Tohoku University

Tohoku University

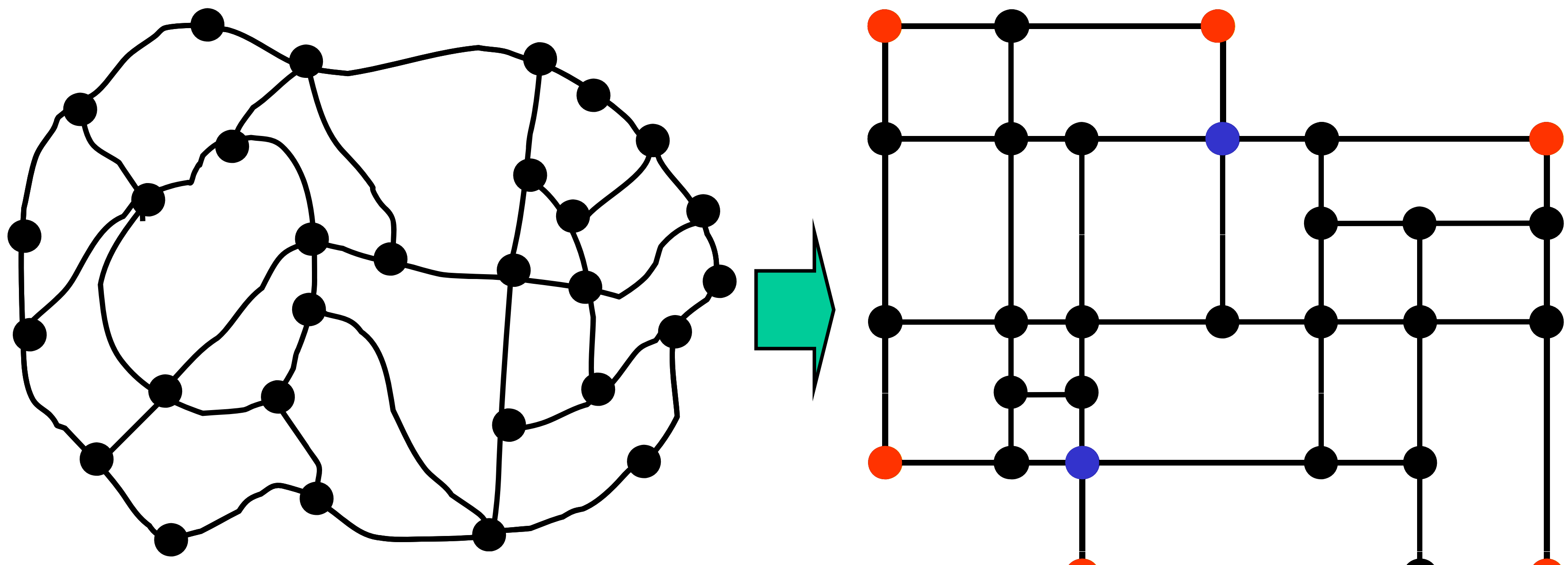
Sendai

360km

Tokyo

Inner Rectangular Drawings of Plane Graphs

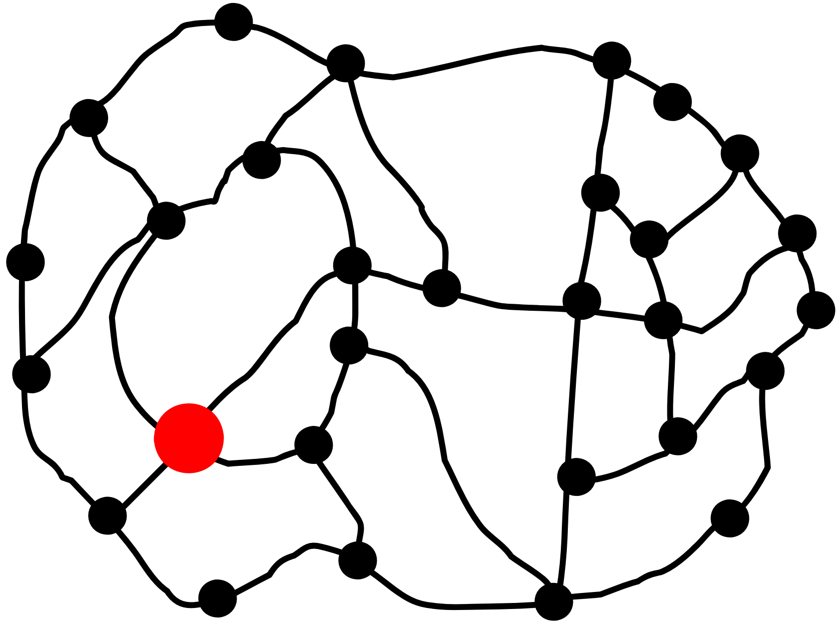
—Application of Graph Drawing to VLSI Layout—



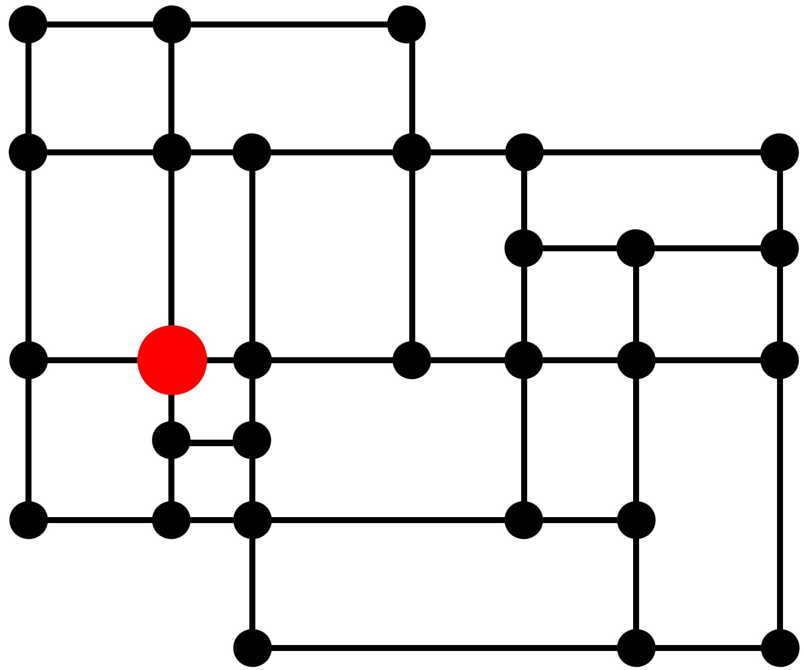
Takao Nishizeki

Tohoku University

Inner Rectangular Drawing



a plane graph *G*



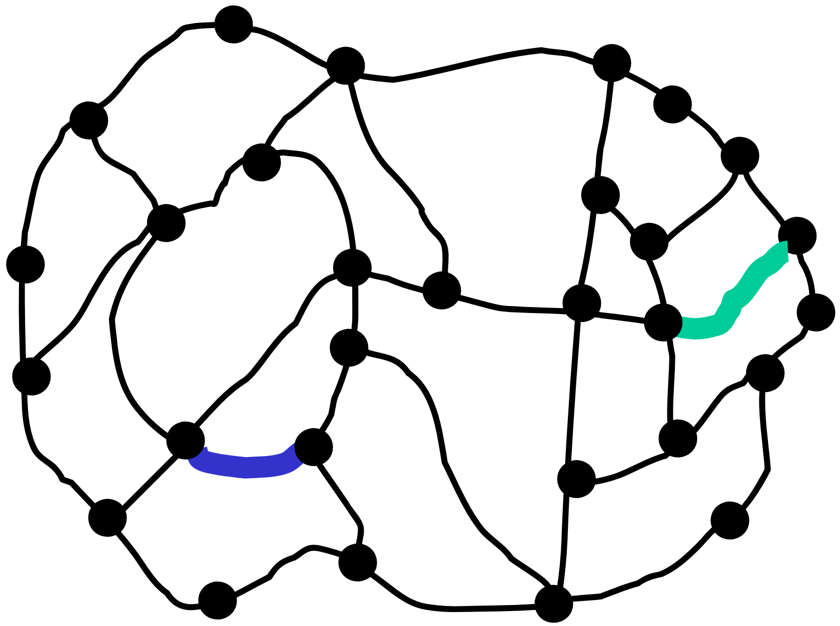
an inner rectangular drawing of *G*

1:each vertex is drawn as a point

2:each edge is drawn as a horizontal or vertical line segment

3:all inner faces are drawn as rectangles

Inner Rectangular Drawing



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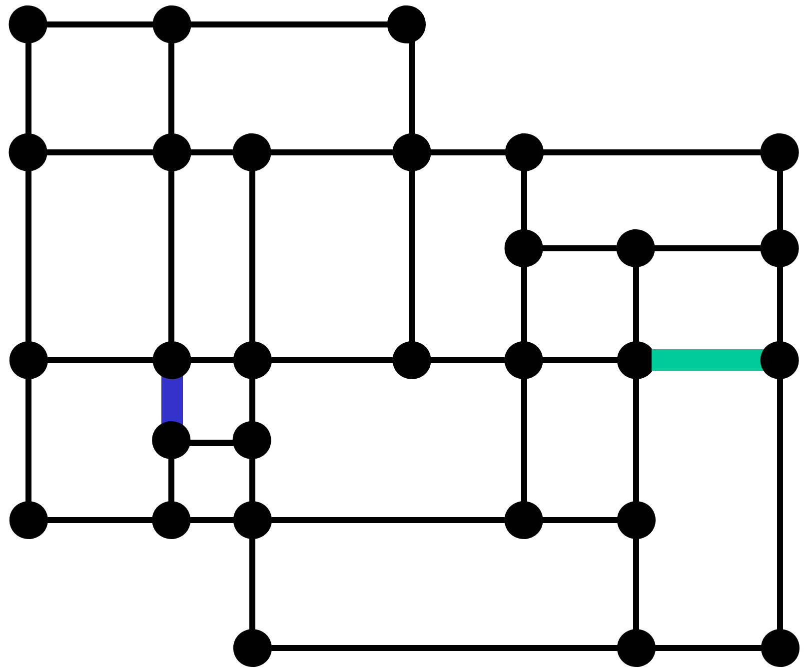

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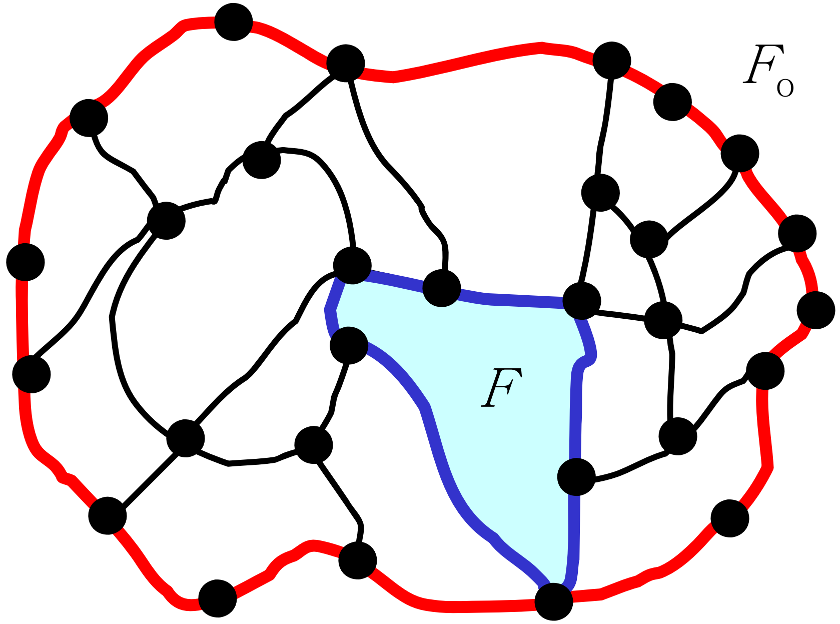
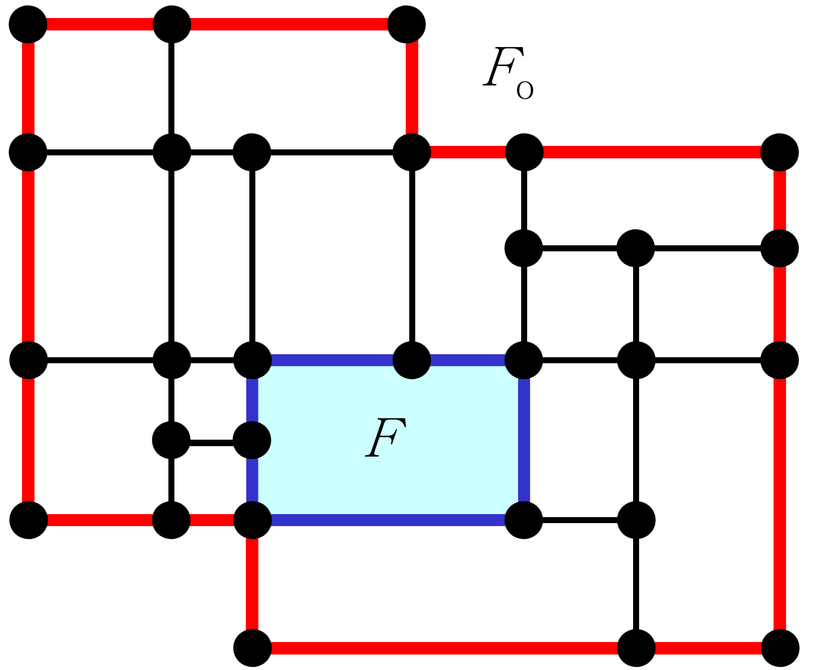


