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Bekki–Nozaki Hole in Traveling Excited Waves on Human Cardiac Interventricular Septum

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We observe some phase singularities in traveling excited waves noninvasively measured by a novel ultrasonic method, on a human cardiac interventricular septum (IVS) for a healthy young male. We present a possible physical model explaining a part of one-dimensional cardiac dynamics of the observed phase defects on the IVS. We show that at least one of the observed phase singularities in the excited waves on the IVS can be explained by the Bekki–Nozaki hole solution in the complex Ginzburg–Landau equation, although the creation and annihilation of phase singularities on the IVS give birth to a variety of complex patterns.

KEYWORDS: Bekki–Nozaki hole, cardiac dynamics, traveling excited waves, phase singularity

The effects of external electric fields on cardiac tissue of rabbits and the propagation of excited waves by producing standing waves of membrane depolarization have been studied both experimentally and numerically.[1,2] Using a method of identifying phase singularities, Gray *et al.* elucidated the mechanisms for the formation and termination of these phase singularities.[3]

Also, a variety of complex patterns which include spiral waves[4] and the spontaneous response of the myocardium to electrical excitation have been observed in human heart by developing an ultrasonic noninvasive novel imaging modality with high temporal and spatial resolutions.[5] Visualizing the propagation of the myocardial response of the electric excitation in human hearts during systole, as is shown in Fig. 1, Kanai[6] observed the velocity components toward the ultrasonic probe as waveforms and their instantaneous phases of 40 Hz components. A velocity component corresponding to the contraction was generated on the septum at a time of T-wave of ECG (end-systole), and propagated slowly (40 mm/ms) in clockwise direction along the left ventricle circumferential direction. Thus, the behavior of phase singularities in a human healthy heart is one of the most interesting subjects in physics and biophysics. In order to explain its behavior in a human healthy heart, a certain model of explanation is therefore needed on the basis of the direct observation of phase defects. We present here a possible physical model explaining a part of cardiac dynamics of these phase singularities on the interventricular septum (IVS).

In this Letter, we show that at least one of the phase singularities in the excited waves on a human cardiac IVS can be explained by the Bekki–Nozaki (BN) hole solution[7] in the complex Ginzburg–Landau equation (CGLE).

The CGLE[8–15] is well known for one of the simplest models that account for the behaviors of nonlinear waves and the spontaneously formed complicated patterns in the spatially extended non-equilibrium systems: the ionization waves in the glow discharge,[16] the chemical oscillations and turbulence,[17] the hydrothermal nonlinear waves in a laterally heated layer,[18] and so on. A wide class of nonlinear

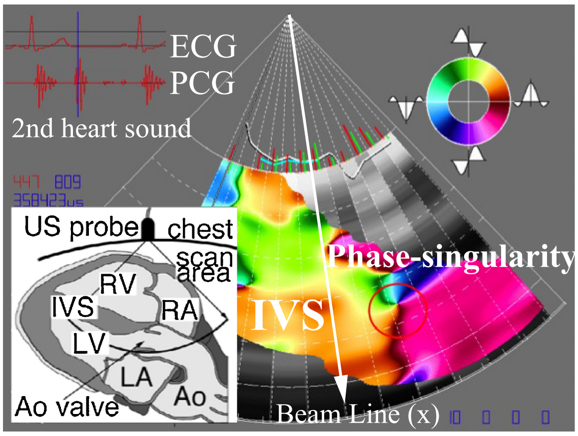

Fig. 1. (Color online) A snapshot of two-dimensional spatial pattern of phase $\Theta(x, y, t)$ in the excited waves on the IVS for a healthy young male, by using the novel ultrasonic measurement technique for myocardial motions *in vivo* found by Kanai. A typical phase singularity of excited waves on IVS is shown by a large circle. The cross-sectional image of the color-coded phase values just before the time of aortic-valve closure at end-systole is shown here. A phase value at a local point on IVS, for example, changes from cyan (+180°) near a certain phase-defect, through green (+90°), and to red (0°) at the different point and time. The left inset shows the scanning range of the ultrasonic beams in this measurement: LV, left ventricle; LA, left atrium; RV, right ventricle; RA, right atrium; US prove, ultrasonic prove; IVS, interventricular septum; Ao, aorta; ECG, electrocardiogram; PCG, phonocardiogram (heart sound).

waves for such strong dispersive systems can be described by the one-dimensional (1D) nonlinear partial differential equation which is called CGLE,

$$\mathrm{i}\frac{\partial}{\partial t}\psi + p\frac{\partial^2}{\partial x^2}\psi + q|\psi|^2\psi = \mathrm{i}\gamma\psi, \qquad (1)$$

where ψ is a complex function of scaled time t and space x, and with the two complex coefficients ($p = p_r + \mathrm{i}p_i$, $q = q_r + \mathrm{i}q_i$) and a real positive constant γ. It is noted that CGLE is represented by the full coefficients without rescaling in order to make a direct comparison between the observed data and the exact solutions of CGLE.

