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COMBINATORIAL PROBLEMS ON SERIES-PARALLEL GRAPHS

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This paper outlines the results and motivation of the paper [1], in which we showed, in a unified manner, that there exist linear time algorithms for many combinatorial problems defined on the class of series-parallel graphs.

A large number of combinatorial problems defined on graphs are NP-complete, and hence there is probably no polynomial-time algorithm for any of them. A number of such problems can be formulated as a "minimum edge (vertex) deletion problem" with respect to some graph property Q. The problem asks for a minimum number of edges (vertices) of a given graph whose deletion results in a graph satisfying Q. Various other problems can be formulated as a "generalized matching problem", in which one would like to find a maximum number of vertex-disjoint copies of a fixed graph B contained in an input graph. Some results have been obtained for these problems. Krishnamoorthy et al. have shown that the minimum vertex deletion problem is NP-complete whenever property Q is nontrivial and hereditary. They have also shown that several minimum edge deletion problems are NP-complete. It should be noted that a hereditary property Q can be characterized by (possibly an infinite number of) "forbidden (induced) subgraphs", that is, a graph G satisfies Q if and only if G contains none of the forbidden graphs as an (induced) subgraph. On the other hand Kirkpatrick and Hell have shown that any generalized matching problem is NP-complete if the graph B has a component with at least three vertices.

Some of the combinatorial problems which are NP-complete for general graphs remains so even for a restricted class of graphs. However it has been shown by ad hoc methods that polynomial-time algorithms are available for some combinatorial problems on special classes of graphs, such as planar graphs, regular graphs, bipartite graphs, or series-parallel graphs. An example is the maximum cut problem, which is NP-complete for nonplanar graphs, but there exists a polynomial-time algorithm for planar graphs, as shown by Hadlock.

In the paper we consider a special class of graphs, called "series-parallel graphs", which can be constructed by recursively applying "series" and "parallel"

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connections. The class of such graphs, which is a well known model of series-parallel electrical networks, is a restricted class of planar graphs. It has been known for some time that many practical problems defined on such graphs can be efficiently solved, for example, "resistance of electrical networks", "reliability of systems", and "scheduling". The following question naturally arises: do there exist polynomial-time algorithms for *all* combinatorial problems defined on such a class of graphs? One can easily see that not every combinatorial problem is polynomial-time computable even if restricted to series-parallel graphs. However we show, in a unified manner, that a number of combinatorial problems are linear time computable for series-parallel graphs. Such a rather broad class of problems includes:

(i) the decision (i.e. yes-no) problem with respect to any property Q characterized by a finite number of "forbidden (induced or homeomorphic) subgraphs", in which one would like to decide whether an input graph satisfies Q;

(ii) the minimum edge (vertex) deletion problem with respect to the same property as above; and

(iii) the generalized matching problem.

Hence the following problems among others prove to be linear time computable for the class of series-parallel graphs:

(1) the minimum vertex cover problem (equivalently the maximum independent vertex set problem);

(2) the maximum (induced) line-subgraph problem:

(3) the minimum edge (vertex) deletion problem with respect to property "without cycles (or paths) of specified length n or any length $\leq n$";

(4) the maximum outerplanar (induced) subgraph problem;

(5) the minimum feedback vertex set problem;

(6) the maximum ladder (induced) subgraph problem ($K_{2,3}$ and its dual are the forbidden homeomorphic subgraphs of a ladder graph);

(7) the minimum path cover problem (in which one would like to find a minimum number of disjoint paths which contain all the vertices of a given graph);

(8) the maximum matching problem; and

(9) the maximum disjoint triangle problem.

Some of these problems have individually been shown to be polynomial-time computable for the class of series-parallel graphs or some larger class containing all such graphs.

The key idea of our algorithms is as follows. The set S of all the subgraphs of "forbidden" graphs is finite if "forbidden" graphs are finite many. Instead of the single original problem, we consider the (finite) collection of all the problems defined by subsets of S as "forbidden" graphs. Our algorithms solve the collection of problems by "divide-and-conquer" based on the recursive definition of series-parallel graphs.

Reference

[1] K. Takamizawa, T. Nishizeki and N. Saito, Linear time computability of combinatorial problems on series-parallel graphs, Tech. Rep. Dept. Electr. Commun., Tohoku Univ., Sendai, Japan (1980).