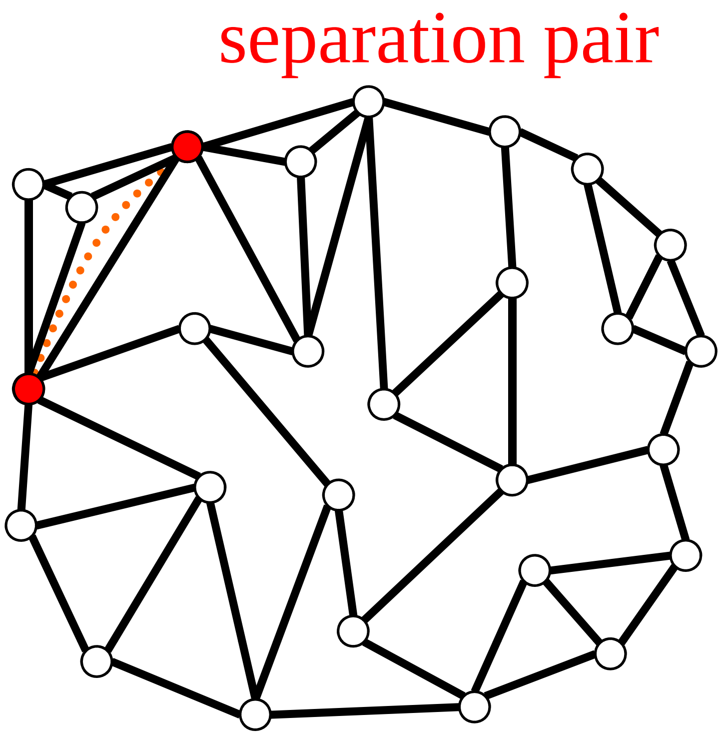
n vertices



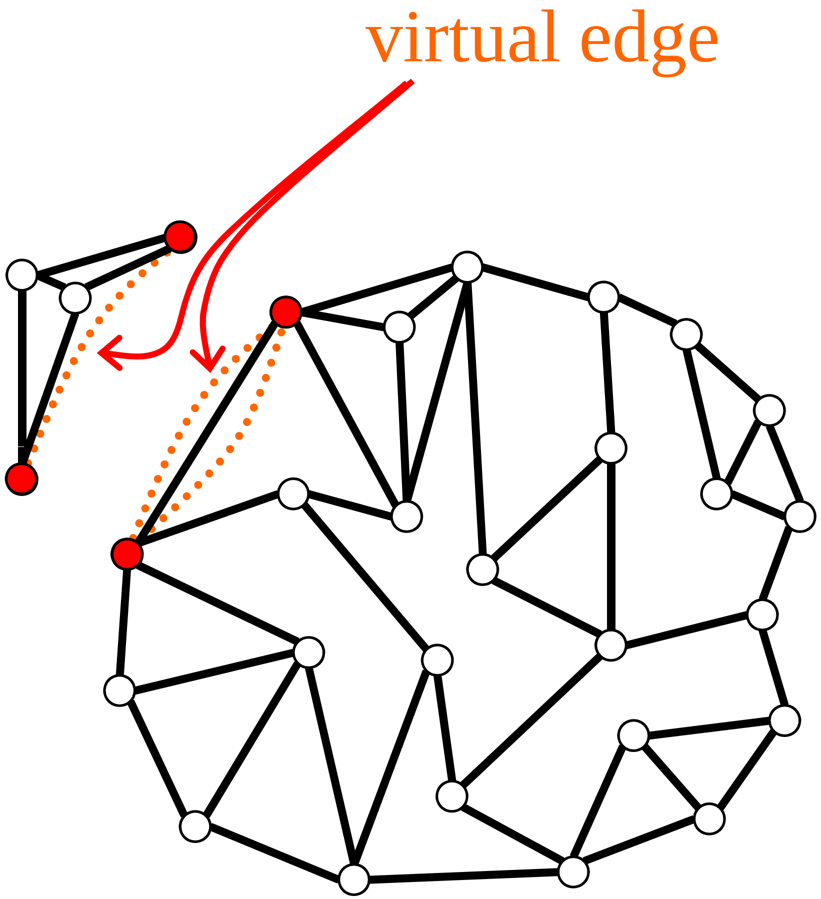
Grid size



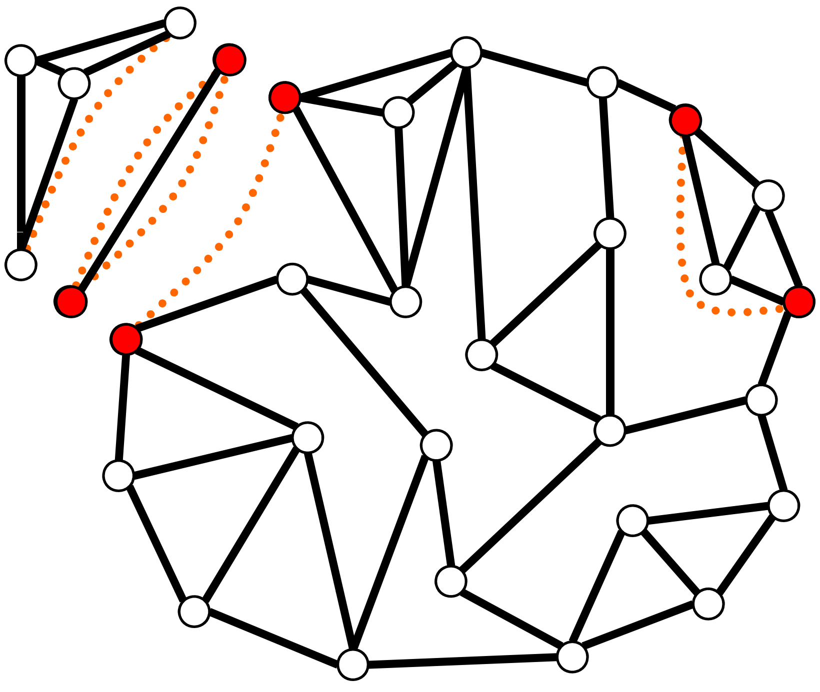
Decomposition tree



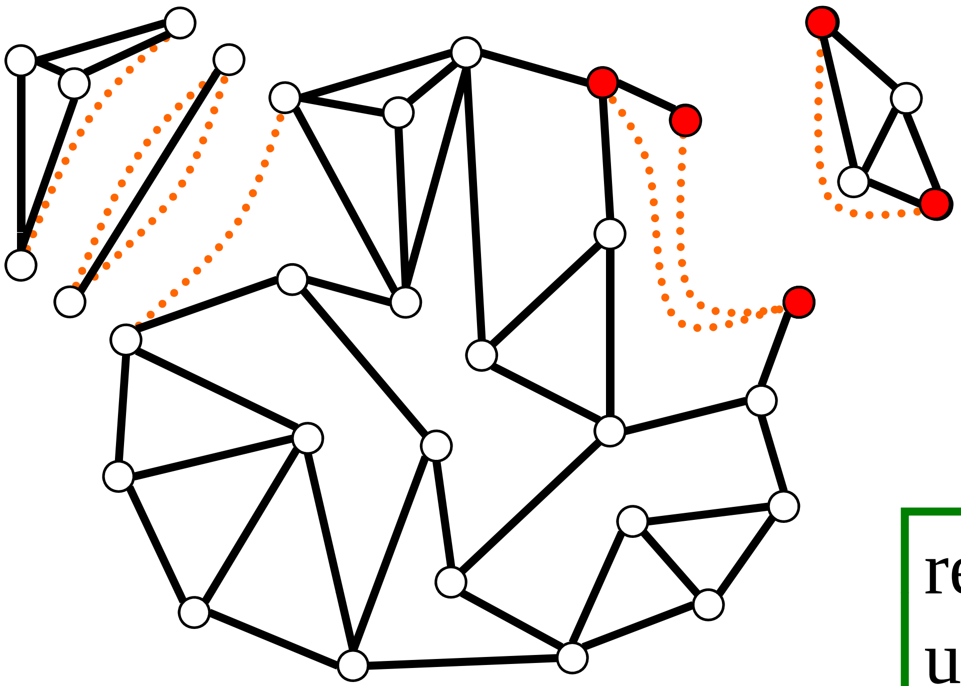
Decomposition tree



Decomposition tree

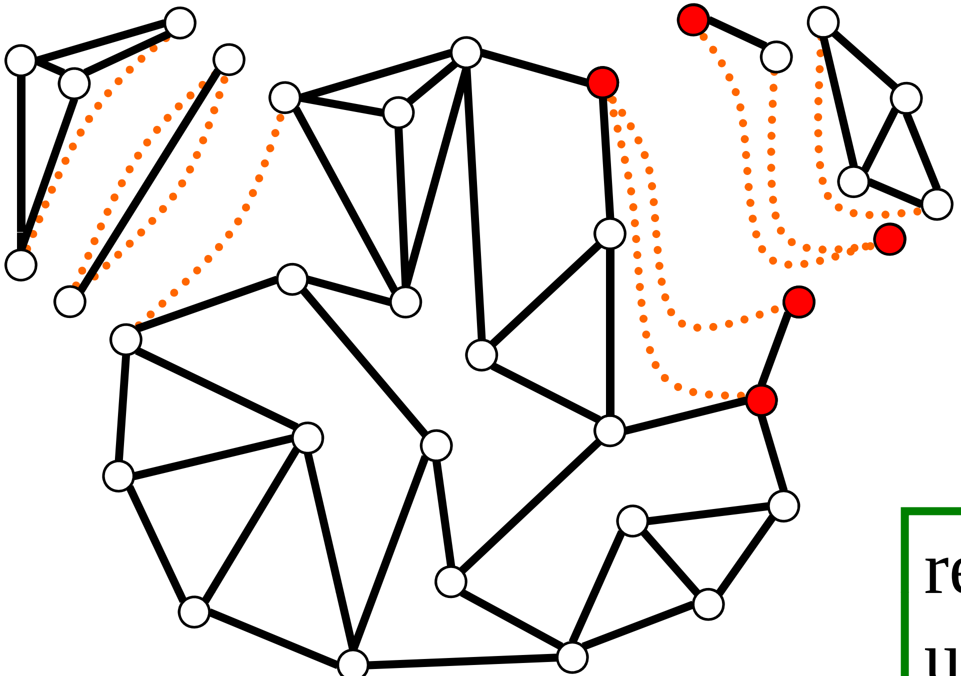


Decomposition tree



repeat this operation