One of the exact solutions of CGLE connects two different patterns specified by the asymptotic wavenumbers

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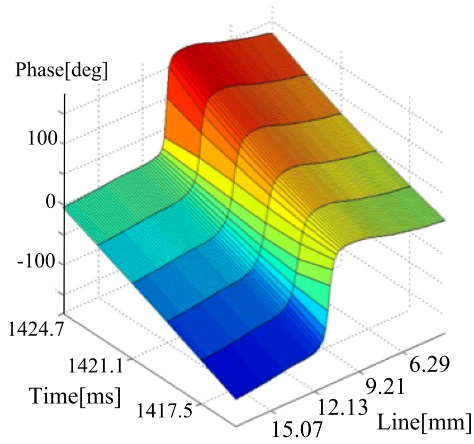

Fig. 2. (Color online) Observed local phase profile $\Theta(x,t)$ (deg) on the IVS with a phase-jump at $x_h = 10.5$ mm in the line direction for $1415.7 < t < 1424.7$ ms and $3.19 < x < 16.95$ mm. We can clearly observe a profile of traveling phase-jump.

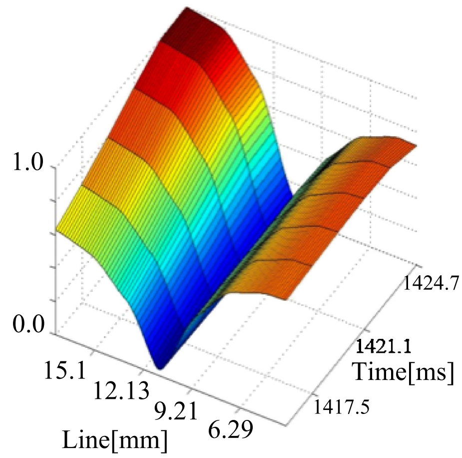

Fig. 3. (Color online) Observed amplitude profile $A(x,t)$ near the phase-jump at $x_h = 10.5$ mm in the line direction for $1415.7 < t < 1424.7$ ms and $3.19 < x < 16.95$ mm. We can clearly observe an amplitude-hole with phase singularity and obtain the velocity of the amplitude-hole [see eq. (5)]: $\tilde{c}_h = -0.08$ mm/ms.

and a phase-jump between two patterns, which is called the BN hole.[7)] However, very few experimental investigations of BN hole have been reported up to now.[18,19)] The estimation of the rescaled coefficients of CGLE from the experimental data has been a difficult task, because some localized amplitude holes have been observed in the hydrothermal nonlinear waves, and not adequately compared with BN hole solution in CGLE.[18)]

Let us first demonstrate a typical observation related to the phase singularities in the excited waves on the IVS for a healthy young male, as is shown in Figs. 1–3, by developing an ultrasonic noninvasive novel imaging modality with high temporal and spatial resolutions[5)] which shows that the propagation of the mechanical wave-front occurs at the end of the cardiac systole by simultaneous measurement of the vibrations at many points (10,000 points) set in the IVS. We obtained 2D patterns of phase and amplitude of the excited waves on the IVS: $\Theta(x,y,t)$ and $A(x,y,t) = |\tilde{\Psi}(x,y,t)|$, where x-axis denotes the line (beam) direction and y-axis does the circumferential (depth) direction. Indeed, the observed data of phases and amplitudes demonstrate a variety of complex patterns, which include a zigzag pattern and the target waves on the IVS.[20)] Our interest is, however, focussed on 1D cardiac dynamics of phase singularities on the IVS, which may be viewed as a 1D generalization of a core of 2D target patterns.[6,20)] For the fixed y (depth direction for long-axis view), therefore, let us define 1D data of measurements of excited waves as follows:

$$\tilde{\Psi}(x,t) = A(x,t)\exp[\mathrm{i}\Theta(x,t)], \tag{2}$$

where t (ms) denotes an observable time of phase singularities ($T_1 < t < T_2$).

