A method for evaluation of regional elasticity of arterial wall with non-uniform wall thickness by measurement of its change in thickness during an entire heartbeat

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Abstract—The average elastic modulus of the artery has been evaluated previously by measurement of the pulse wave velocity and the change in arterial diameter during one heartbeat. On the other hand, we aim to measure the regional elastic modulus of the arterial wall by measurement of its small change in thickness during an entire heartbeat [1]. In this paper, a method is proposed for measurement of the regional elastic modulus even if the artery has non-uniform wall thickness such like an atherosclerotic plaque. In basic experiment, the proposed method is validated using the silicone rubber tube as a model of the artery. In in vivo experiments, the spatial distribution of the elastic modulus is measured around the human carotid atherosclerotic plaque. From this result, difference in the regional elasticity around the atherosclerotic plaque can be observed and the soft region is found in the plaque. Such information seems to be useful for diagnosis whether the plaque easily ruptures or not.

I. INTRODUCTION

The steady increase in cases of myocardial infarction and cerebral infarction, which are considered to be mainly caused by atherosclerosis, becomes a more and more serious problem. Therefore, it is important to diagnose atherosclerosis in early stage, and noninvasive measurement of the arterial elasticity is one of effective methods.

Various methods for measurement of the arterial elasticity have been proposed previously. Measurements of the velocity of the pulse wave propagating along the artery and the change in diameter due to heartbeat are some of these methods, and the elastic moduli can be obtained by the Moens-Korteweg’s equation [2] and the pressure elasticity has been proposed previously. Measurements of the arterial elasticity is one of effective methods.

In our study, for the purpose of noninvasive diagnosis of atherosclerosis, we aim to measure the elasticity in each local region of a few millimeters in size on the arterial wall using ultrasound. It is realized by measurement of the small change in thickness of the arterial wall due to heartbeat [4]. It can be measured at each local region which corresponds to the focal area of the ultrasonic beam. In this paper, we propose a method for calculation of the regional elastic modulus of the arterial wall from the measured change in thickness even if the artery has non-uniform wall thickness such like an atherosclerotic plaque.

II. A METHOD FOR CALCULATING ELASTIC MODULUS FROM CHANGE IN WALL THICKNESS

Under in vivo condition, the artery is strongly restricted in the axial direction, therefore, two dimensional stress-strain relationship can be assumed. We assume that the pressure distribution in the arterial wall changes linearly due to the distance from the inner surface of the arterial wall (Fig. 1). When we consider the small region of the arterial wall divided into N layers (Fig. 2), the pressure, \( p_n(t) \), at the inner surface of the nth layer is defined as:

\[
p_n(t) = \frac{N - n + 1}{N} p_i(t) - \frac{n}{N} \rho_a,
\]

where \( p_i(t) \) and \( p_a \) are the pressure in lumen and the atmospheric pressure. According to the region PP’QQ’ in Fig. 2, y-axis component, \( F_y \), of the force caused by pressures is expressed as follows:

\[
F_y = \left[ \{p_n(t) - p_a\} \rho_n \cos \theta + \{p_{n+1} - p_a\} \left( \frac{h_0}{N} \right) \cos \theta \right] \cos \theta
\]

where \( \rho_n \) and \( h_0 \) are the inner radius and the thickness of the nth layer, respectively. In Eq. (2), the first term and the second term are the radial forces applied to PP’ and QQ’ in Fig. 2, respectively, and \( \cos \theta_0 \) means that the right hand side of Eq. (2) shows the y-axis component. The y-axis component, \( T_n \sin \theta_0 \), of the tension, \( T_n \), of the nth layer balances with the integration of \( F_y \) with respect...
to $\theta$ from $-\theta_0$ to $\theta_0$.

$$
\int_{-\theta_0}^{\theta_0} \left( \frac{\rho_n \omega - N - n}{h_0} \right) \frac{h_0}{N} \left( p_i(t) - p_a \right) \cos \theta d\theta
$$

$$
= 2 \left( \frac{\rho_n \omega - N - n}{h_0} \right) \frac{h_0}{N} \left( p_i(t) - p_a \right) \sin \theta_0
$$

$$
= 2T_n \sin \theta_0.
$$

The circumferential stress, $\sigma_{\theta n}(t)$, of the $n$th layer is obtained by dividing the tension, $T_n$, by the thickness, $h_0/N$, of the layer.

