Measurement of Change in Thickness of Cylindrical Shell Caused by Remote Actuation for Assessment of Viscoelasticity of Arterial Wall

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Abstract—In this paper, for assessment of the viscoelastic property of the arterial wall, a method is proposed for measuring elastic moduli at multiple frequencies by generating the change in internal pressure due to remote cyclic actuation. From basic experiments using a silicone rubber tube, it was found that the change in internal pressure at the ultrasonic beam position (for measurement of the elastic modulus) can be generated by remotely applied actuation. Furthermore, from resultant minute changes in wall thickness of less than 10 $\mu$m measured by the phased tracking method, elastic moduli were obtained at multiple actuation frequencies.

Keywords—Ultrasonics, viscoelasticity, atherosclerosis

I. INTRODUCTION

To characterize tissues in atherosclerotic plaque, we have developed a method, namely, the phased tracking method, for measuring the strain (change in wall thickness) and elasticity of the arterial wall. However, some types of tissue, such as lipids and blood clots, cannot be discriminated from each other based only on elasticity because of the small difference in their elasticity.

For more precise tissue characterization, we are attempting to measure the regional viscoelasticity. To obtain the viscoelastic property, in this paper, a method is proposed for measurement of elastic moduli at multiple frequencies by generating the change in internal pressure due to remote cyclic actuation.

II. METHODS

A. Measurement of change in arterial wall thickness

To measure the small change in thickness, the phase shift of the echo signal due to its both way propagation between the object and the ultrasonic transducer is estimated from two consecutive echo signals [1]. For this purpose, quadrature demodulation is applied to the echo signal, which is received by a 7.5 MHz-linear probe, and then the quadrature and the inphase signals are A/D converted with a 12-bit A/D converter at a sampling frequency of 10 MHz. From the demodulated signal, the phase shift, $\Delta \theta(t)$, between two consecutive echo signals at a time $t$ is obtained from the complex cross correlation function. From the estimated phase shift, $\Delta \theta(t)$, the velocity, $v(t)$, is obtained as follows:

$$ v(t) = -\frac{c_0}{2\omega_0} \frac{\Delta \theta(t)}{T}, \quad (1) $$

where $\omega_0$ and $c_0$ are the center angular frequency of the ultrasonic pulse and the speed of sound, respectively.

By subtracting the displacement, $x_{in}(t)$, of the intimal side of the arterial wall from that, $x_{ad}(t)$, of the adventitial side, the change in thickness, $\Delta h(t)$, of the arterial wall is obtained as follows:

$$ \Delta h(t) = x_{ad}(t) - x_{in}(t) = \int_0^t \{ v_{ad}(t) - v_{in}(t) \} dt. \quad (2) $$

B. Elastic modulus obtained by change in wall thickness

Under in vivo conditions, the artery is strongly restricted in the axial direction. Therefore, deformation in the axial direction can be neglected. By assuming that the arterial wall is incompressible and elastically isotropic, the elastic modulus, $E_0^h$, obtained from the change in wall thickness is defined as follows [2]:

$$ E_0^h = \frac{1}{2} \left( \frac{r_0}{h_0} + 1 \right) \frac{\Delta p(t)}{\Delta h(t)} \frac{h_0}{h_0} \quad \text{ (3)} $$

where $r_0$ and $h_0$ are the inner radius and the wall thickness at the end diastole.

When we describe the change in wall thickness, $\Delta h(t)$, and the change in internal pressure, $\Delta p(t)$, as complex sinusoidal functions, $\Delta h_0 e^{j(2\pi f_{ac} t - \phi)}$ and $\Delta p_0 e^{j2\pi f_{ac} t}$, at an actuation frequency, $f_{ac}$, eq. (3) can be rewritten as the complex elastic modulus as follows:

$$ E_0^h = \frac{1}{2} \left( \frac{r_0}{h_0} + 1 \right) \frac{\Delta p_0}{\Delta h_0} e^{j\phi}, \quad (4) $$

where $\Delta h_0$ and $\Delta p_0$ are the amplitude of the change in wall thickness and that of the change in internal pressure, respectively, and $\phi$ is the phase difference between changes in the internal pressure, $\Delta p(t)$, and the wall thickness, $\Delta h(t)$.