As is shown in Fig. 2, we can obtain a pair of asymptotic local wavenumbers $\tilde{k}_j$ ($j = 1, 2$) defined by

$$\tilde{k}_j = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \frac{\Theta(x_2,t) - \Theta(x_1,t)}{x_2 - x_1}\,\mathrm{d}t, \tag{3}$$

where $\tilde{k}_1$ for $x_1 < x_2 < x_h$ and $\tilde{k}_2$ for $x_h < x_1 < x_2$ during $T_1 < t < T_2$, respectively. A position of phase singularity (hole) is denoted by x_h, and a life-time of hole in our case is about 10 ms for a very slow speed of hole. We also define a phase-jump $\tilde{\sigma}_{\mathrm{ob}}$,

$$\tilde{\sigma}_{\mathrm{ob}} = \lim_{\epsilon\to+0}\sup_{x\in\mathbb{R}} |\Theta(x - x_h - \epsilon, t) - \Theta(x - x_h + \epsilon, t)|, \tag{4}$$

where the phase $\Theta(x,t)$ is linearly extrapolated at a fixed time. Figure 2 shows a typical one-dimensional phase $\Theta(x,t)$ (deg) at a certain small region ($3.19 < x < 16.95$ mm and $1415.7 < t < 1424.7$ ms). From Fig. 2 and eq. (4), we can observe a phase-jump $\tilde{\sigma}_{\mathrm{ob}} = 2.9$ rad.

Next, let us define a position x' of minimum amplitude at time t_1 and a position x'' of minimum amplitude at time t_2, then, in a uniform linear motion of phase singularities, we have its velocity $\tilde{c}_h$ (mm/ms) defined by

$$\tilde{c}_h = \frac{x'' - x'}{t_2 - t_1}. \tag{5}$$

As is shown in Fig. 3, we can observe a propagation of the phase singularity on the IVS and we have $\tilde{c}_h = -0.08$ mm/ms from eq. (5).

Finally, from eqs. (3) and (4), as is shown in Fig. 4, we have obtained the following fundamental physical quantities related to BN hole: (i) a pair of asymptotic local wavenumbers $\tilde{k}_1 = 0.13 \pm 0.01\,\mathrm{mm}^{-1}$ and $\tilde{k}_2 = -0.05 \pm 0.005\,\mathrm{mm}^{-1}$, (ii) the phase-jump $\tilde{\sigma}_{\mathrm{ob}} = 2.9 \pm 0.1$ rad, and the curvature defined by eq. (11) near the hole $|\tilde{\kappa}| = 0.39 \pm 0.02\,\mathrm{mm}^{-1}$. Here the curvature does not mean the reciprocal of its radius. On the other hand, from eq. (5), (iii) we have also obtained the velocity of phase singularity $\tilde{c}_h = -0.08 \pm 0.01$ mm/ms for $1415.7 < t < 1424.7$ ms, as is shown in Fig. 3. A set of observations obtained from the data (2) is represented by

$$\tilde{K}_{\mathrm{ob}} = \{\tilde{k}_1, \tilde{k}_2, \tilde{c}_h, \tilde{\sigma}_{\mathrm{ob}}, |\tilde{\kappa}|\} \in \mathbb{R}^5. \tag{6}$$

Similarly, we have observed another traveling amplitude holes at many points in y-axis (circumferential direction) as well as in x-axis (beam direction) and two-dimensional target waves on the IVS.[20)]

Solving the reality condition in the bilinear form of CGLE,[7,12)] we have an important parameter α mentioned previously,

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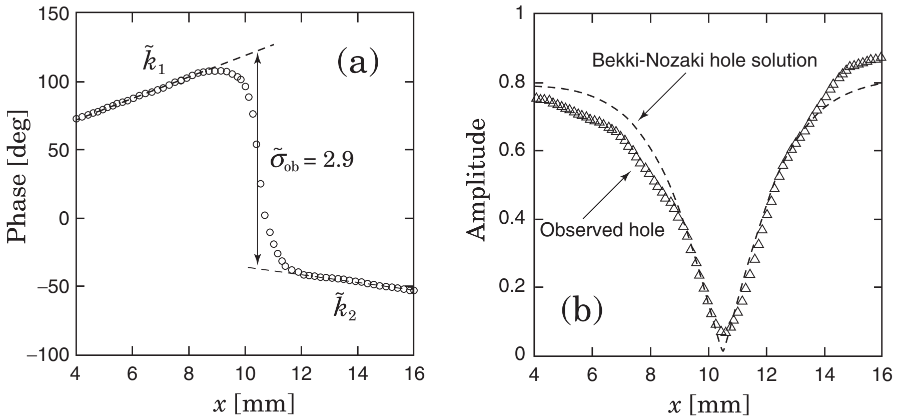