until no more splits are possible

Decomposition tree



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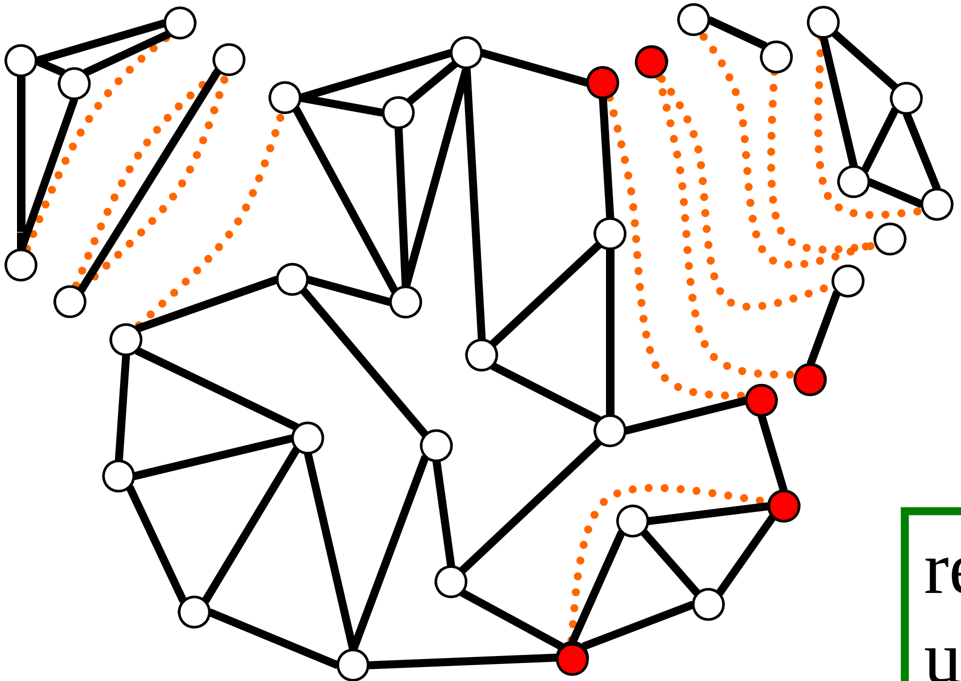

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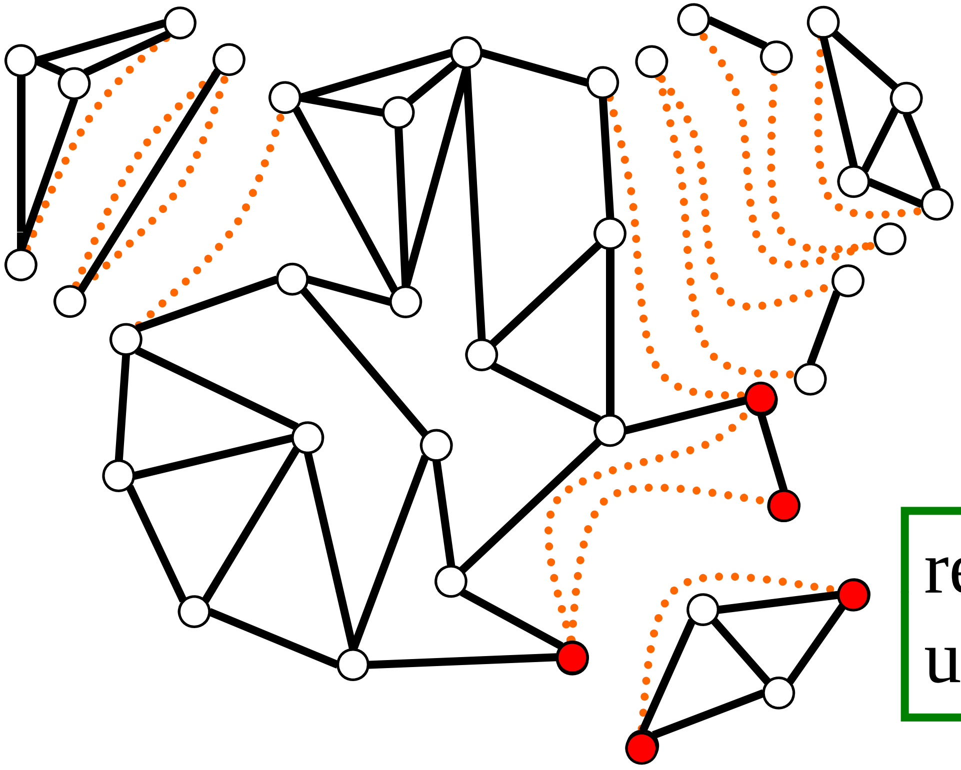
Decomposition tree



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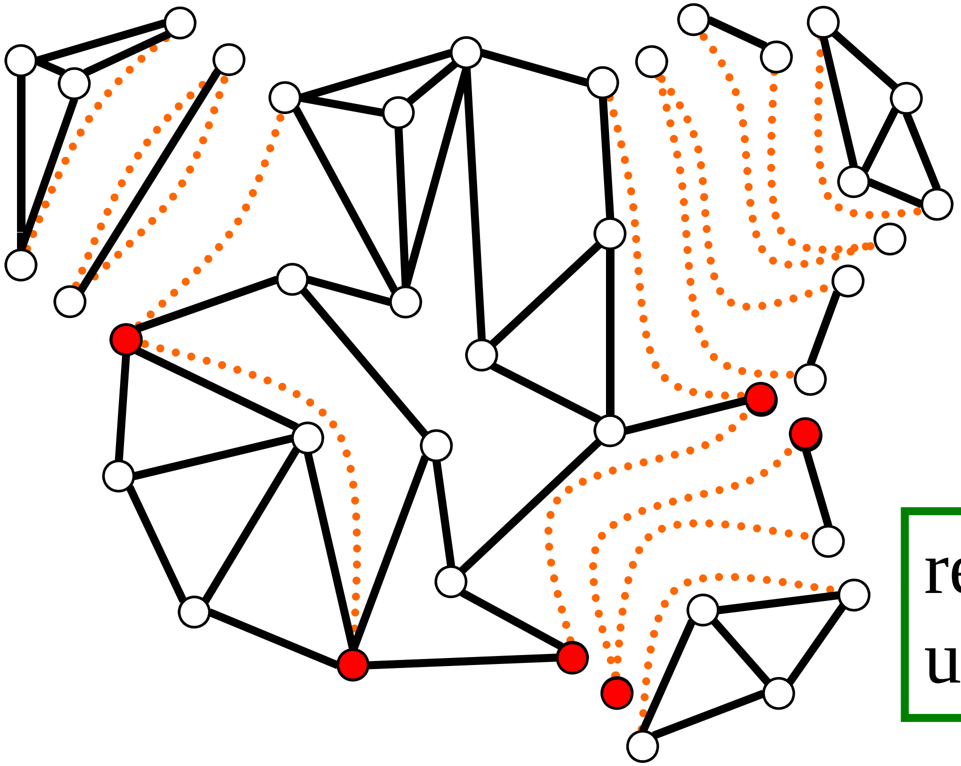