a plane graph G

rectilinear polygon



an inner rectangular drawing of G

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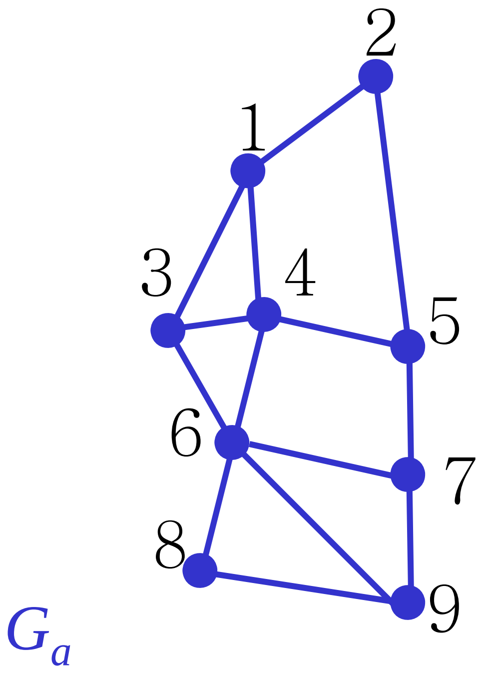
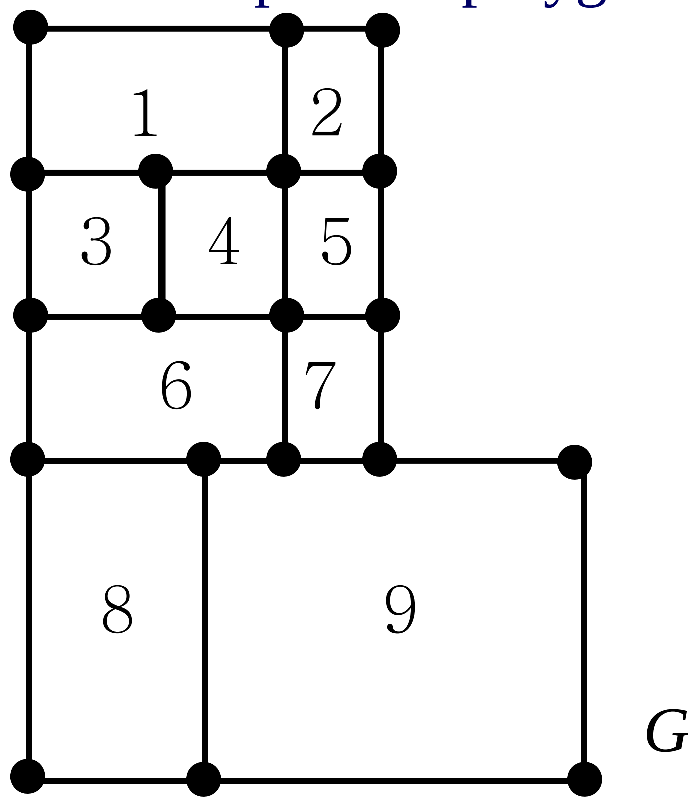
3:all inner faces are drawn as rectangles

Application

VLSI floor planning

The outer boundary of a VLSI chip
is often an axis-parallel polygon





Vertex: module
edge : adjacency among modules

Inner rectangular drawing

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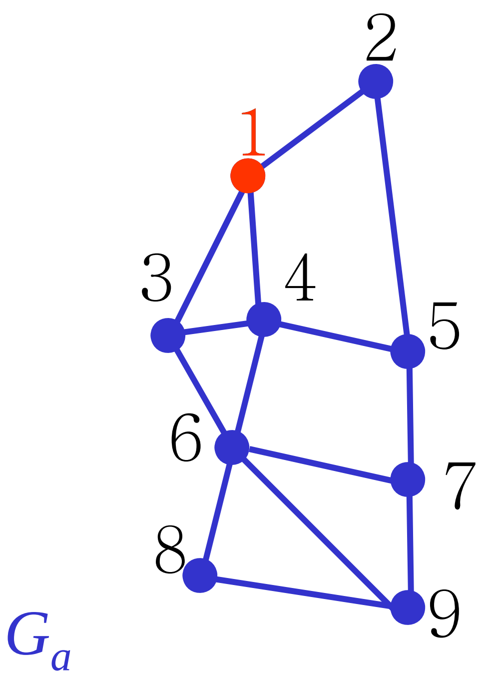
VLSI floor planning



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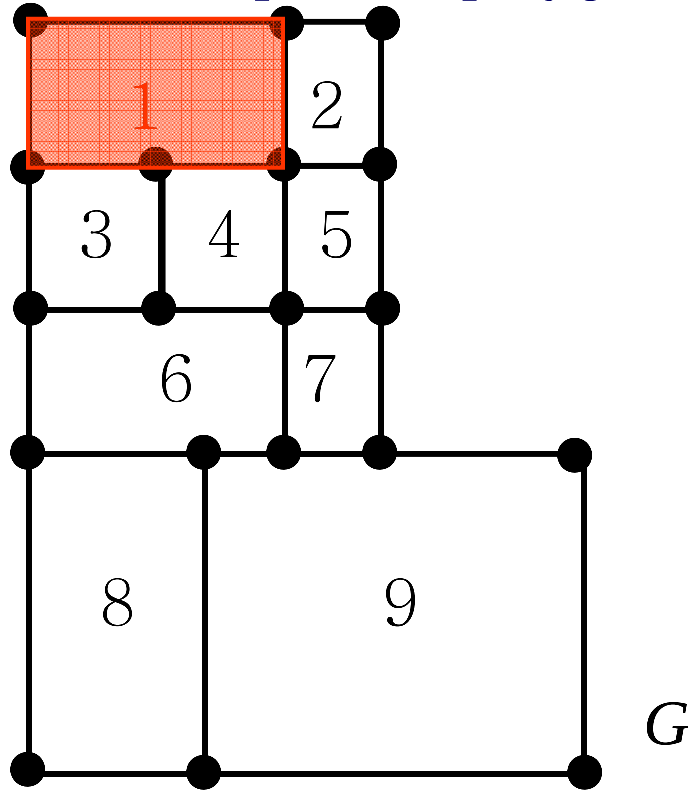

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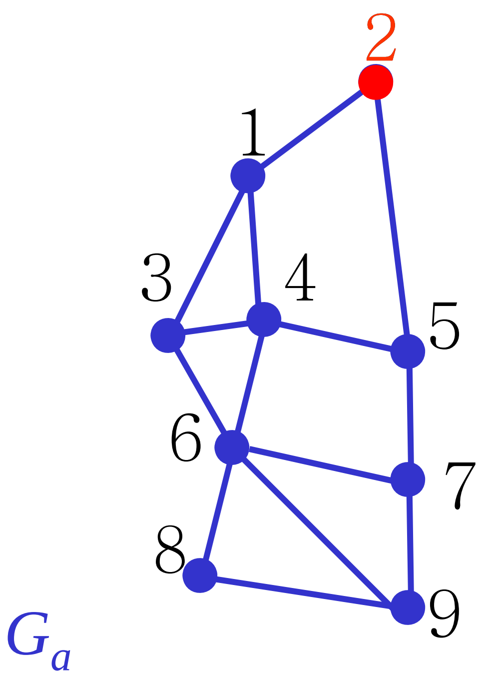

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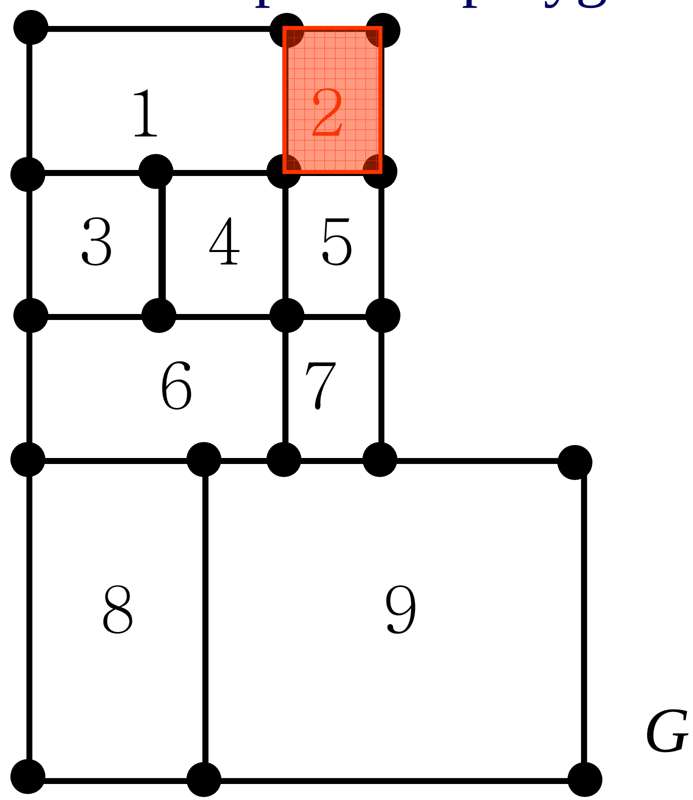

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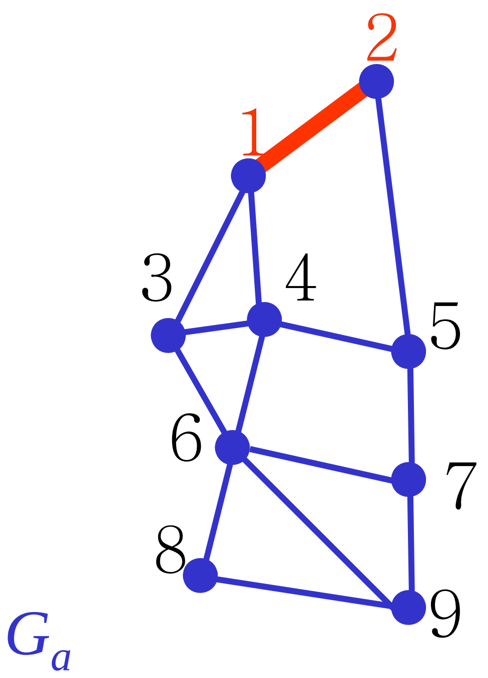

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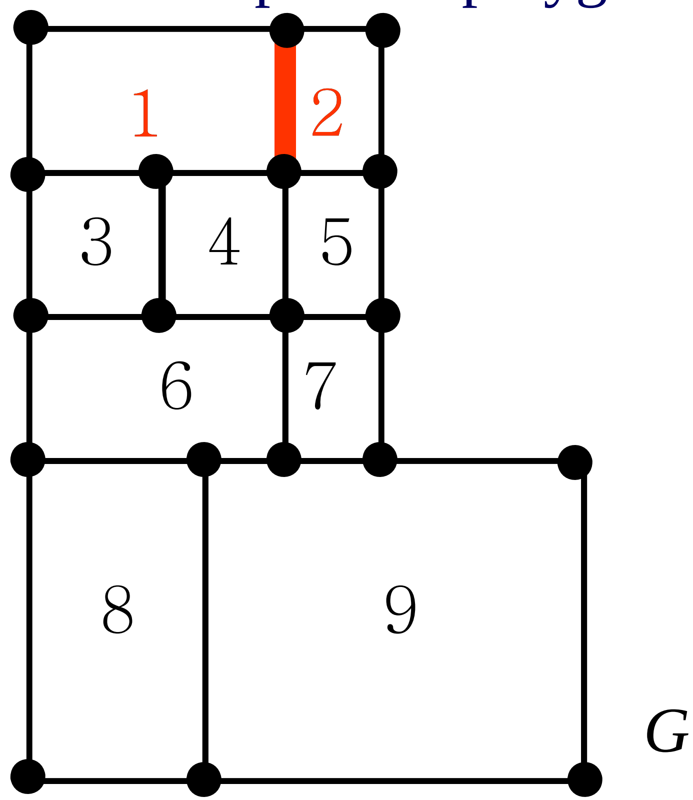

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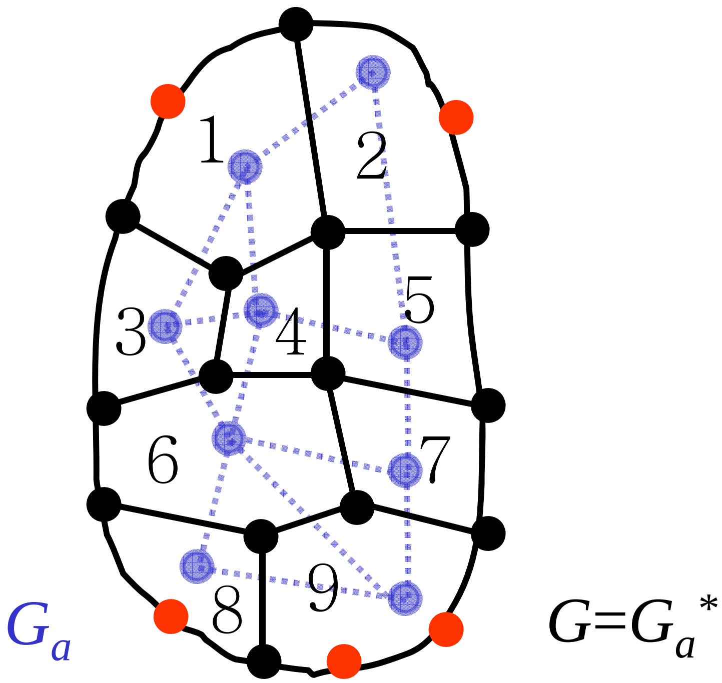