Fig. 4. (a) A snapshot of observed phase-jump near the local point $x_h = 10.5$ mm at a fixed time $t = 1421.1$. Circle denotes the phase of excited waves $\Theta(x, t)$ (deg). We obtain the asymptotic wave-numbers $\tilde{k}_1 = 0.13\,\mathrm{mm}^{-1}$, $\tilde{k}_2 = -0.05\,\mathrm{mm}^{-1}$, and the phase-jump $\tilde{\sigma}_{\rm ob} = 2.9$ rad. (b) A snapshot of observed amplitude hole and the BN hole with the coefficients $\mathfrak{C}_{\mathfrak{h}}$. Triangle denotes the amplitude hole $A(x, t)$ for the fixed time $t = 1421.1$. The curvature near the observed hole defined by eq. (11) is $|\tilde{\kappa}| = 0.39\,\mathrm{mm}^{-1}$ and the curvature near BN hole is $|\kappa| = 0.4031$.

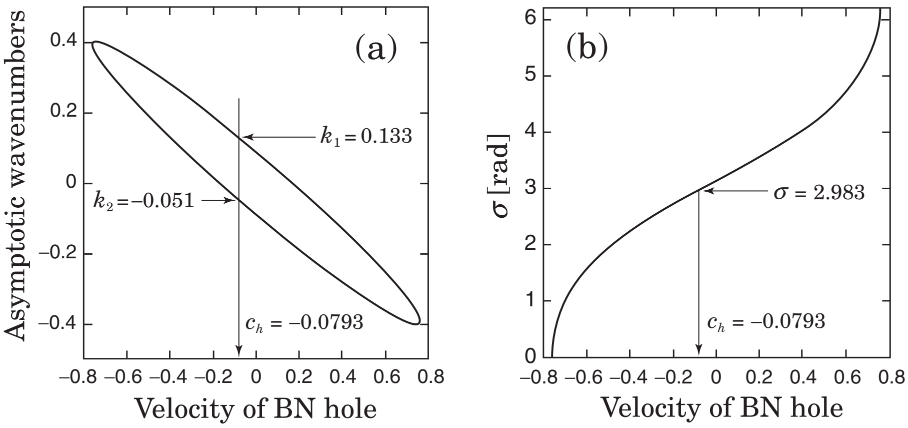

Fig. 5. (a) Asymptotic wavenumbers k_1 and k_2 versus the velocity (c_h) of BN hole [$-0.5 < k_1$ (or k_2) < 0.5 and $-0.8 < c_h < 0.8$]. From the analytic form of BN hole solution, we obtain $k_1 = 0.133$, $k_2 = -0.051$, and $c_h = -0.0793$, respectively. We have $\kappa_+ (= k_1 + k_2) = 0$ in case of $c_h = 0$. (b) Phase-jump (σ) versus the velocity (c_h) of BN-hole ($0 \leqq \sigma \leqq 2\pi$ and $-0.8 < c_h < 0.8$). From eqs. (7) to (13) with the coefficients $\mathfrak{C}_{\mathfrak{h}}$, we can obtain the value of the phase-jump $\sigma = 2.983$ rad for $c_h = -0.0793$. We have $\sigma = \pi$ in case of $c_h = 0$.

$$\alpha = -\beta \pm \sqrt{\beta^2 + 2} \quad \left(\beta \equiv \frac{3}{2}\frac{p_r q_r + p_i q_i}{p_r q_i - p_i q_r}\right). \tag{7}$$

A set of all the coefficients in CGLE is written by

$$\mathfrak{C} = \{p_r, p_i, q_r, q_i, \gamma\} \in \mathbb{R}^5, \tag{8}$$

where $p_i < 0$ and $q_i > 0$. Algebraic condition in CGLE uniquely determines k_1, k_2, c_h, $\arg(b_2/b_1)$ and $|b_2/b_1|$ after algebraic manipulations.