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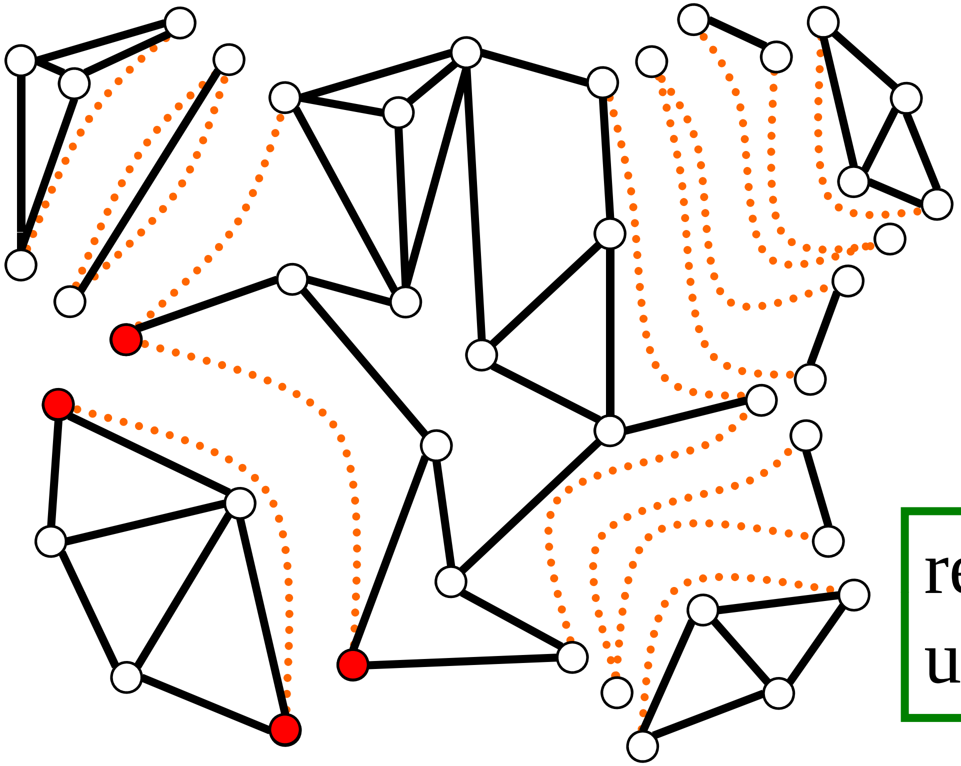

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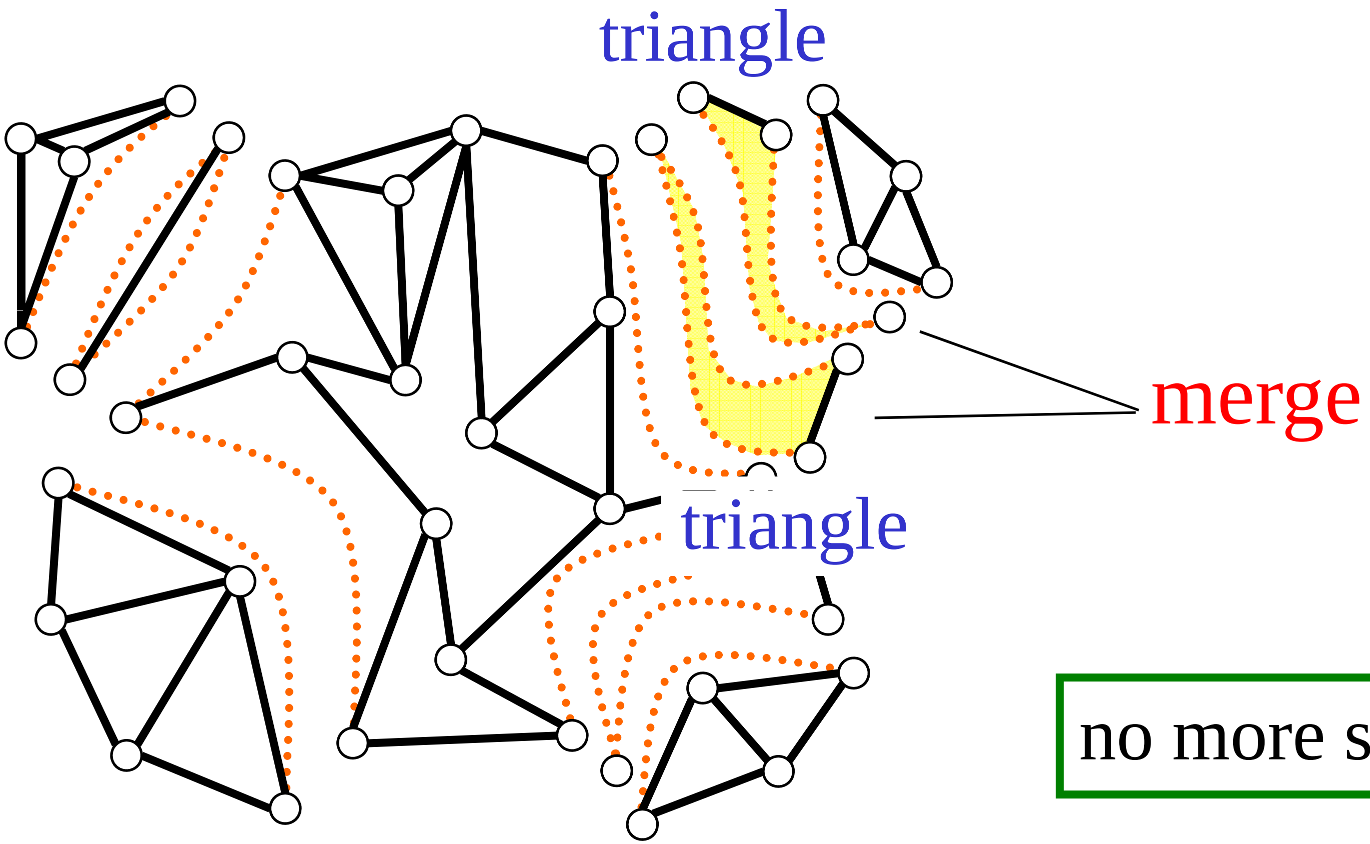
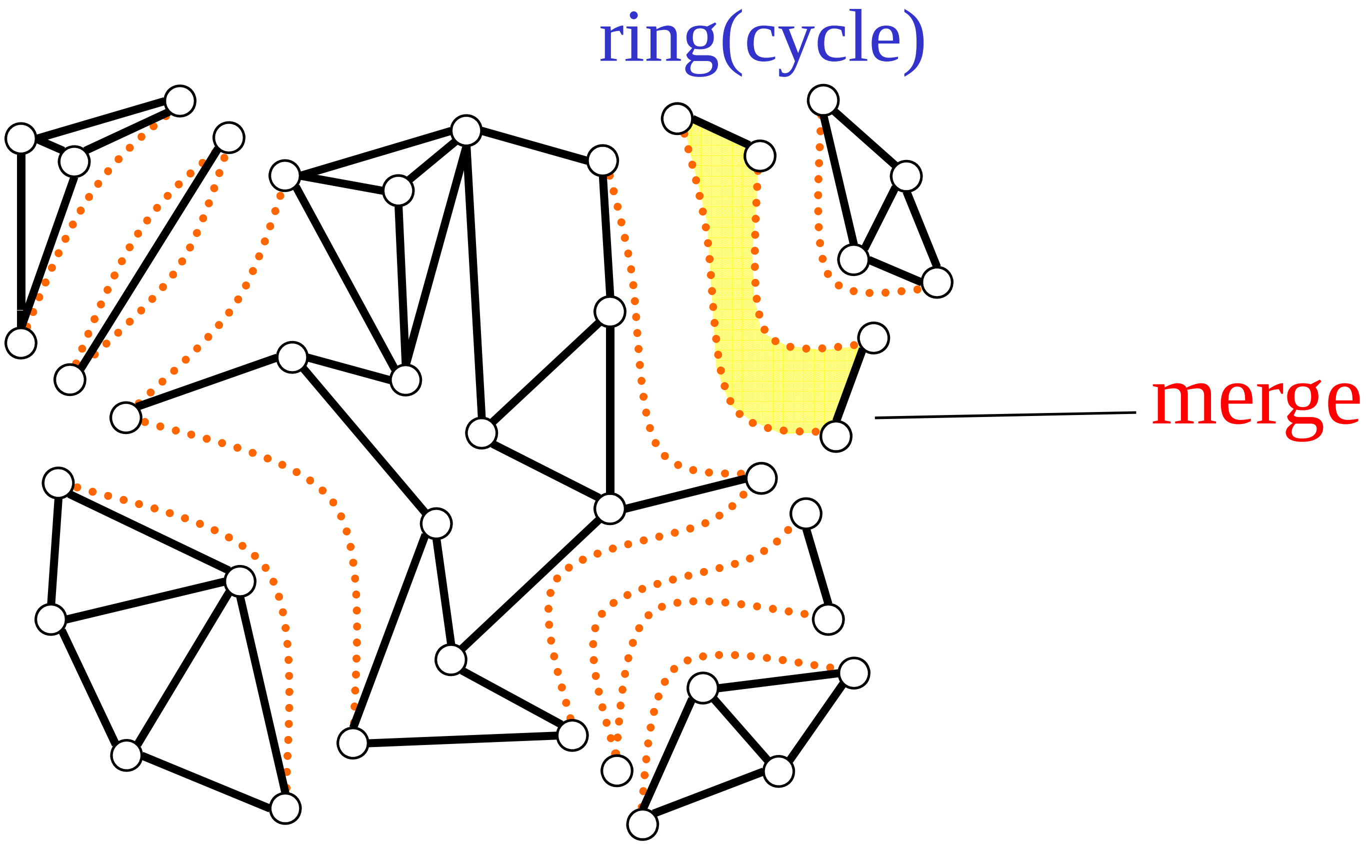
Decomposition tree