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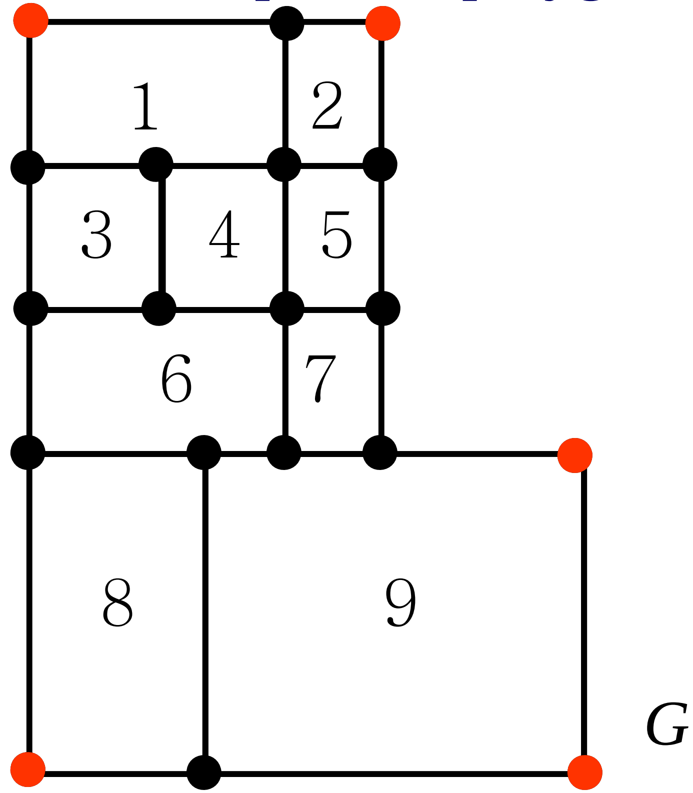

Inner rectangular drawing

Application

VLSI floor planning

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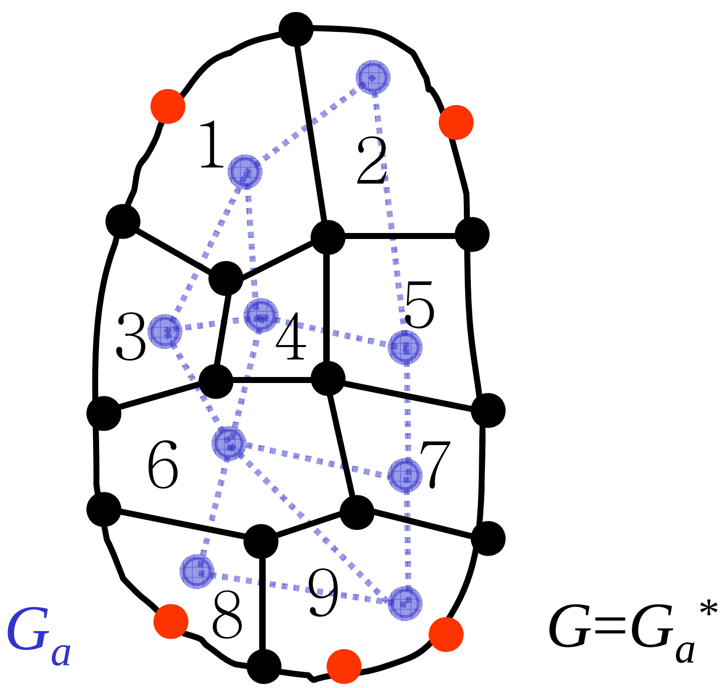

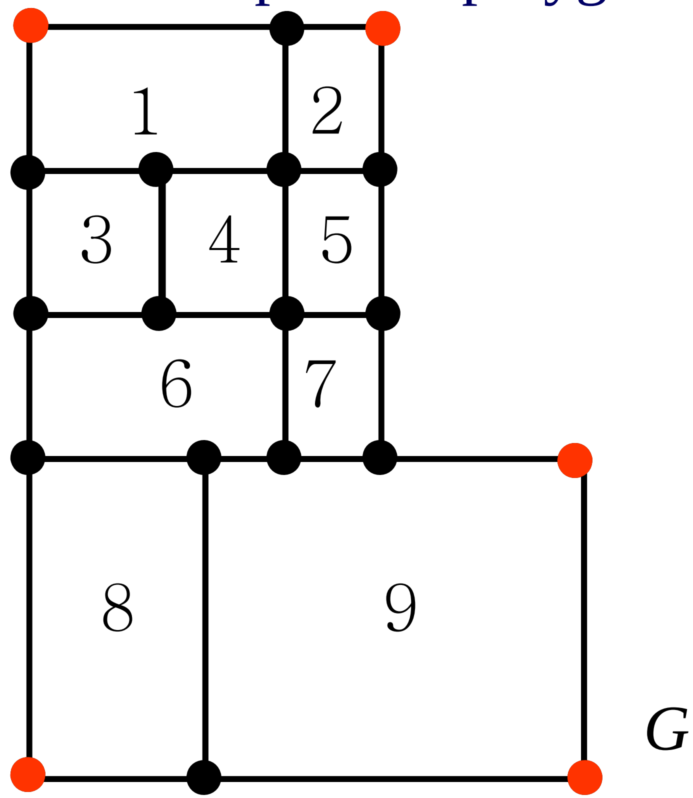

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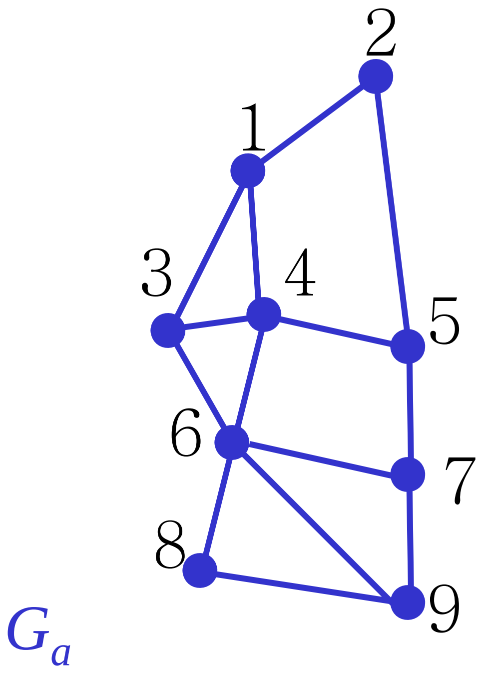
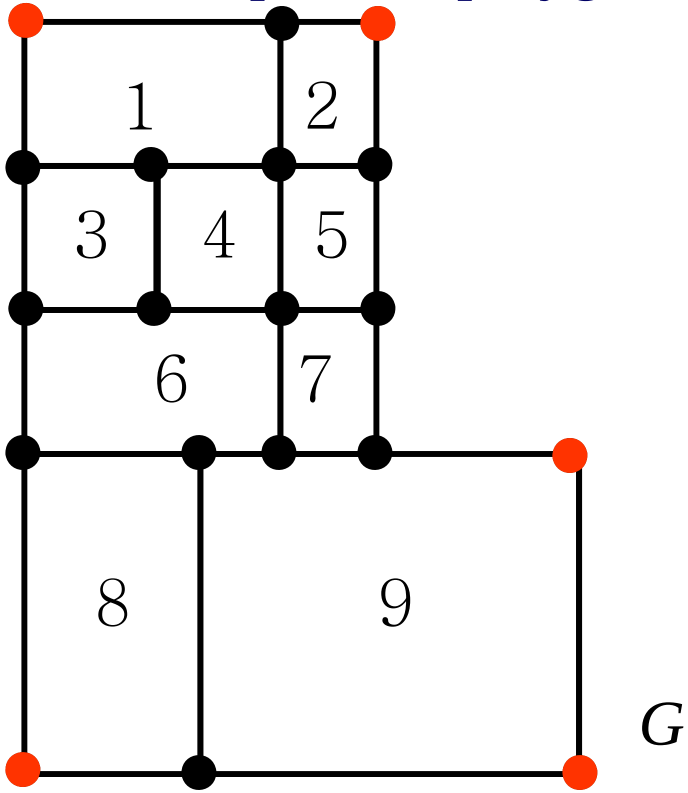




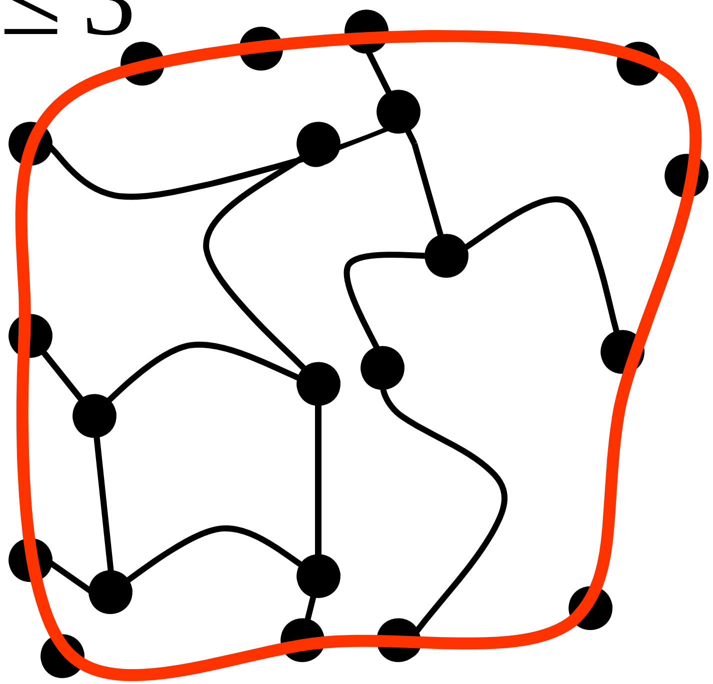
Vertex: module
edge : adjacency among modules

Inner rectangular drawing

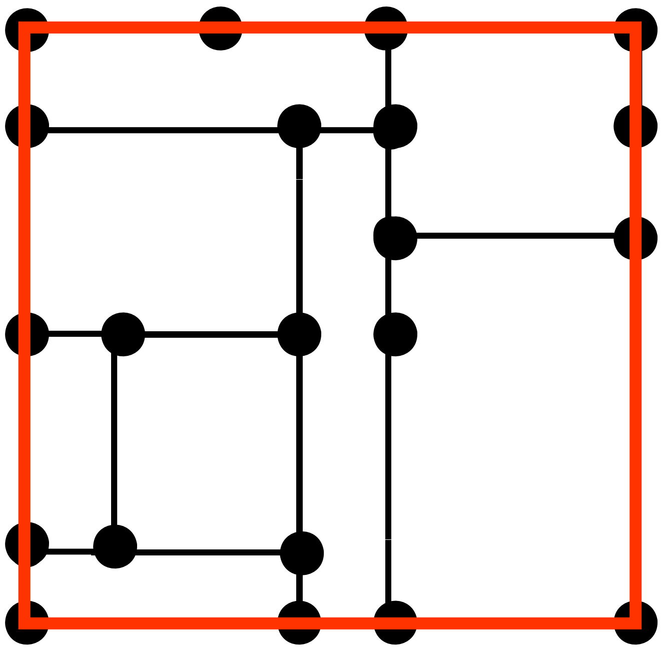
Known Result

a necessary and sufficient condition for the existence of a rectangular drawing of G with $\Delta \leq 3$ [T84,RNN98] and a linear algorithm for $\Delta \leq 3$ [RNN98,BS88,KH97]