The velocity of propagating BN hole is given by

$$c_h = \frac{p_r q_i - p_i q_r}{q_i}(k_1 + k_2). \tag{9}$$

We have a phase-jump ($0 \leq \sigma \leq 2\pi$)

$$\sigma \equiv \arg\left(\frac{b_2}{b_1}\right) = \arctan\left\{\frac{2\dfrac{p_i}{q_i}\left[(p_r q_i - p_i q_r) + \dfrac{1}{\alpha}(p_r q_r + p_i q_i)\right]\kappa_+ \kappa_-}{\left(\dfrac{p_i}{q_i}\right)^2 |q|^2 \kappa_+^2 - \left(1 + \dfrac{1}{\alpha^2}\right)|p|^2 \kappa_-^2}\right\}, \tag{10}$$

where $\kappa_\pm = k_1 \pm k_2$. It is noted that this phase-jump connects discontinuously two different patterns specified by wavenumbers k_1 and k_2.

The curvature near the hole is given by

$$\kappa = -\frac{\kappa_-}{2\alpha}. \tag{11}$$

We also obtain analytically the ratio

$$\left|\frac{b_2}{b_1}\right| = \left[\frac{(s_1 + s_2)^2 + (t_1 - t_2)^2}{(s_1 - s_2)^2 + (t_1 + t_2)^2}\right]^{1/2}, \tag{12}$$

where

$$s_1 = p_i \kappa_+, \qquad s_2 = \left(\frac{p_r}{\alpha} - p_i\right)\kappa_-,$$
$$t_1 = p_i \frac{q_r}{q_i}\kappa_+, \quad t_2 = \left(p_r + \frac{p_i}{\alpha}\right)\kappa_-.$$

Asymptotic wavenumbers k_1 and k_2 satisfy

$$\frac{(\kappa_+)^2}{a_1^2} + \frac{(\kappa_-)^2}{a_2^2} = 1, \tag{13}$$

$$a_1^2 = \frac{4K_m^2}{1 + \dfrac{3\alpha p_i |q|^2}{(1 + \alpha^2) q_i (p_r q_i - p_i q_r)}},$$

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$$a_2^2 = \frac{4K_m^2}{1 + \dfrac{3q_i|p|^2}{\alpha p_i(p_r q_i - p_i q_r)}},$$

where a_1^2 and a_2^2 are positive constants on account of the real condition and $K_m = \sqrt{\gamma/(-p_i)}$ ($p_i < 0$).

The Bekki–Nozaki hole solution[7] is given by

$$\psi(x,t) = \frac{b_1 \exp(\kappa\xi) + b_2 \exp(-\kappa\xi)}{\exp(\kappa\xi) + \exp(-\kappa\xi)} \times \exp\left[\frac{\mathrm{i}}{2}\int^{\xi}(\kappa_+ + \kappa_- \tanh \kappa x)\,\mathrm{d}x - \mathrm{i}\Omega t\right], \quad (14)$$

where $\xi = x - c_h t$ and $\Omega = p_r K_m^2 - c_h(k_1 k_2 + K_m^2)/(k_1 + k_2)$. Equations (9) and (13) give the algebraic relation between the velocity (c_h) and the asymptotic wavenumbers of BN hole solution with $\mathfrak{C}$. It is noted that the family of BN hole solution can be parametrized by the velocity of BN hole, as is shown in Fig. 5.

Also, eq. (14) gives the following inequality

$$\frac{|b_1|^2}{4}\mathrm{sech}^2(\kappa\xi)\left[\exp(\kappa\xi) - \left|\frac{b_2}{b_1}\right|\exp(-\kappa\xi)\right]^2 \leqq |\psi|^2 \leqq \frac{|b_1|^2}{4}\mathrm{sech}^2(\kappa\xi)\left[\exp(\kappa\xi) + \left|\frac{b_2}{b_1}\right|\exp(-\kappa\xi)\right]^2. \quad (15)$$

We have observed two different patterns specified wavenumbers $\tilde{k}_1$ and $\tilde{k}_2$ near the phase defect, and the phase-jump between two patterns. The phase-jump occurs at $x_h = 10.5$ mm, as is shown in Figs. 2 and 3, the amplitude $A(x,t) = |\psi(x,t)|$ of excited waves decreases and forms a dip shaped like a hole. In order to make a direct comparison between the observed data and the exact solutions of CGLE, we must find a set of all the coefficients in CGLE, taking into account the Benjamin–Feir instability.[14] Finally, we find all the coefficients in CGLE after much trial and error[7]