$$
\sigma_{\theta n}(t) = \frac{T_n}{(h_0/N)} = \left\{ \frac{\rho_n \omega}{h_0} - (N - n) \right\} \frac{p_i(t) - p_a}{N}.
$$

The radial stress, $\sigma_{r n}(t)$, of the $n$th layer is defined as the average of the pressures in the radial direction.

$$
\sigma_{r n}(t) = \frac{2N - 2n + 1}{2N} \{ p_i(t) - p_a \}.
$$

By expressing $p_i(t) - p_a$ by the sum of the diastolic pressure, $p_d$, and the pressure increment, $\Delta p(t)$, from $p_d$, which is described by

$$
p_i(t) - p_a = p_d + \Delta p(t),
$$

the circumferential and radial incremental stresses, $\Delta\sigma_{\theta n}(t)$ and $\Delta\sigma_{r n}(t)$, are expressed as follows:

$$
\Delta\sigma_{\theta n}(t) = \left\{ \frac{\rho_n \omega + \Delta \rho_n(t)}{h_0} + \Delta h_n(t) \right\} \frac{p_d + \Delta p(t)}{N} - \left\{ \frac{\rho_n \omega}{h_0} - (N - n) \right\} \frac{p_d}{N},
$$

$$
\approx \left\{ \frac{\rho_n \omega}{h_0} - (N - n) \right\} \frac{\Delta p(t)}{N},
$$

$$
\Delta\sigma_{r n}(t) = \frac{2N - 2n + 1}{2N} \{ p_d + \Delta p(t) \} + \frac{2N - 2n + 1}{2N} p_d
$$

$$
= -\frac{2N - 2n + 1}{2N} \Delta p(t),
$$

where $\rho_n \omega + \Delta \rho_n(t)$ and $h_0/N + \Delta h_n(t)$ are approximated by $\rho_n \omega$ and $h_0/N$ because the change in inner radius, $\Delta h_n(t)$, and the change in thickness, $\Delta h_n(t)$, of the $n$th layer are much smaller than $\rho_n \omega$ and $h_0/N$, respectively.

The radial incremental strain, $\Delta\varepsilon_{r n}(t)$, of the $n$th layer is expressed by the radial and circumferential incremental stresses, $\Delta\sigma_{r n}(t)$ and $\Delta\sigma_{\theta n}(t)$, as follows:

$$
\Delta\varepsilon_{r n}(t) = \frac{\Delta\sigma_{r n}(t)}{E_{r n}} - \nu \frac{\Delta\sigma_{\theta n}(t)}{E_{\theta n}},
$$

where $E_{r n}$, $E_{\theta n}$, and $\nu$ are the radial and circumferential elastic moduli and the Poisson’s ratio, respectively.

By substituting Eqs. (7) and (8) into Eq. (9), we define the elastic modulus, $E_{\theta n}$, obtained from the change in thickness by

$$
E_{\theta n} = \frac{1}{2} \left\{ \frac{\rho_n \omega}{h_0} + (N - n + 1) \right\} \frac{\Delta p(t)}{N},
$$

where the arterial wall is assumed to be incompressible ($\nu=0.5$) and elastically isotropic ($E_{r n} = E_{\theta n}$).

![Fig. 1. The assumed pressure distribution in the arterial wall.]

![Fig. 2. The small region of the arterial wall.]

**III. Basic experiments**

The proposed formula of the elastic modulus, $E_{\theta n}$, of Eq. (10) is validated by the following basic experiments employing a silicone rubber tube (tube-A), which has non-uniform wall thickness and two layers, as a model of the artery. Figure 3 shows the schematic diagram of the experimental setup. In this system, the pulse pressure is
generated by the artificial heart. The change in wall thickness caused by the pulse pressure is measured by ultrasonic imaging utilizing the phased tracking method [4], and the pressure inside the silicone rubber tube is simultaneously measured by the pressure transducer (NEC, 9E02-P16).