$\Delta \leq 3$



a plane graph G

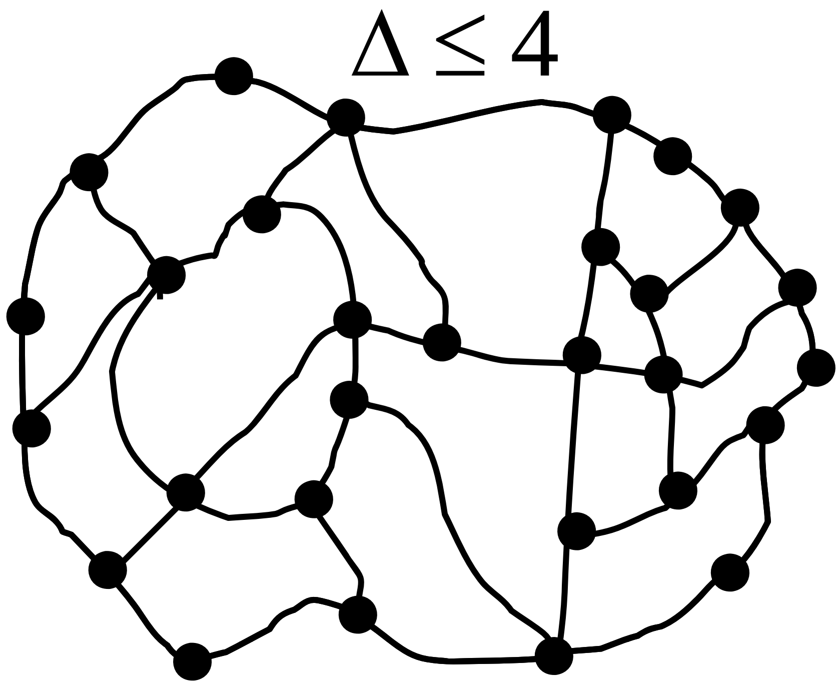


a rectangular drawing of G

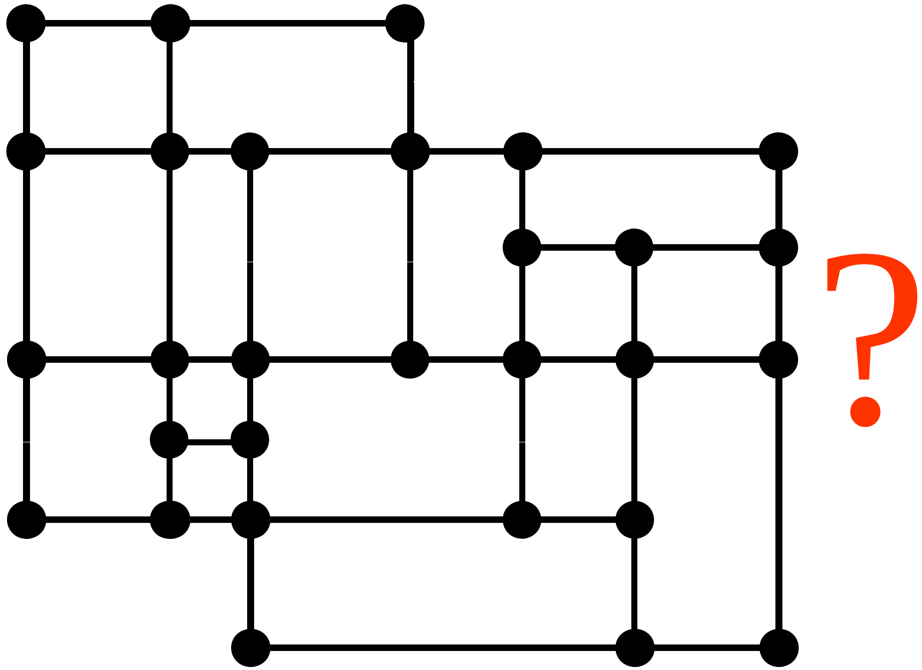
Open Problem

a necessary and sufficient condition for the existence of an inner rectangular drawing of *G* (with $\Delta \leq 4$)?

efficient algorithm to find an inner rectangular drawing of *G* (with $\Delta \leq 4$)?



a plane graph *G*



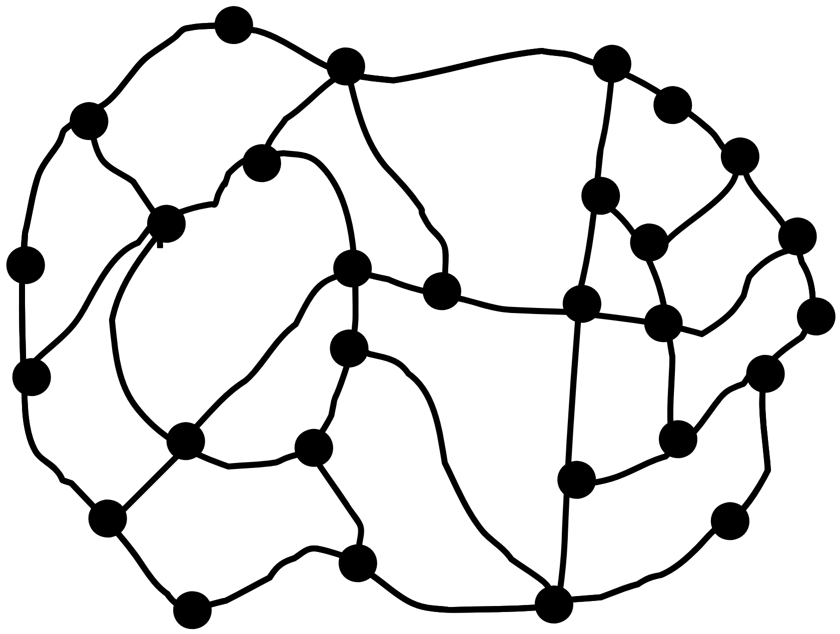
an inner rectangular drawing of *G*

Our Results

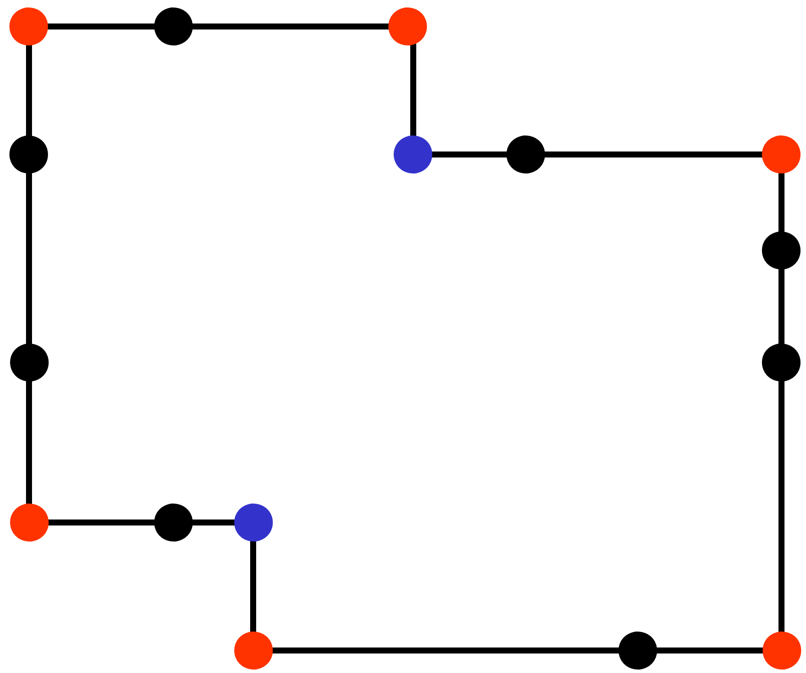
1: a necessary and sufficient condition for the existence of an inner rectangular drawing of G.

Our Results

2: $O(n^{1.5}/\log n)$ algorithm to find an inner rectangular drawing of G if a "sketch" of the outer face is given.



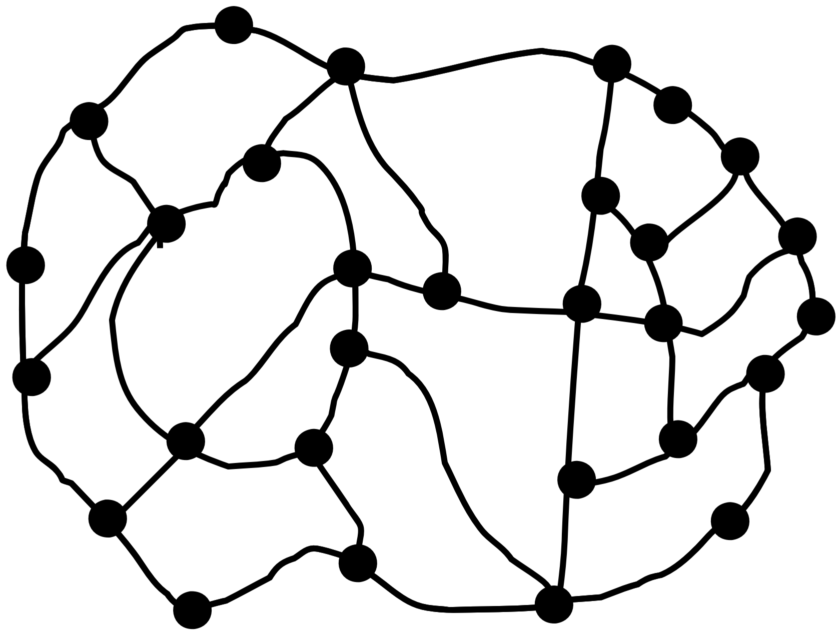
a plane graph G



a "sketch" of the outer face

Our Results

2: $O(n^{1.5}/\log n)$ algorithm to find an inner rectangular drawing of G if a "sketch" of the outer face is given.



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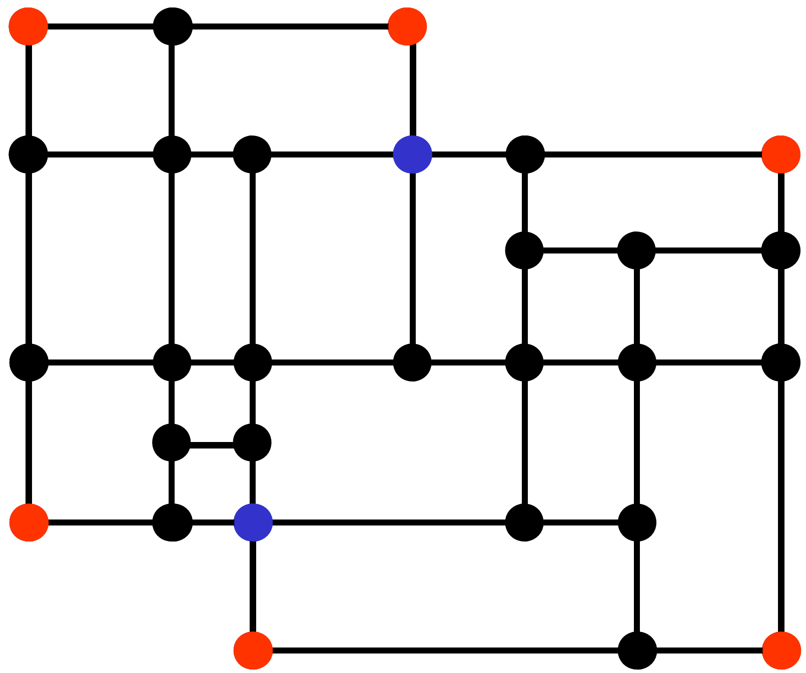
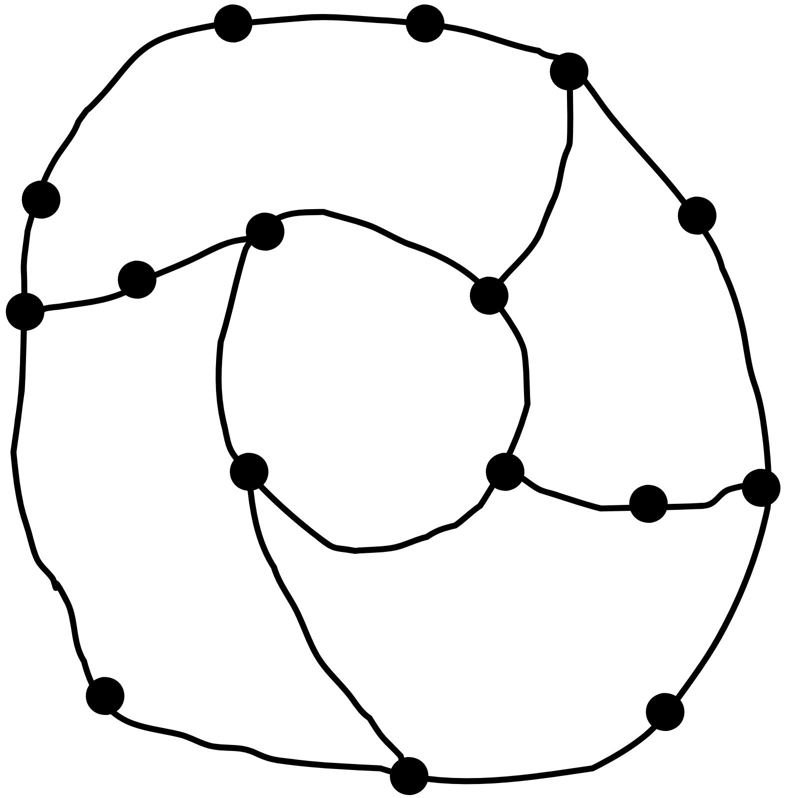


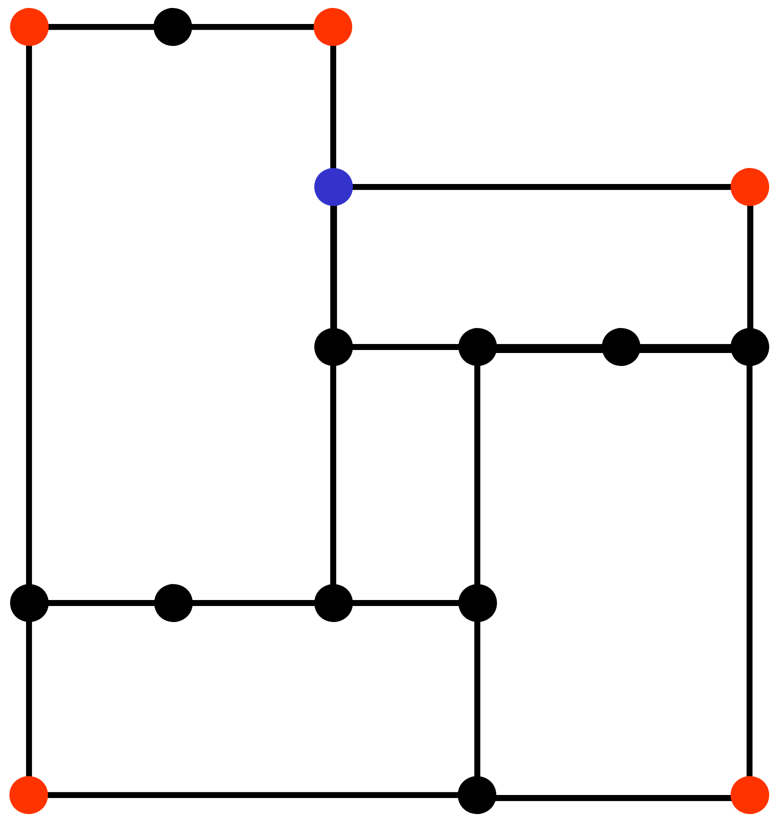
an inner rectangular drawing of G

Our Results

3: a polynomial time algorithm to find an inner rectangular drawing of *G* in a general case, where a sketch is not always given.