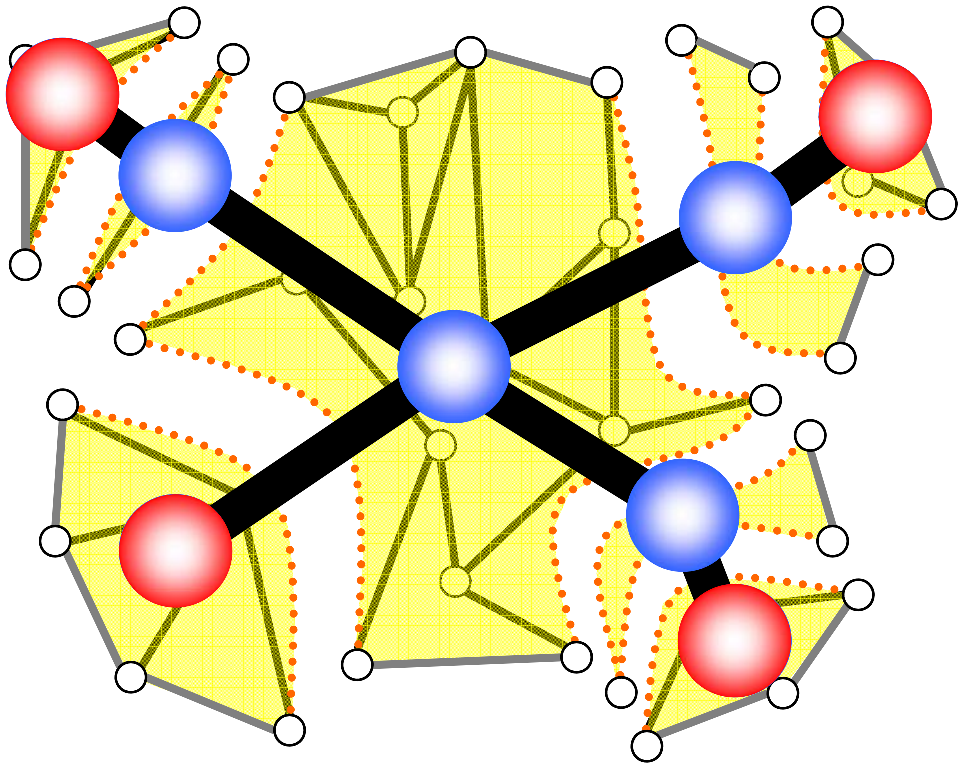
no more splits are possible

Decomposition tree



Decomposition tree

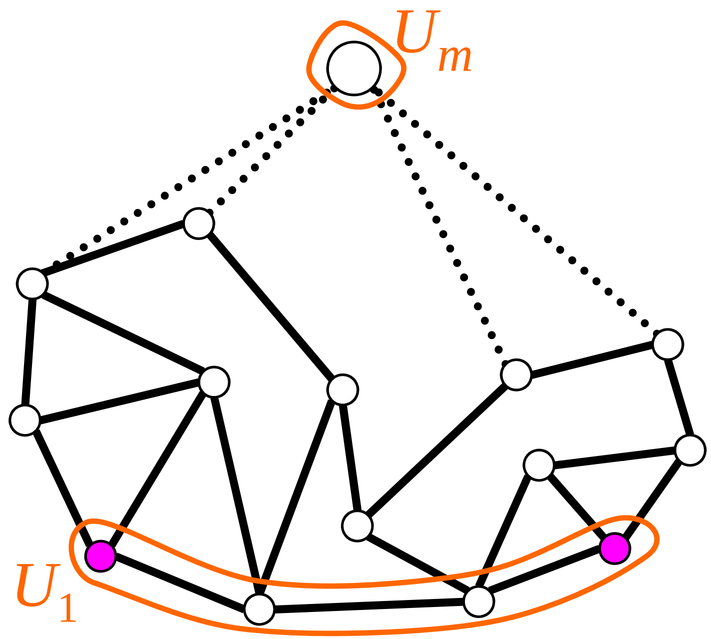
3-connected component decomposition tree [HT73]