Figure 4(a) is the cross sectional B-mode image of the silicone rubber tube obtained by the standard ultrasonic diagnostic equipment. The M-mode image shown in Fig. 4(b) is obtained by repeatedly transmitting the ultrasonic pulses to the silicone rubber tube. Figures 4(c) and 4(d) are the drive signal of the artificial heart and the pressure pulses to the silicone rubber tube. Figures 4(e) and 4(f) are obtained by repeatedly transmitting the ultrasonic beams utilizing the phased tracking method [4], and the pressure inside the silicone rubber tube is simultaneously measured by the pressure transducer (NEC, 9E02-P16).

By manually setting the points, \( x_1 \) and \( x_2 \), at the timing of the rising of the drive signal (just before the artificial heart begins to pump), velocities, \( v_1(t) \) and \( v_2(t) \), of points \( x_1 \) and \( x_2 \) are measured as shown in Fig. 4(e) and 4(f), and the change in thickness, \( \Delta h_1(t) \), of the inner layer is obtained as shown in Fig. 4(h) by integrating the difference between these two velocities. In the same manner, the change in thickness, \( \Delta h_2(t) \), of the external layer, which is shown in Fig. 4(i), is obtained by measuring velocities, \( v_2(t) \) and \( v_3(t) \), at the points \( x_2 \) and \( x_3 \).

From the measurements of changes in thickness, \( \Delta h_1(t) \) and \( \Delta h_2(t) \), the elastic modulus of each layer is obtained at each insonification angle, \( \theta \), from \(-20^\circ\) to \(20^\circ\) as shown in Fig. 5. In Figs. 5(a) and 5(b), the elastic moduli, \( E_{\theta_1}^h \) and \( E_{\theta_2}^h \), of the inner and the external layer are plotted as diamonds. From the results of Figs 5(a) and 5(b), \( E_{\theta_1}^h \) and \( E_{\theta_2}^h \) of tube-A measured at each insonification angle agree well with the circumferential elastic modulus obtained from the static pressure-diameter test as plotted by dashed line. In the pressure-diameter test, the silicone rubber tube (tube-B) with uniform wall thickness made of the same material as tube-A is used as a specimen.

On the other hand, the pressure elastic modulus, \( E_p \), is defined by

\[
E_p = \frac{r_0}{2h_0} \frac{\Delta p(t)}{\Delta d(t)},
\]

where \( d_0 \) and \( \Delta d(t) \) are the inner diameter and the change in inner diameter, respectively. \( E_p \), which is obtained from the change in diameter as measured previously, is 2.1 MPa with respect to tube-A. It is different from the results of the pressure-diameter test.

From these results, it is shown that the proposed method has an advantage in measurement of the regional elastic modulus of the tube with non-uniform wall thickness.

Fig. 3. The experimental setup of the basic experiment.

Fig. 4. Measurement of the change in wall thickness of tube-A. (a) B-mode image. (b) M-mode image. (c) B-mode image of the human carotid artery. (d) The drive signal. (e) and (f) are the drive signal of the artificial heart. (g) Velocities of points \( x_1 \), \( x_2 \), and \( x_3 \). (h) and (i) Changes in thickness of the inner and the external layer.

IV. In vivo Experimental Results

The proposed method is applied to in vivo measurement at the human carotid atherosclerotic plaque. Figure 6(a) shows the B-mode image of the human carotid artery. Figure 6(b) is obtained by scanning the ultrasonic beam
around the atherosclerotic plaque shown by the region surrounded by dashed line in Fig. 6(a). Figure 7(a) is the M-mode image of the scanned position, $l_{10}$, shown in Fig. 6. Figures 7(b) and 7(c) are the electrocardiogram and phonocardiogram, respectively. The arterial wall is divided into 7 layers by manually setting the points, where the velocities are obtained, at the timing of the R-wave of the electrocardiogram. From the velocities of these points, the change in thickness, $\Delta h_n(t)$, of each layer is obtained, and the elastic modulus, $E^{h}_n$, is calculated from the resultant change in thickness.

By applying the same procedure to each scanned position, the elastic modulus of each region is color-coded and superimposed on the reconstructed B-mode image as shown in Fig. 8. In Fig. 8, the difference in the regional elasticity around the atherosclerotic plaque can be displayed by the proposed method.

V. Conclusions

In this paper, we proposed a method to obtain the regional elastic modulus of the arterial wall by measuring the change in wall thickness due to heartbeat. The spatial distribution of the regional elastic modulus measured by the proposed method will be useful for diagnosis whether the atherosclerotic plaque easily ruptures or not.

REFERENCES