a plane graph *G*



an inner rectangular drawing of *G*

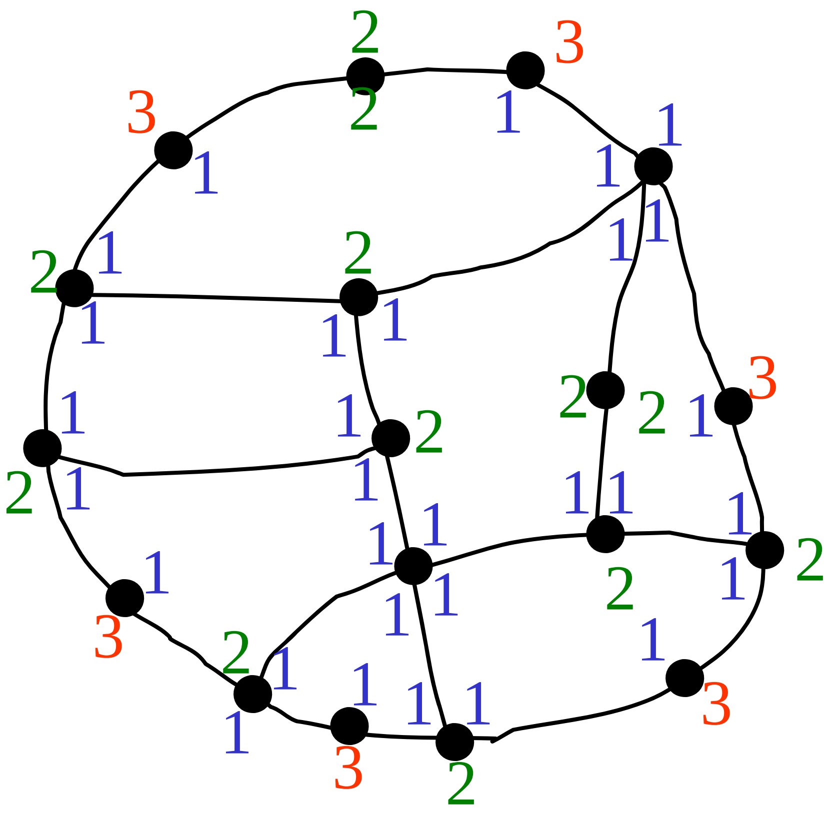
1:A necessary and sufficient condition for the existence of an inner rectangular drawing of G.

2: $O(n^{1.5} / \log n)$ time algorithm to find an inner rectangular drawing of G if a sketch of the outer face is given.

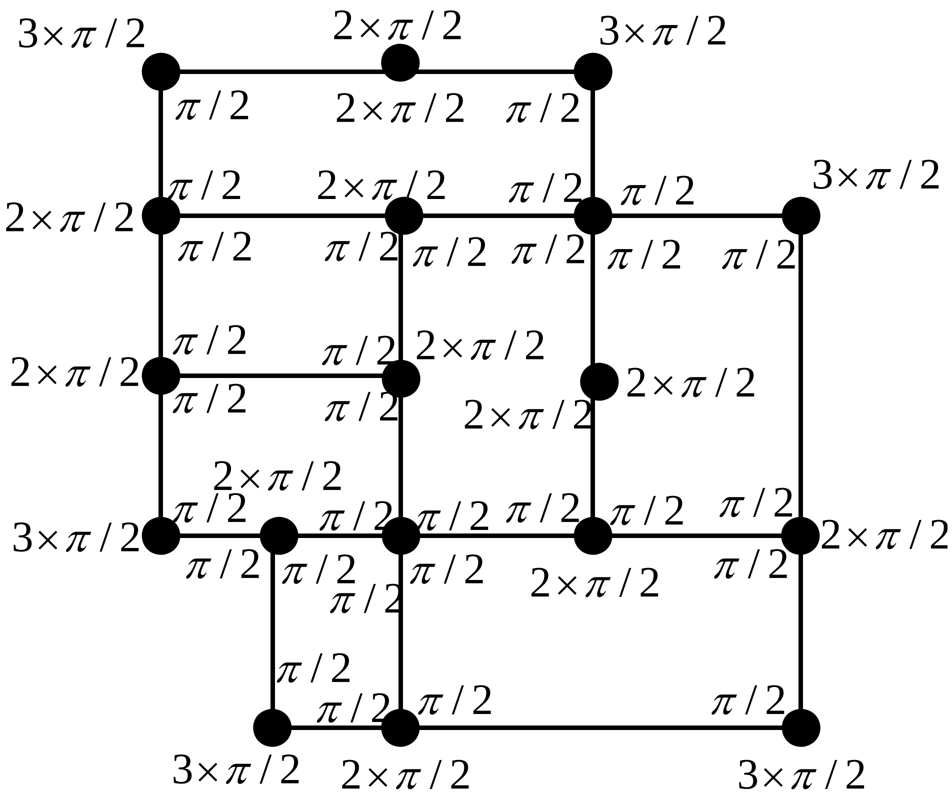
3: a polynomial time algorithm to find an inner rectangular drawing of G in a general case, where a sketch is not always given.

Definition of Labeling

$1 \times \pi / 2 \quad 2 \times \pi / 2 \quad 3 \times \pi / 2$



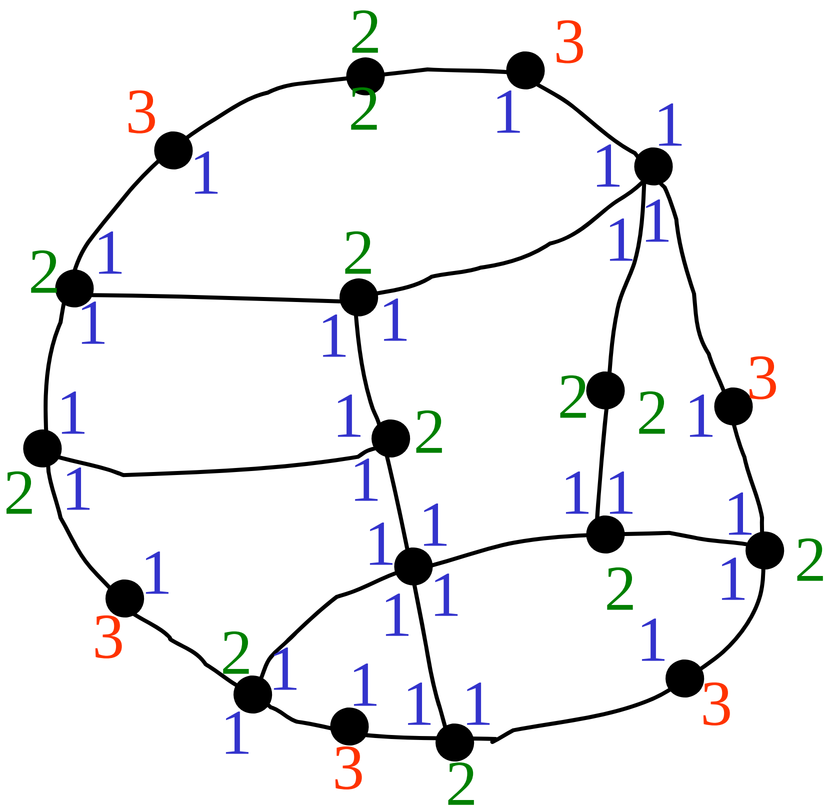
a plane graph G



an inner rectangular drawing of G

Consider a labeling which assigns label 1,2 or 3 to every angle of G

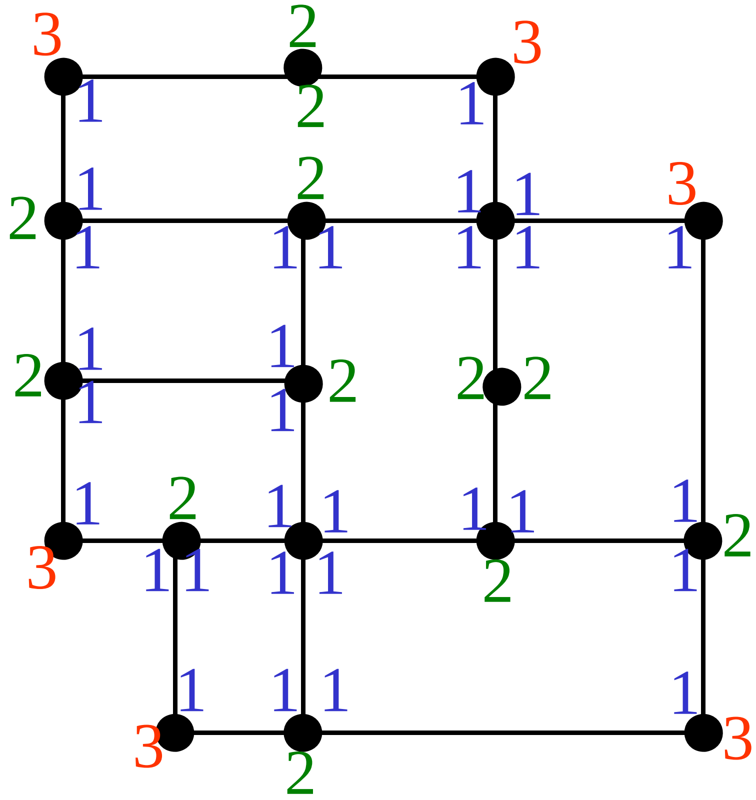
Definition of Labeling



a plane graph G

$1\times\pi/2$ $2\times\pi/2$ $3\times\pi/2$

1 2 3



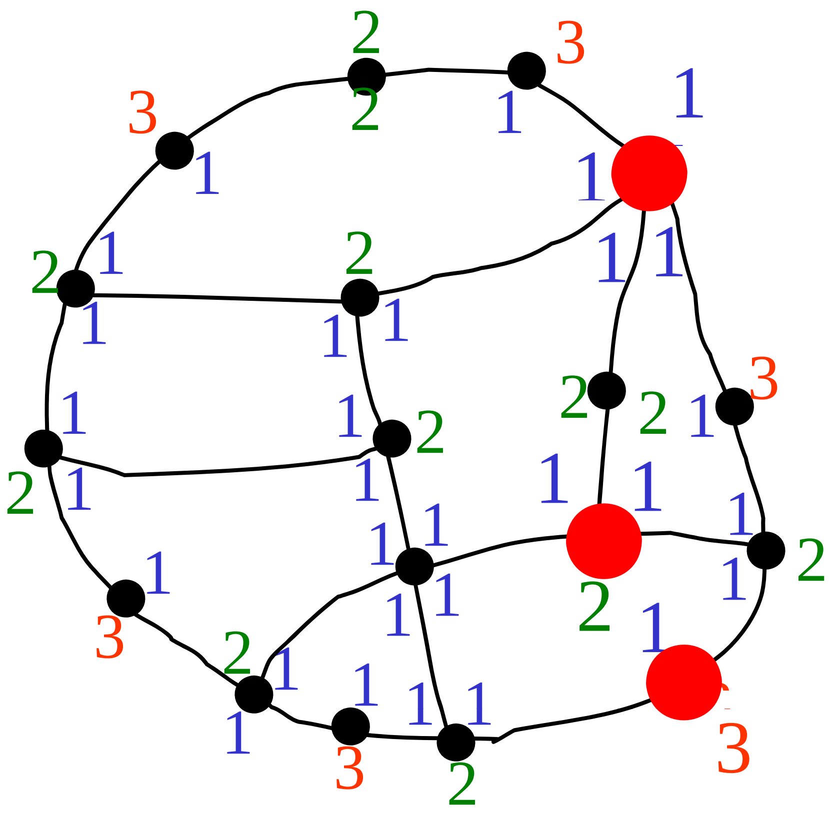
an inner rectangular drawing of G

Consider a labeling which assigns label 1,2 or 3 to every angle of G

Regular labeling

A regular labeling satisfies the following three conditions (a)-(c)

(a) the labels of all the angles of each vertex v total to 4;