Canonical decomposition

canonical decomposition

$$\Pi = (U_1, U_2, \cdots, U_{m-1}, U_m)$$



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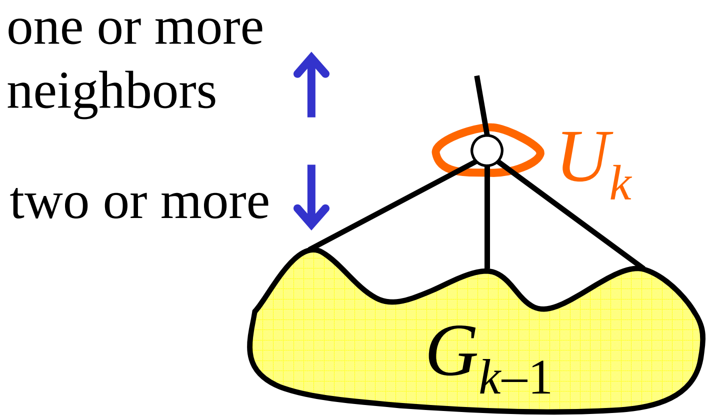
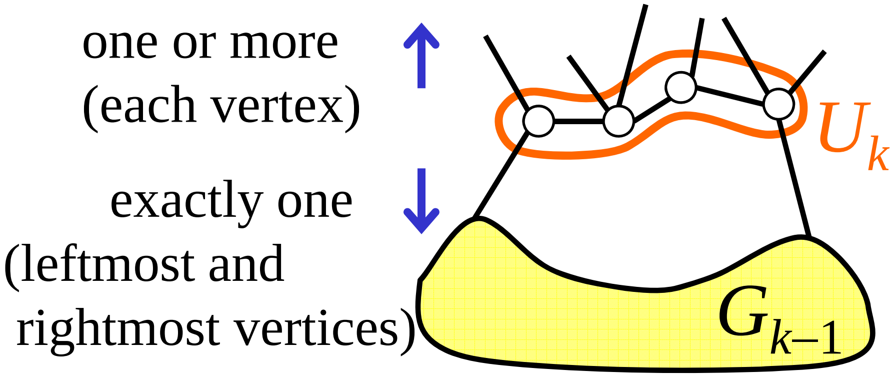
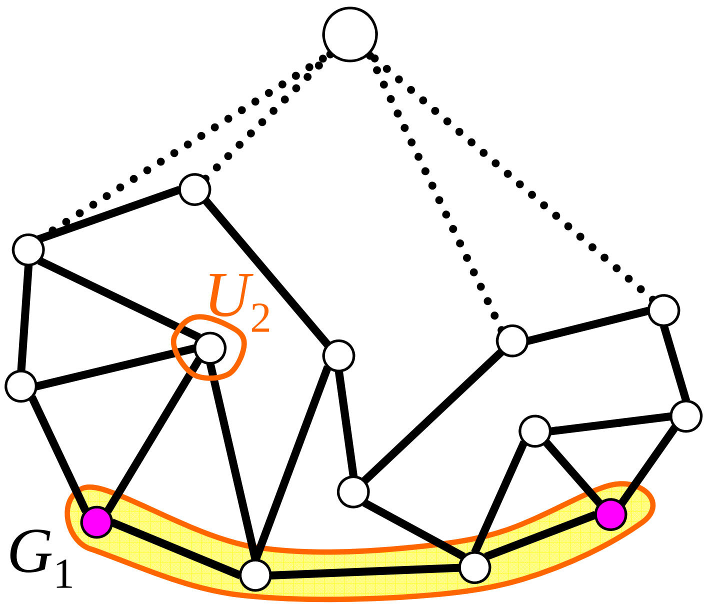




G_k is induced by
$U_1 \cup U_2 \cup \cdots \cup U_k$

canonical decomposition

$$\Pi = (\, U_1 \,,\, U_2 \,,\, \cdots \,,\, U_{m-1} \,,\, U_m \,)$$



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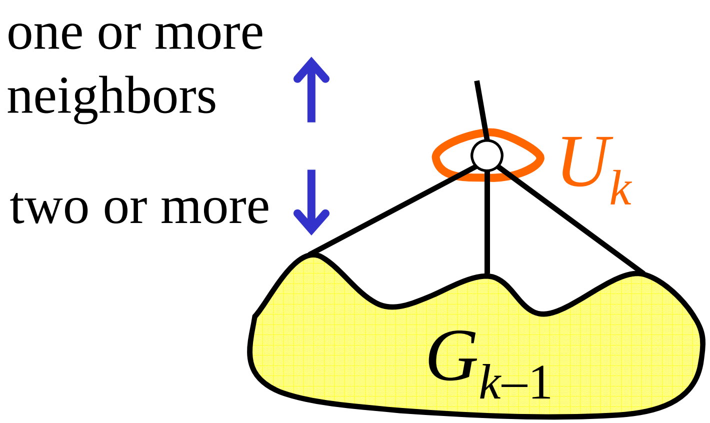

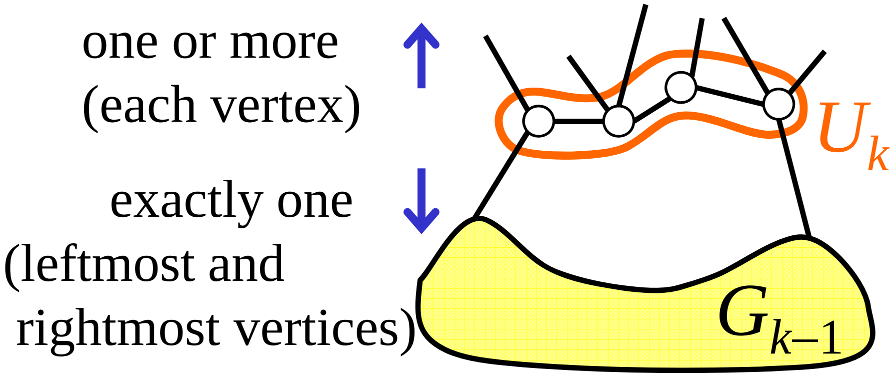

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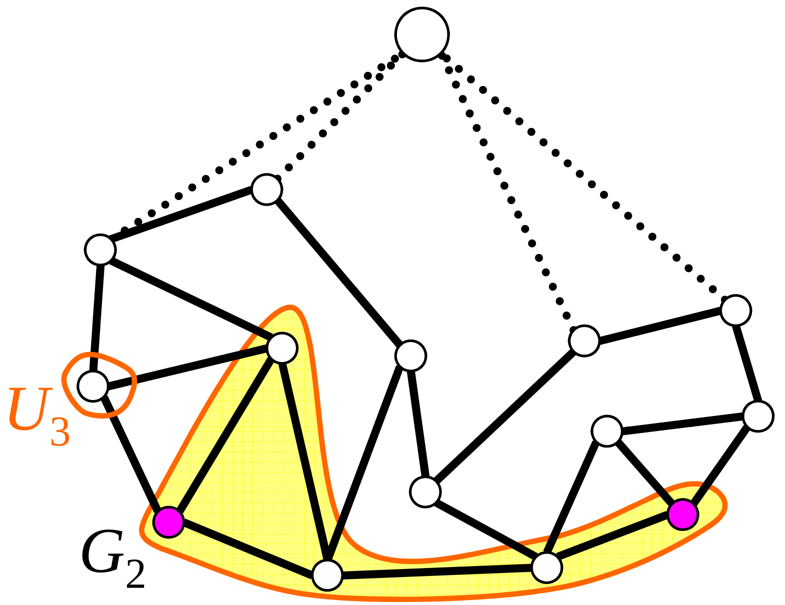

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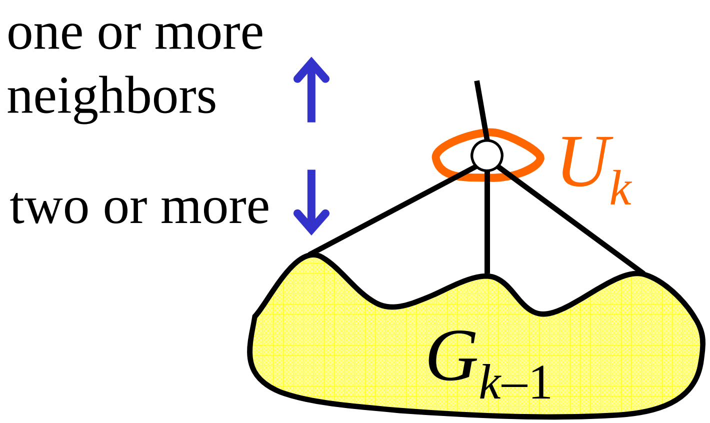

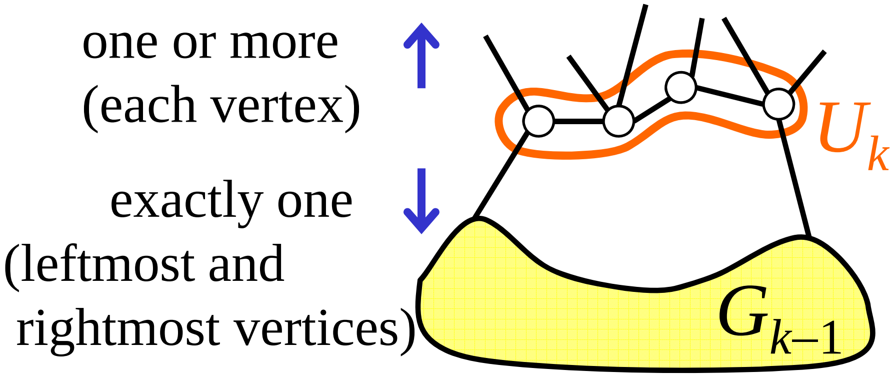

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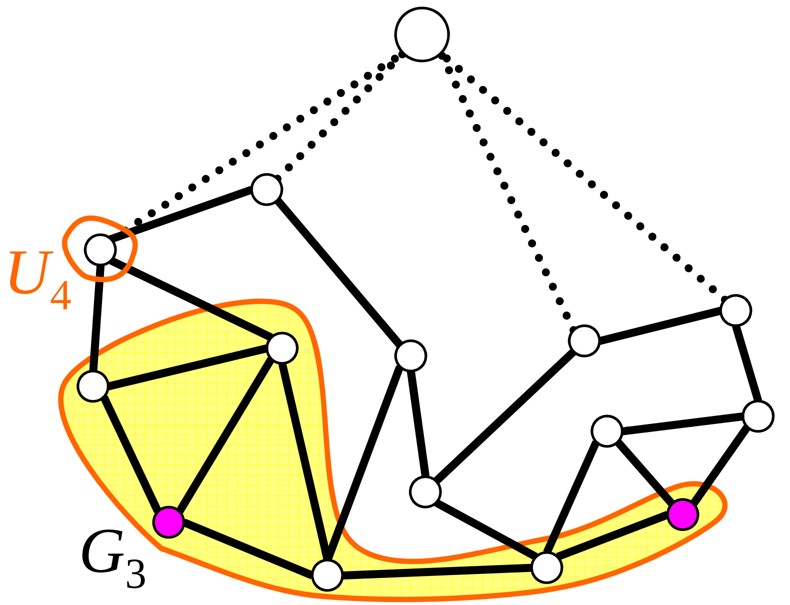