In this paper, the absolute value, $|E_0^h|$, of the complex elastic modulus defined by eq. (4) is obtained from $\Delta h_0$ and $\Delta p_0$.

III. EXPERIMENTAL SYSTEM FOR BASIC EXPERIMENTS

The change in pressure inside a silicone rubber tube is generated by compressing a rubber balloon, which is placed at a position of 40 cm away from the ultrasonic...
probe, with an actuator. This system simulates measurement of the change in wall thickness of the carotid artery due to remote actuation applied at the brachial artery. The change in wall thickness due to the resulting change in internal pressure is measured using ultrasound, and the internal pressure is also measured by a pressure transducer (NEC 9E02-P16) placed inside the tube.

IV. Basic Experimental Results

Fig. 1(a) shows the M-mode image of the silicone rubber tube, and Fig. 1(b) shows the measured internal pressure. From Fig. 1(b), it is found that the change in internal pressure at the measurement position can be generated by remotely applied actuation. By setting two points, A and B, along the ultrasonic beam at a time $t = 0$ in the M-mode image, the velocities, $v_A(t)$ and $v_B(t)$, of these points were obtained by the phased tracking method as shown in Figs. 1(c) and 1(d). The change in thickness, $\Delta h(t)$, of the anterior wall was obtained by integrating the difference between these two velocities. In Fig. 1(e), a minute change in thickness of less than 10 $\mu$m was measured at an actuation frequency of 7.5 Hz. Furthermore, changes in the wall thickness, $\Delta h(t)$, and the internal pressure, $\Delta p(t)$, were measured at multiple frequencies from 7.5 Hz to 20 Hz.

From the amplitude of the change in internal pressure, $\Delta p_0$, and that of the change in wall thickness, $\Delta h_0$, the elastic modulus, $|E_0^h|$, was obtained by eq. (4) at each actuation frequency, $f_{ac}$.

In Fig. 2, the elastic moduli, $|E_0^h|$, measured by the ultrasonic phased tracking method are shown by diamonds, which are plotted as a function of actuation frequency, $f_{ac}$. The elastic modulus, $|E_0^h|$, was measured three times at each actuation frequency, and the elastic modulus, $|E_0^p|$, was measured with high reproducibility. In Fig. 2, it can be seen that the elastic modulus, $|E_0^h|$, increases with actuation frequency, $f_{ac}$.

To validate ultrasonic measurements, in Fig. 2, the incremental elastic moduli, $|E_{inc}|$ [5], are measured using the laser line gauge are shown by squares, which are superimposed on results of the ultrasonic measurement shown by diamonds. In Fig. 2, the elastic moduli measured by the laser line gauge have a tendency similar to those measured by ultrasound. From these results, it is found that the change in the elastic modulus due to the increase in the actuation frequency, $f_{ac}$, can be measured accurately using the ultrasonic phased tracking method.

V. Discussion and Conclusion

In Fig. 2, the elastic modulus, $|E_0^h|$, is observed to increase with actuation frequency, $f_{ac}$. The reason for this can be considered as follows: When we assume the Voigt model as a viscoelastic model of the silicone rubber, the complex elastic modulus, $E_{Voigt}^*$, is expressed as follows:

$$E_{Voigt}^* = \sqrt{E_0^2 + (2\pi f_{ac}\eta)^2} \cdot e^{i\phi},$$

where $E_0$ and $\eta$ are the static elastic modulus and the viscosity constant, respectively.

In eq. (5), it is found that the absolute value of the complex elastic modulus of the Voigt model increases with the frequency of the applied stress. Therefore, measurement of elastic moduli, $|E_0^h|$, at multiple frequencies has potential for assessment of the viscoelastic property.

![Fig. 1. (a) M-mode image of the silicone rubber tube. (b) Internal pressure. (c) Velocity, $v_A(t)$, of point A. (d) Velocity, $v_B(t)$, of point B. (e) Change in thickness, $\Delta h(t)$, of the anterior wall (actuation frequency of 7.5 Hz).](image1)

![Fig. 2. Elastic moduli, $|E_0^h|$ and $|E_{inc}|$, at each actuation frequency, $f_{ac}$. Diamonds and squares show results obtained by ultrasound and those obtained by the laser line gauge, respectively.](image2)

REFERENCES