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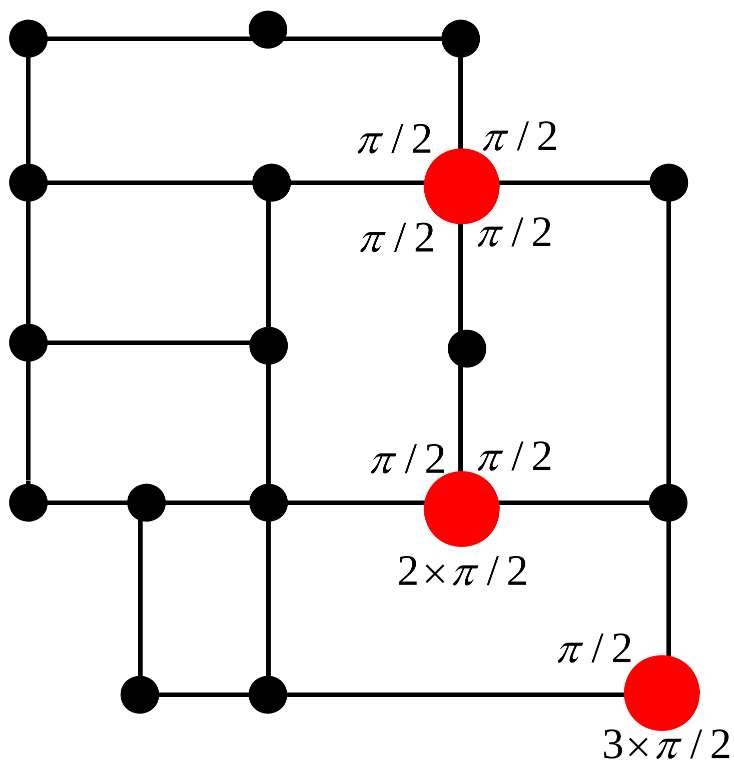
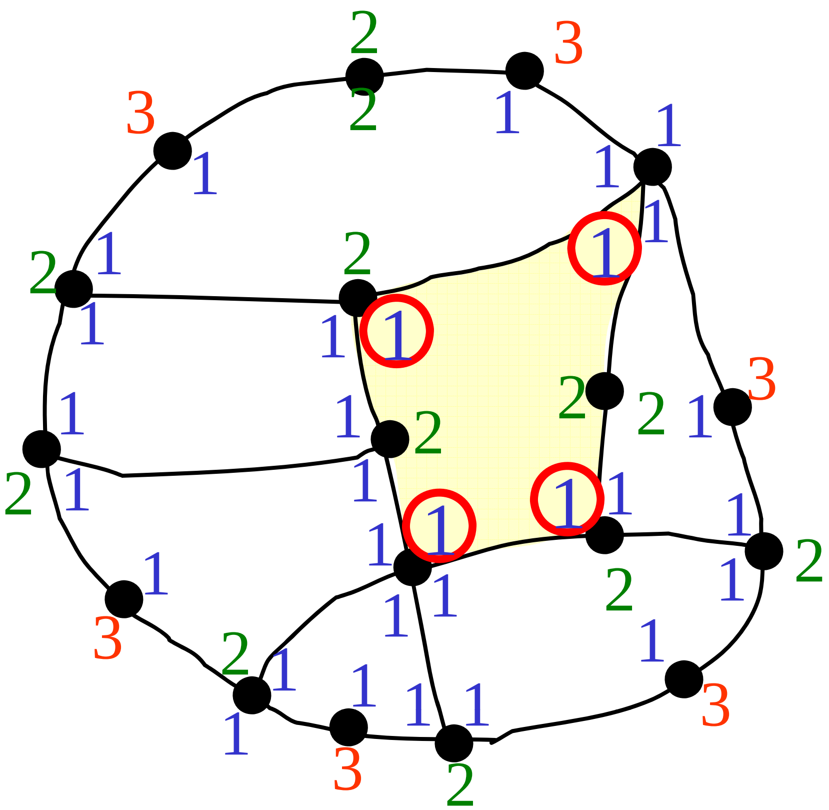


an inner rectangular drawing of G

Regular labeling

(b) the labels of any inner angles is 1 or 2, and any inner face has exactly four angles of label 1;



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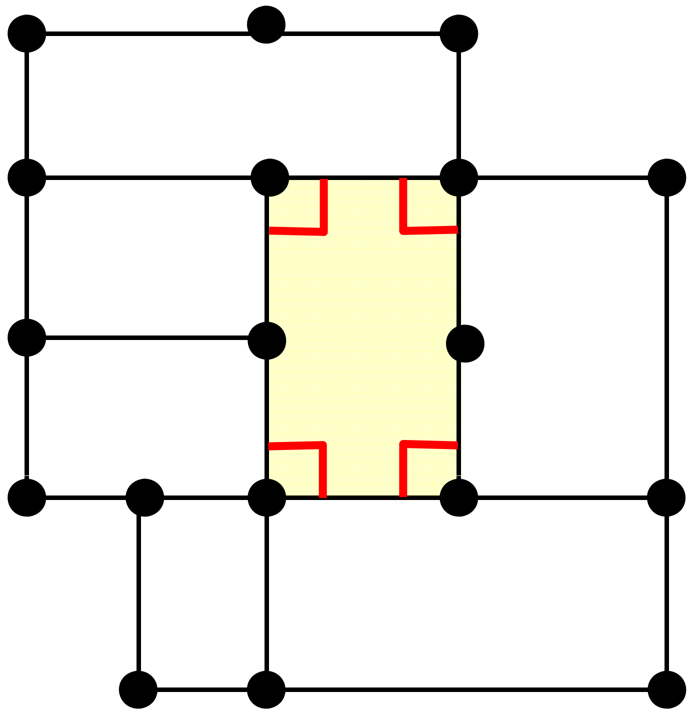


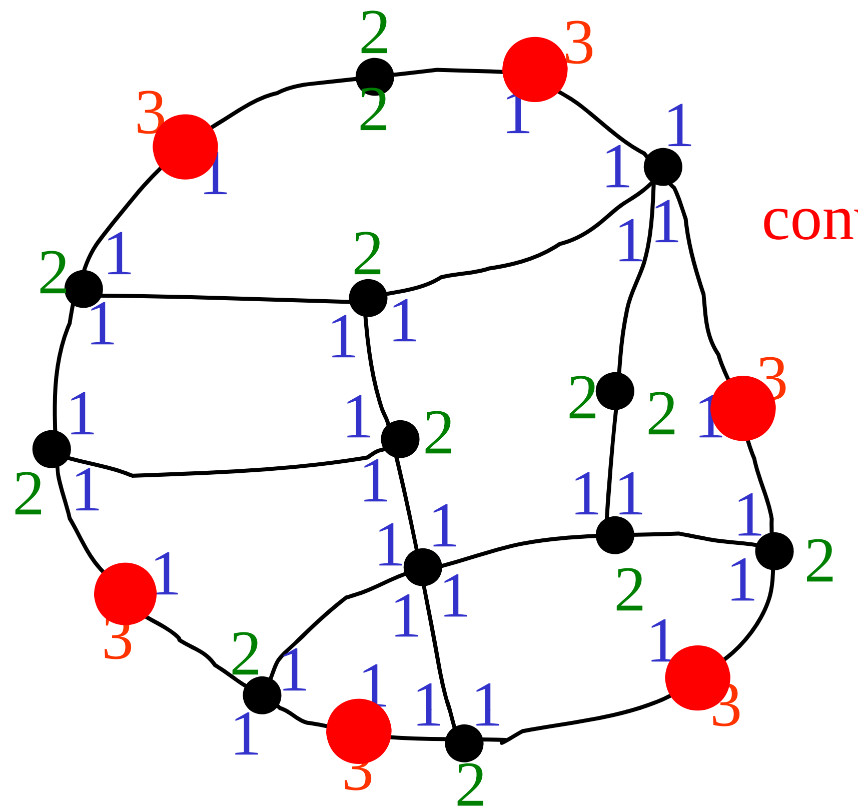
an inner rectangular drawing of *G*

Regular labeling

(c) n_{cv} - n_{cc} = 4.

n_{cv}: the number of outer angles having label 3

n_{cc} : the number of outer angles having label 1

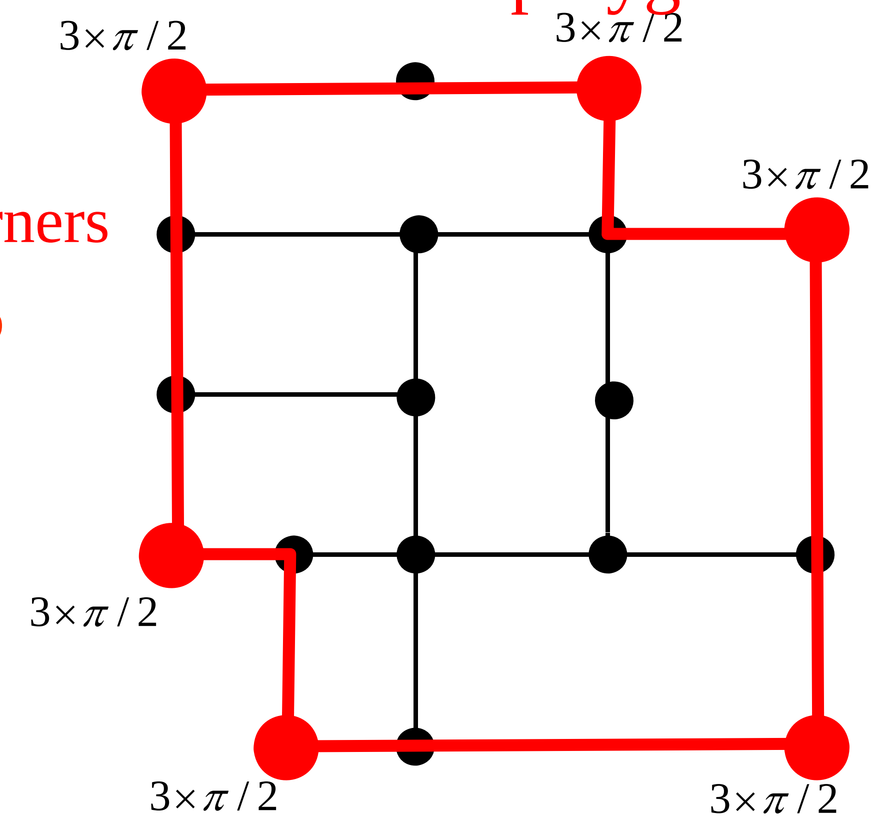


a plane graph G

convex corners

$n_{cv}=6$

rectilinear polygon



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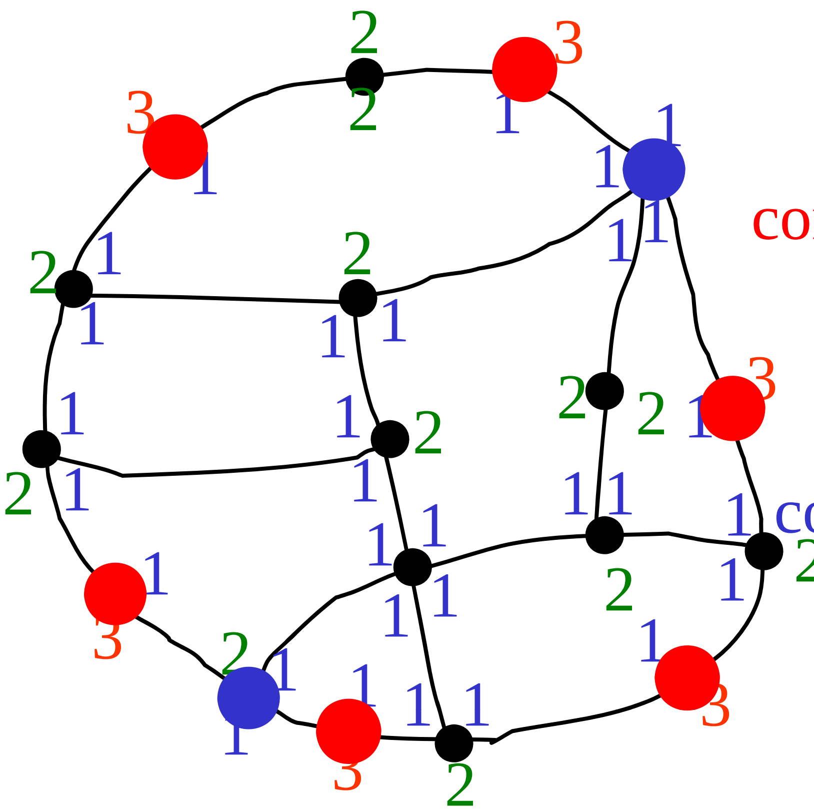