$$\mathfrak{C}_{\mathfrak{h}} = \{-1.8, -2.0, 0.5, 1.2, 0.8\}. \quad (16)$$

Let us show numerically that a set of data of BN hole solution (K_{BN}) is almost equivalent to that of $\tilde{K}_{\mathrm{ob}}$. From eqs. (7) and (15), we have $\alpha = 0.228$. Since a pair of wavenumbers k_1 and k_2 is chosen so that eq. (13) is satisfied, as is shown in Fig. 5(a), we can obtain $k_1 = 0.133$ and $k_2 = -0.051$. Therefore, from eq. (9), we obtain the velocity of BN hole $c_h = -0.0793$. It is noted that the selection of wavenumber occurs so that the phase-jump turns into BN hole. From eq. (11), we have the curvature near BN hole $|\kappa| = 0.4031$, as is shown in Fig. 4(b). Substituting these parameters obtained above into eq. (10), we obtain $\sigma = 2.983$ rad, as is shown in Fig. 5(b). From eq. (12), we have also $|b_2/b_1| = 1.02$. As is shown in Fig. 5, we can obtain consistently K_{BN}:

$$K_{\mathrm{BN}} = \tilde{K}_{\mathrm{ob}}. \quad (17)$$

Since the observation of max $|\tilde{c}_h| = 1$ mm/ms for several faster holes is corresponding to eq. (13), as is shown in Fig. 5, the model of BN hole solution is consistent to the observed data (2). This suggests that BN hole solution plays an important role in understanding of one-dimensional cardiac dynamics of phase singularities as a criterion of a healthy heart.

Although it is difficult to estimate all the coefficients in CGLE from the observed data, we found all the corresponding coefficients in our case. Substitution of these coefficients into eq. (15) gives an amplitude profile of BN hole as in Fig. 4(b). Different boundary condition of observed holes from BN hole explains the large deviation from BN hole for $|\kappa\xi| \gg 1$ since we can observe BN holes only in local finite small regions on the IVS. Thus, we have shown that at least one of the observed phase singularities in the excited waves on the IVS can be explained by BN hole solution of CGLE with the coefficients $\mathfrak{C}_{\mathfrak{h}}$.

A variety of complex patterns of phase-defects on the IVS except for observations of amplitude holes will be published elsewhere.[20)]

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1) R. A. Gray, O. A. Mornev, J. Jalife, O. V. Aslanidi, and A. M. Pertsov: Phys. Rev. Lett. **87** (2001) 168104.
2) S. Takagi, A. Pumir, L. Kramer, and V. Krinsky: Phys. Rev. Lett. **90** (2003) 124101.
3) R. A. Gray, A. M. Pertsov, and J. Jalife: Nature **392** (1998) 75.
4) J. Cysyk and L. Tung: Biophys. J. **94** (2008) 1533.
5) H. Kanai: IEEE Trans. Ultrason. Ferroelectr. Freq. Control **52** (2005) 1931.
6) H. Kanai: Ultrasound Med. Biol. **35** (2009) 936.
7) N. Bekki and K. Nozaki: Phys. Lett. A **110** (1985) 133.
8) K. Nozaki and N. Bekki: Phys. Rev. Lett. **51** (1983) 2171.
9) K. Nozaki and N. Bekki: Phys. Rev. Lett. **50** (1983) 1226.
10) K. Nozaki and N. Bekki: Phys. Lett. A **102** (1984) 383.
11) K. Nozaki and N. Bekki: J. Phys. Soc. Jpn. **54** (1985) 2363.
12) K. Nozaki and N. Bekki: J. Phys. Soc. Jpn. **53** (1984) 1581.
13) I. S. Aranson and L. Kramer: Rev. Mod. Phys. **74** (2002) 99.
14) H. Chaté and P. Manneville: Phys. Lett. A **171** (1992) 183.
15) M. C. Cross and P. C. Hohenberg: Rev. Mod. Phys. **65** (1993) 851.
16) N. Bekki: J. Phys. Soc. Jpn. **50** (1981) 659.
17) Y. Kuramoto: *Chemical Oscillations, Waves, and Turbulence* (Springer, Berlin, 1984) Springer Series in Synergetics.
18) J. Burguete, H. Chaté, F. Daviaud, and N. Mukolobwiez: Phys. Rev. Lett. **82** (1999) 3252.
19) K. L. Babcock, G. Ahlers, and D. S. Cannell: Phys. Rev. Lett. **67** (1991) 3388.
20) Y. Harada: Private communication.

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