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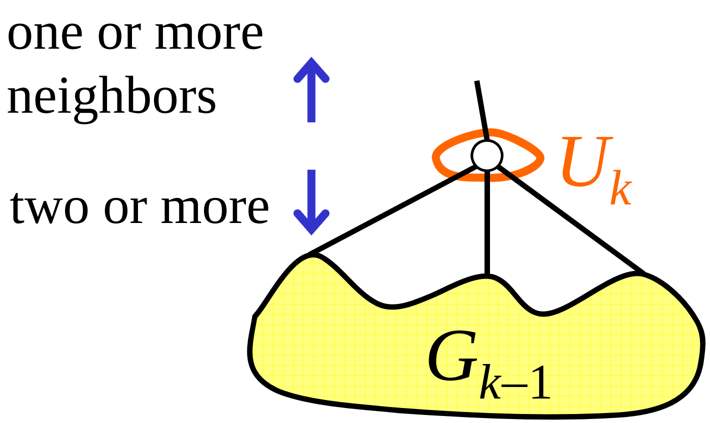

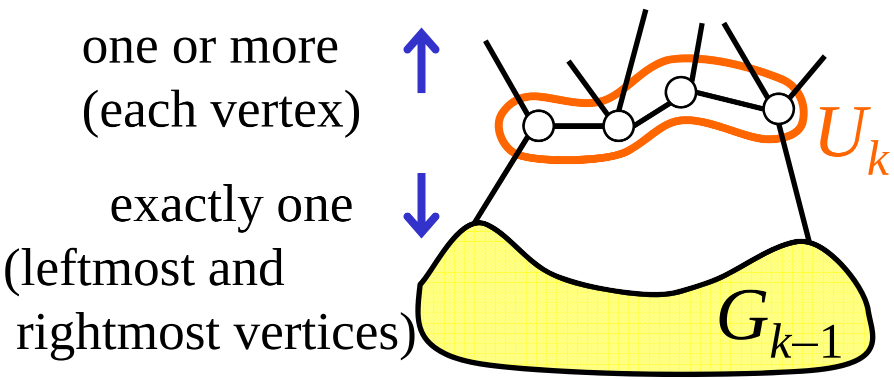

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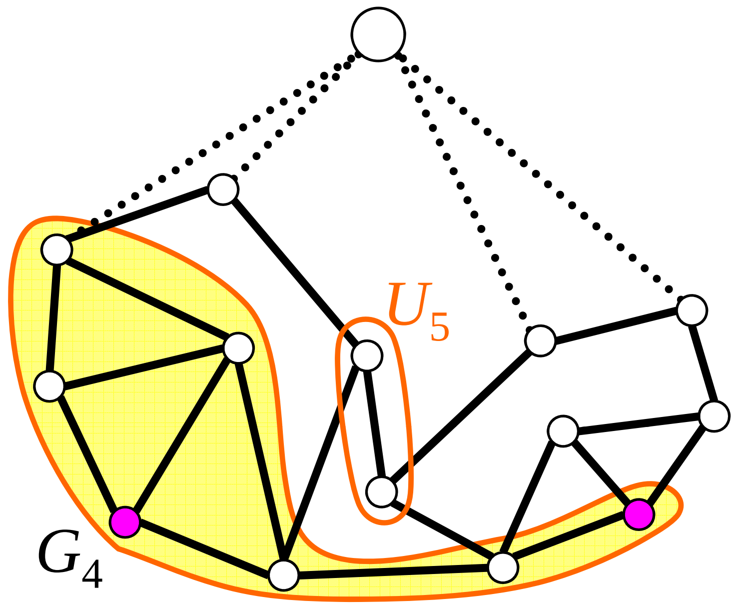

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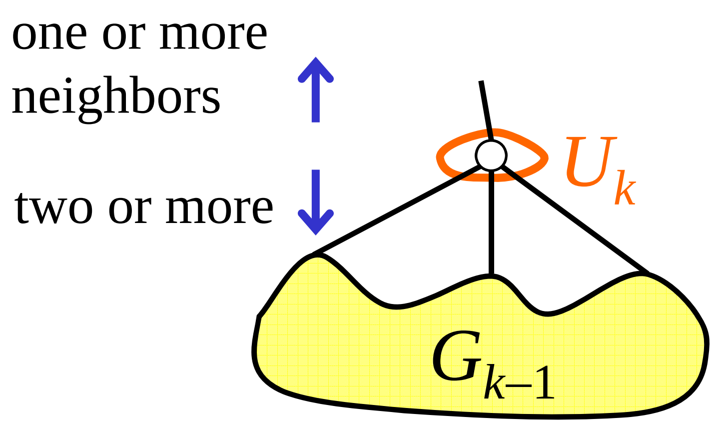

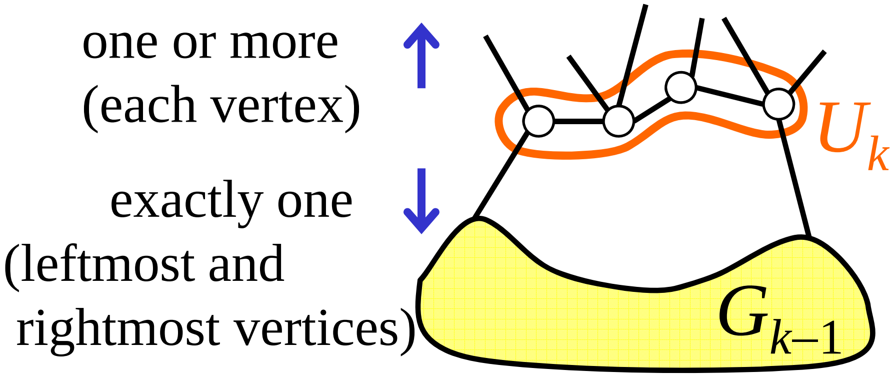

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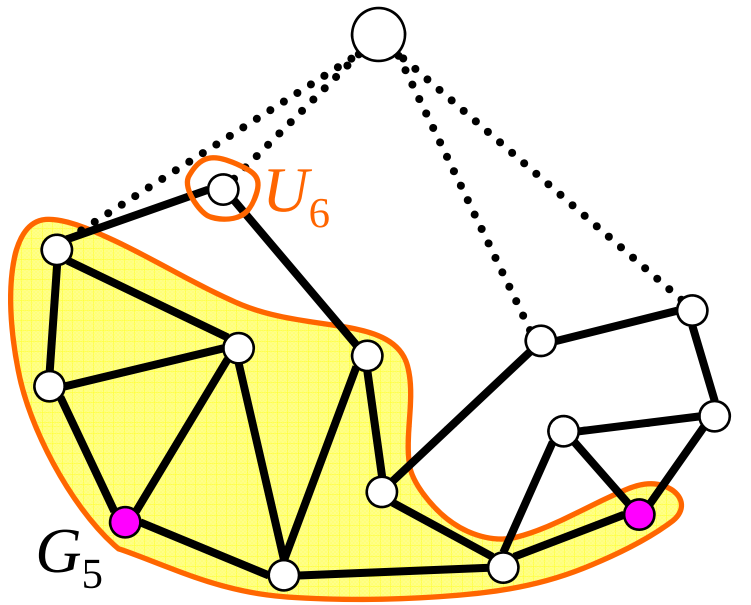