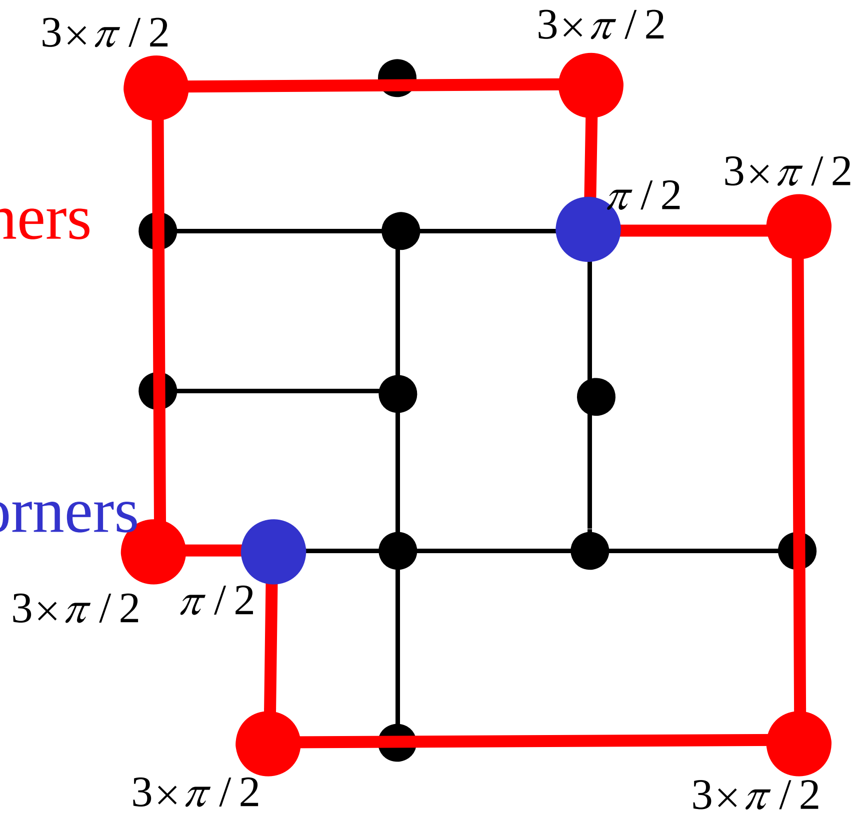
a plane graph *G*

convex corners

n_{cv}=6

n_{cc}=2

concave corners



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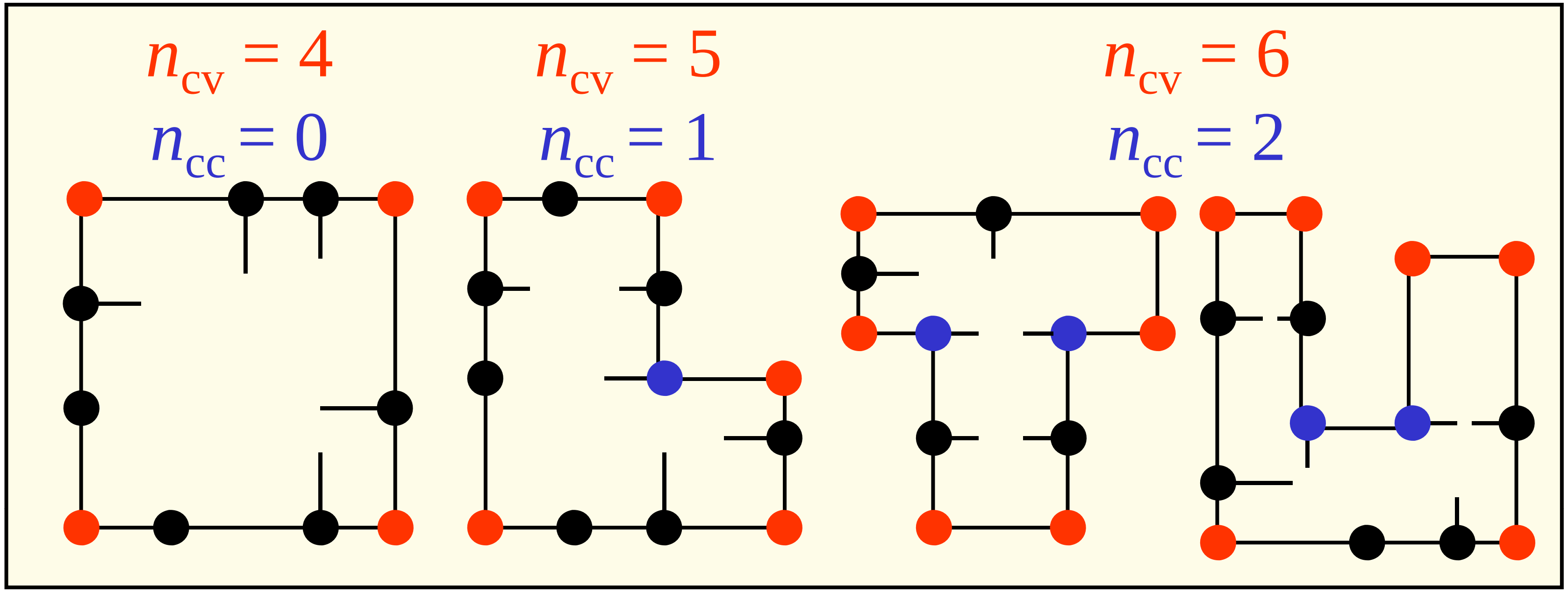
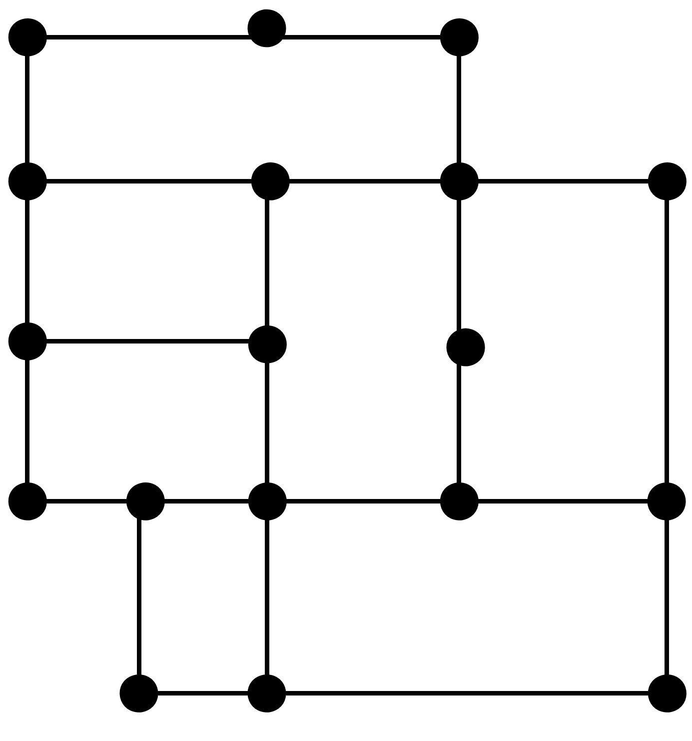


A necessary and sufficient condition for the existence of an inner rectangular drawing of *G*

A plane graph *G* has an inner rectangular drawing ⇒ *G* has a regular labeling



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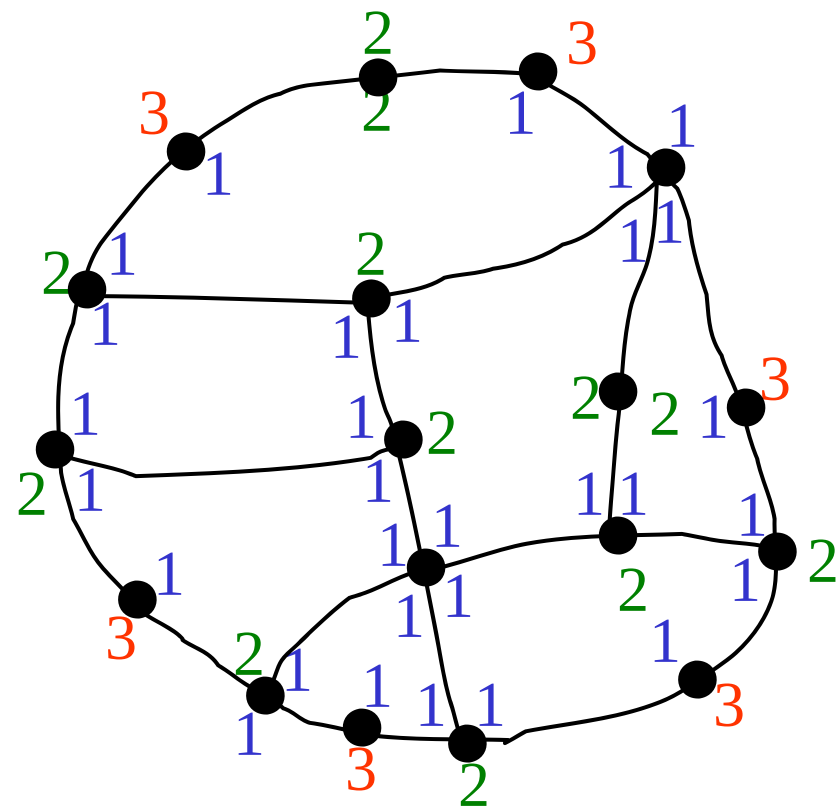

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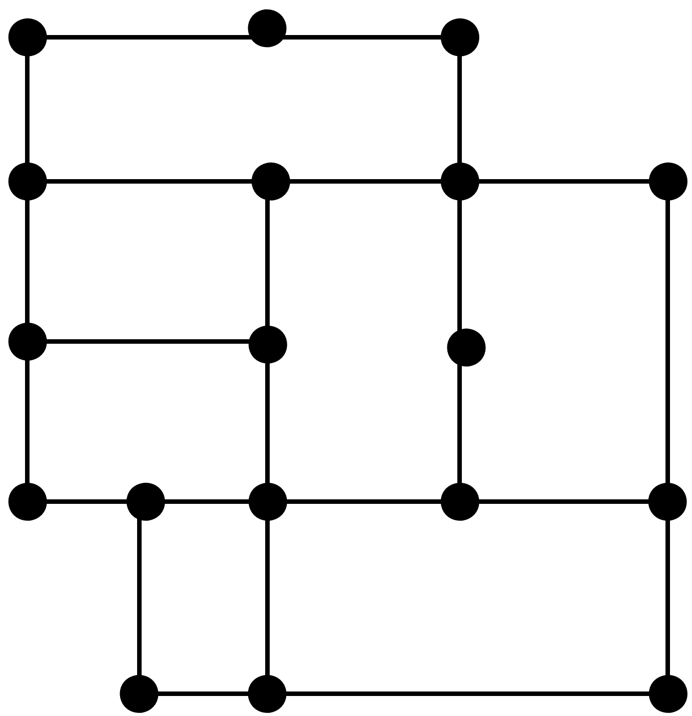

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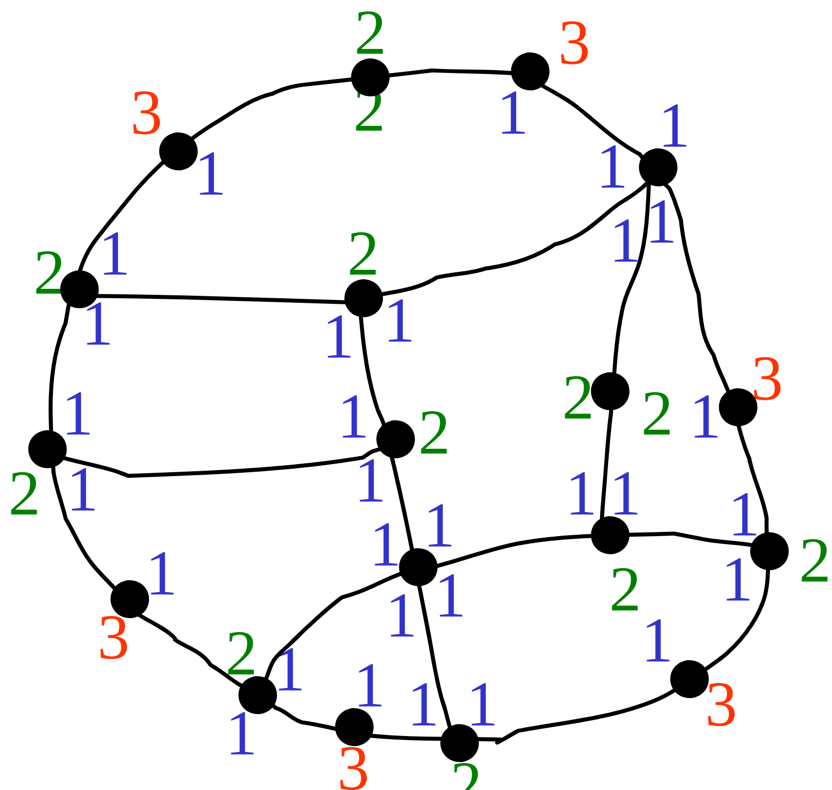
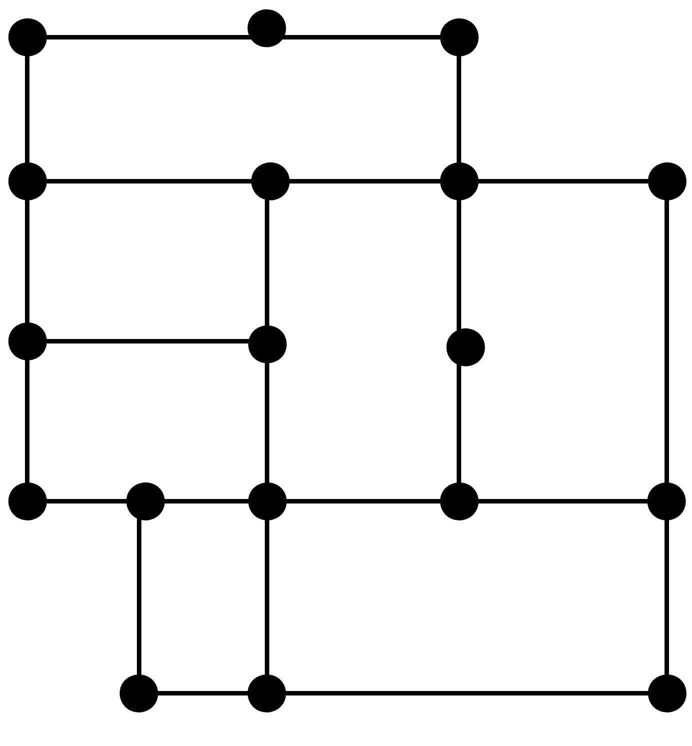


a plane graph *G*

A necessary and sufficient condition for the existence of an inner rectangular drawing of G

A plane graph G has an inner rectangular drawing ⟺ G has a regular labeling



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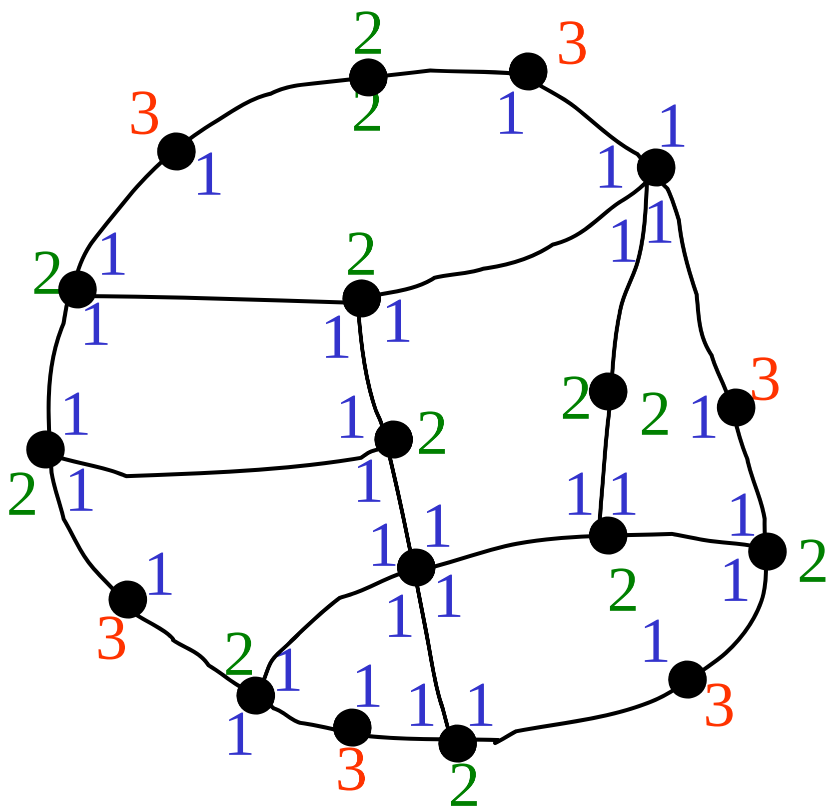

a plane graph G

1:A necessary and sufficient condition for the existence of an inner rectangular drawing of G.