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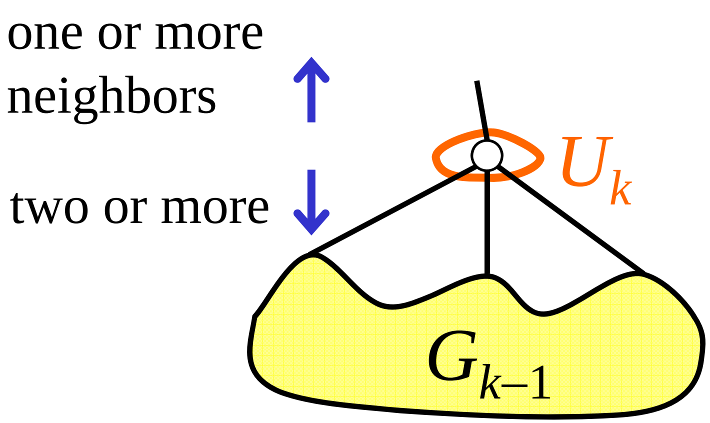

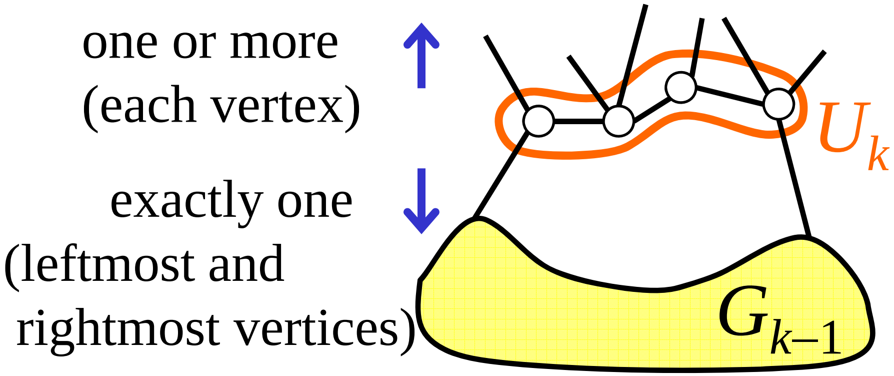

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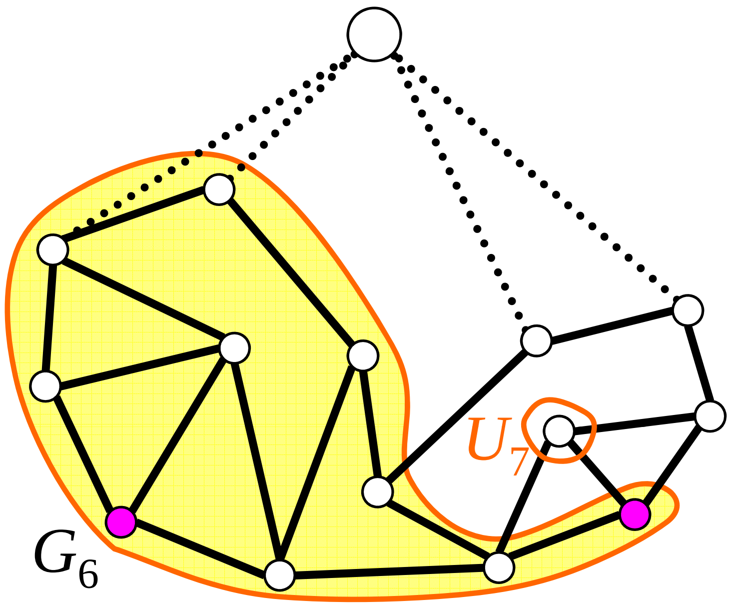
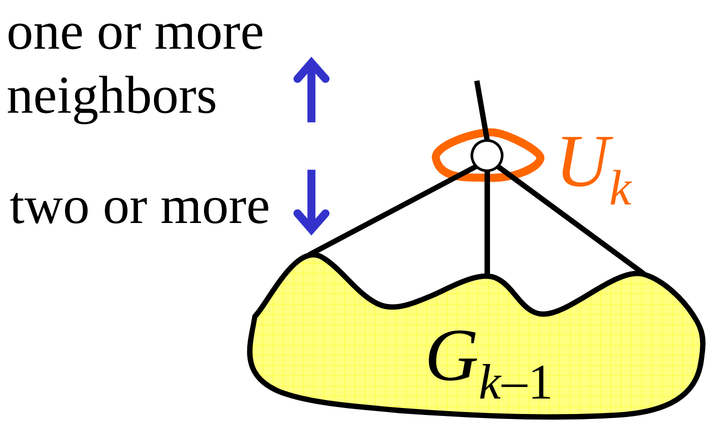
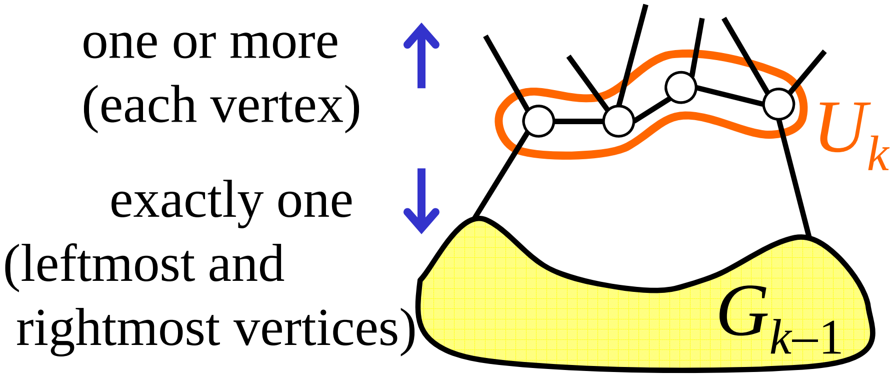


Canonical decomposition





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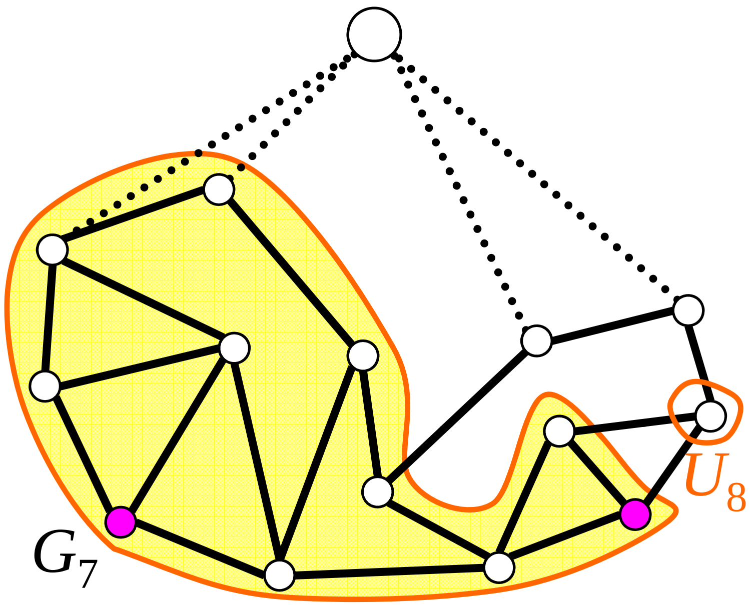
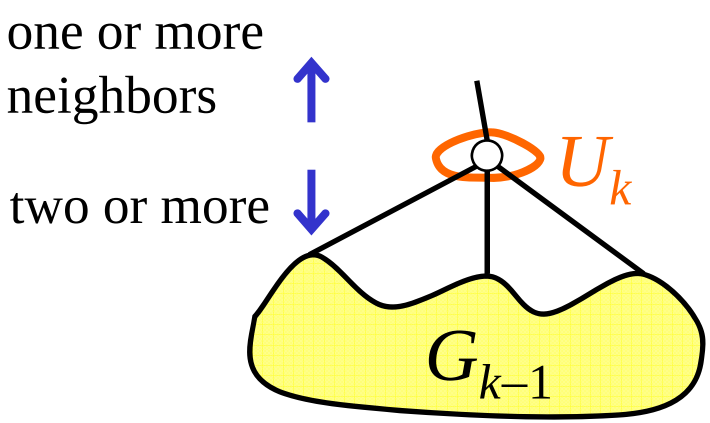
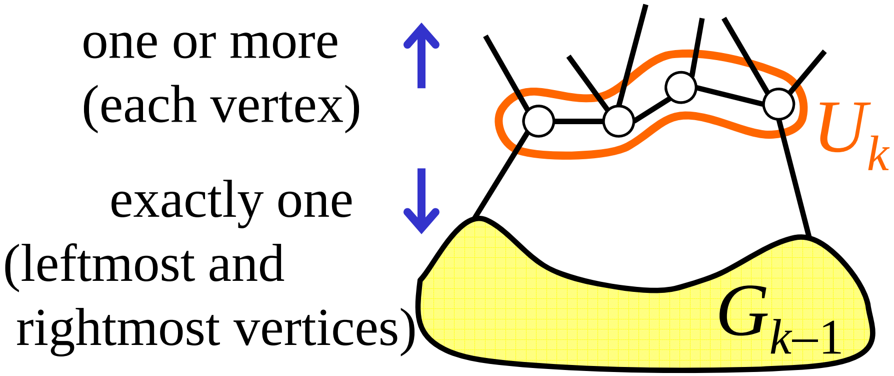


Canonical decomposition





canonical decomposition

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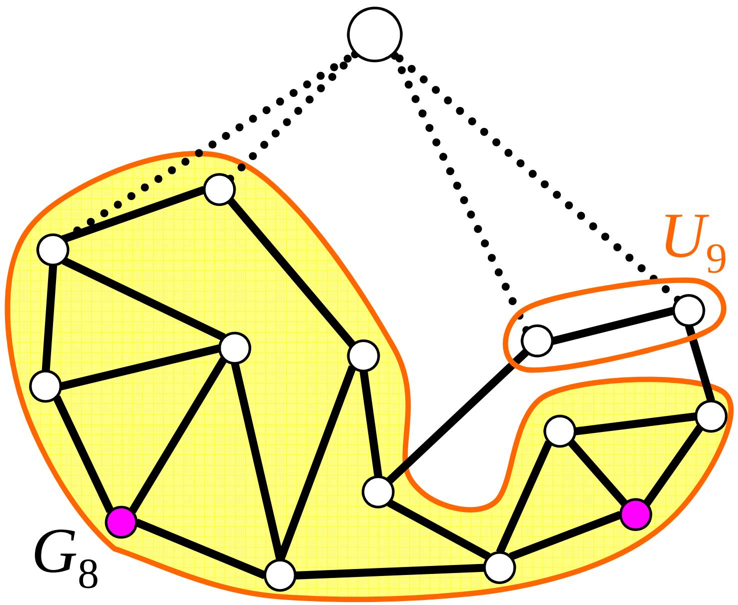
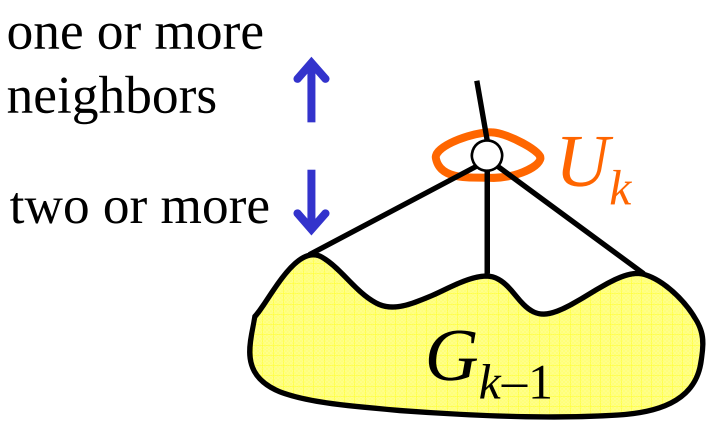
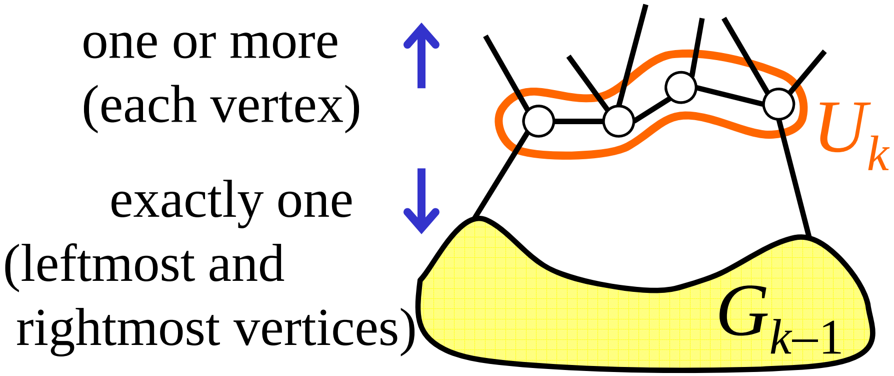


Canonical decomposition





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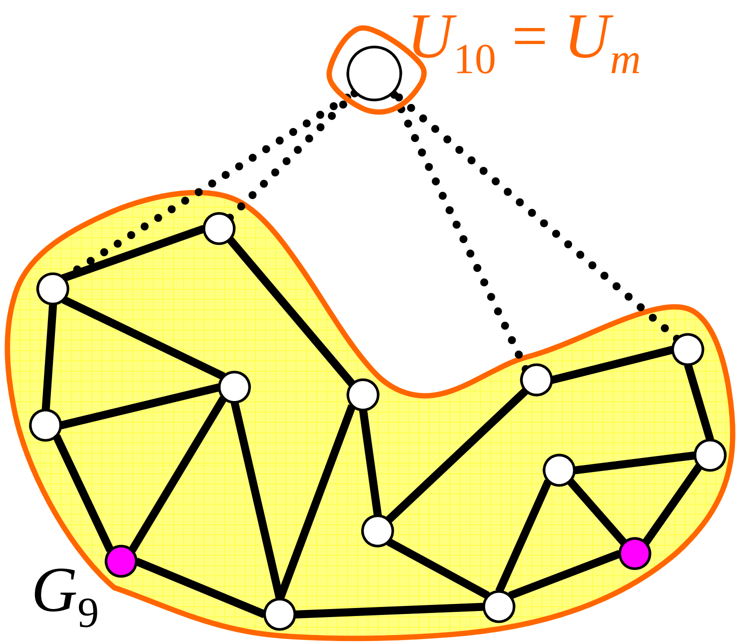

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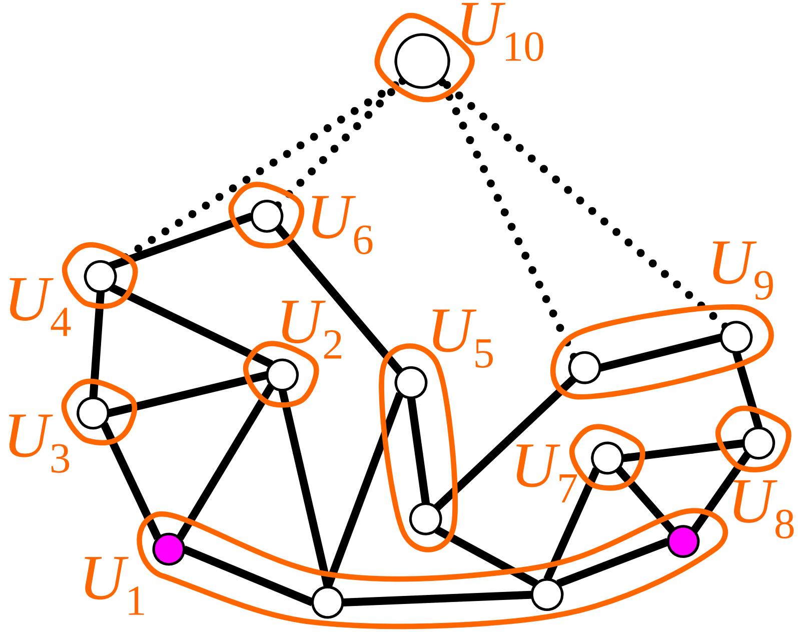

$2n \times n^2$ size

$$n_1^2 + n_2^2 + 2n+1$$

$$= (n_1 + n_2)^2 - 2n_1 n_2 + 2n+1$$

$$\leq n^2$$

$$n = n_1 + n_2$$

$$n_1, n_2 \geq 5$$

convex grid drawing