2: $O(n^{1.5}/\log n)$ time algorithm to find an inner rectangular drawing of G if a sketch of the outer face is given.

3: a polynomial time algorithm to find an inner rectangular drawing of G in a general case, where a sketch is not always given.

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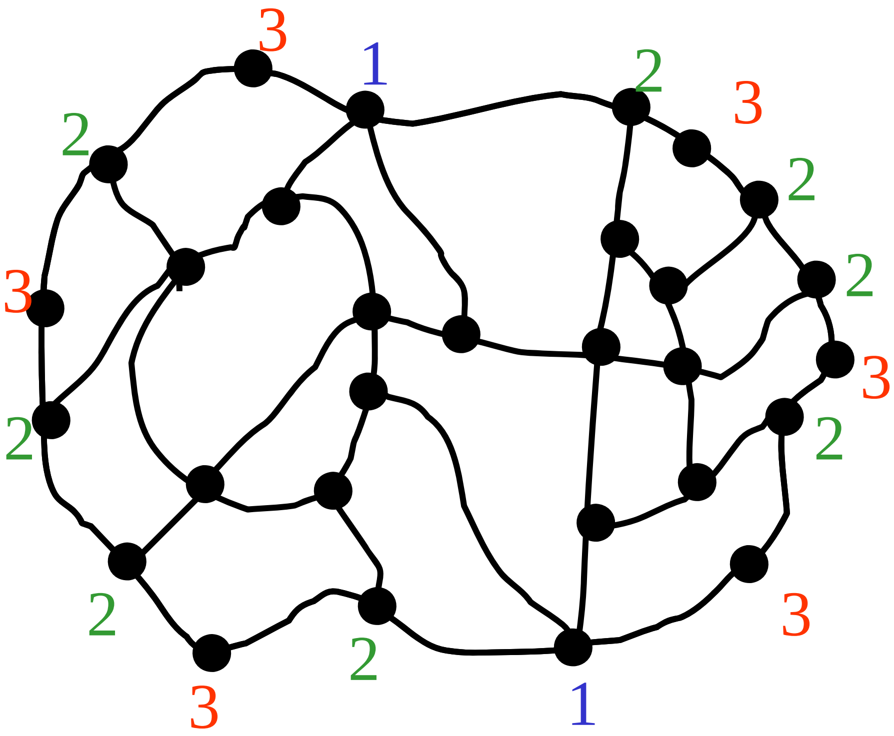
Inner rectangular drawing with sketched outer face



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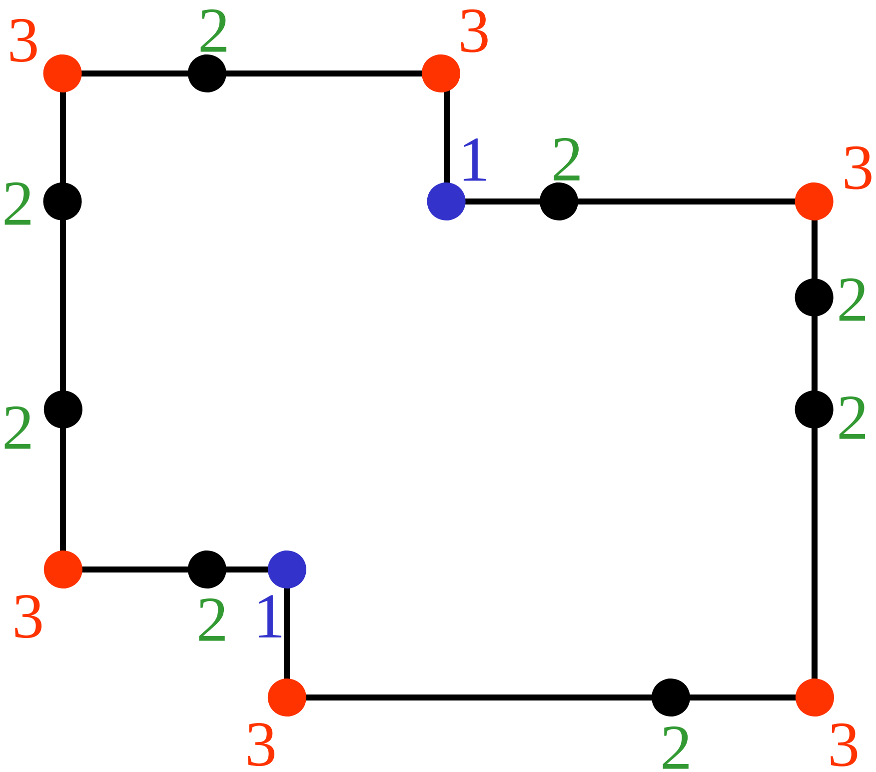
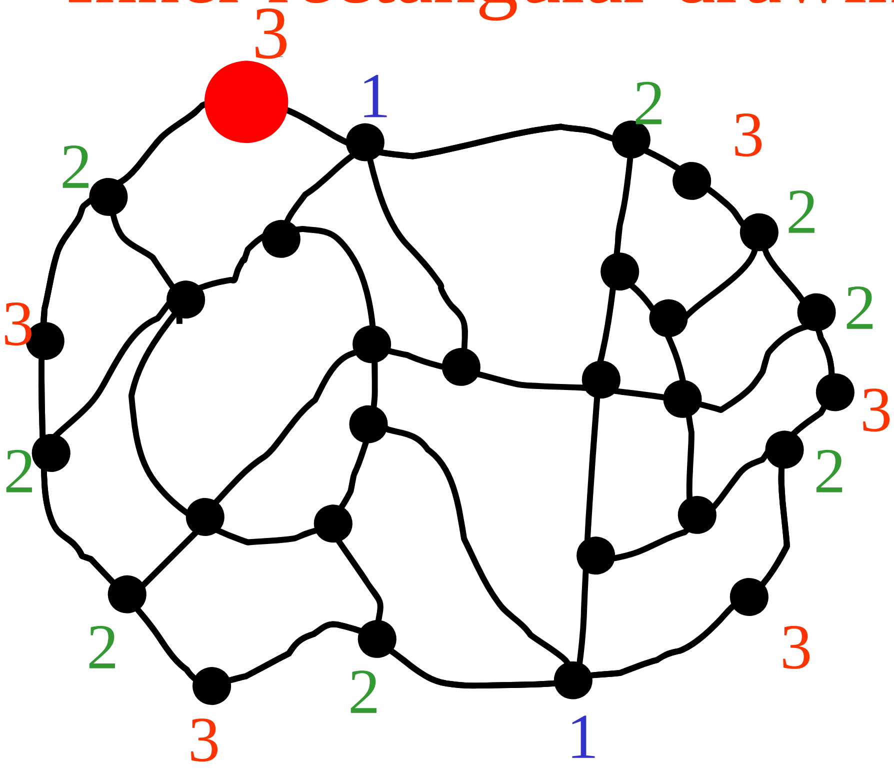


a sketch of the outer face of *G*

Suppose that a sketch of the outer face of *G* is prescribed, that is, all the outer angles of *G* are labeled with 1, 2 or 3

Inner rectangular drawing with sketched outer face



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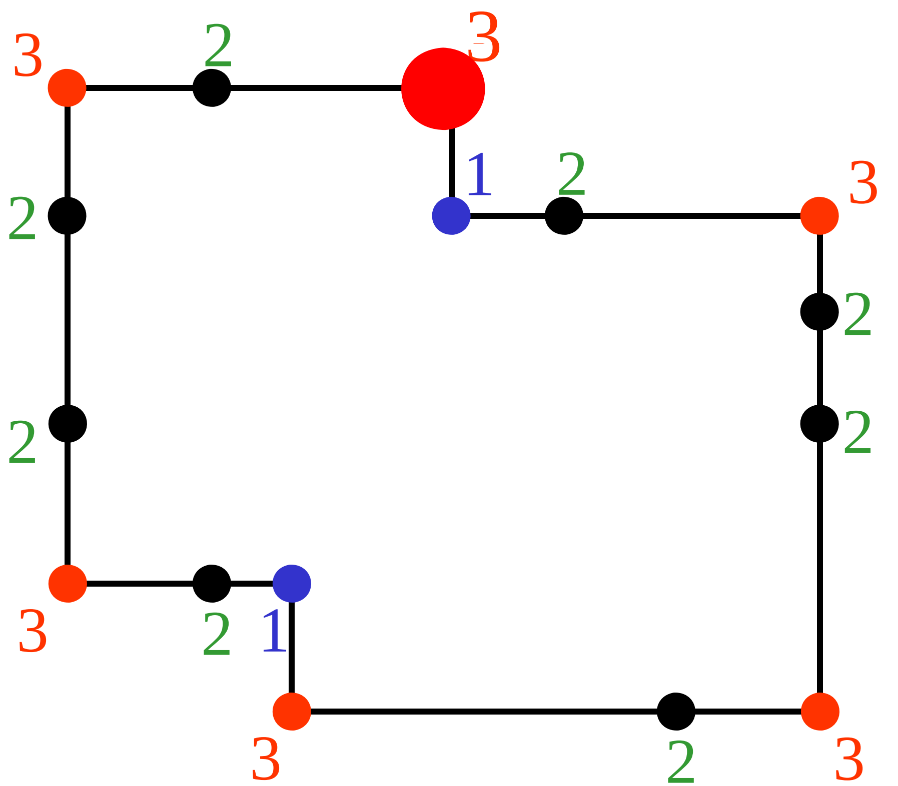

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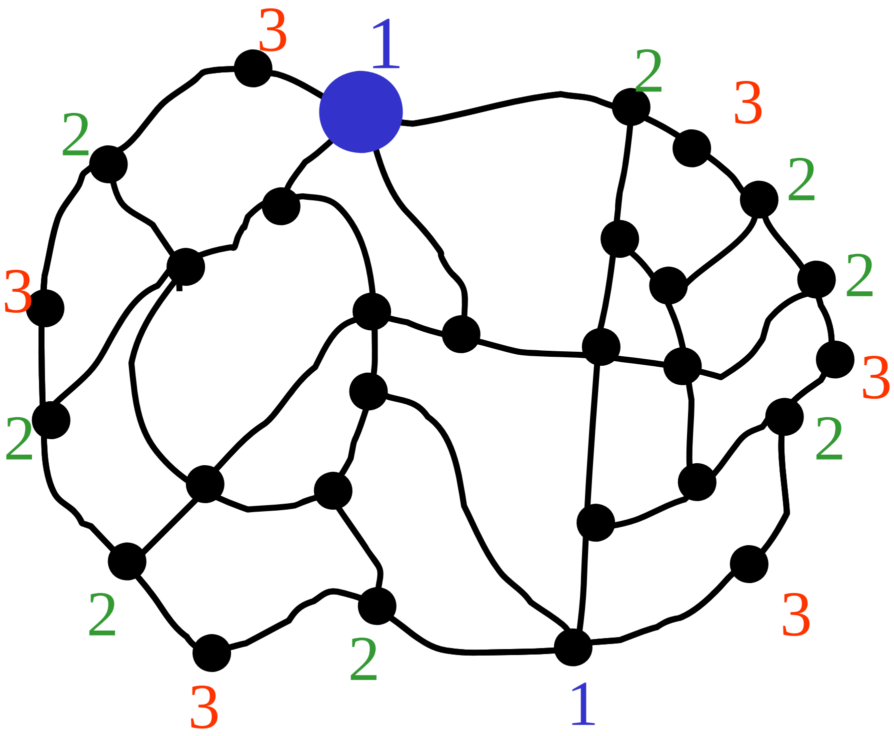

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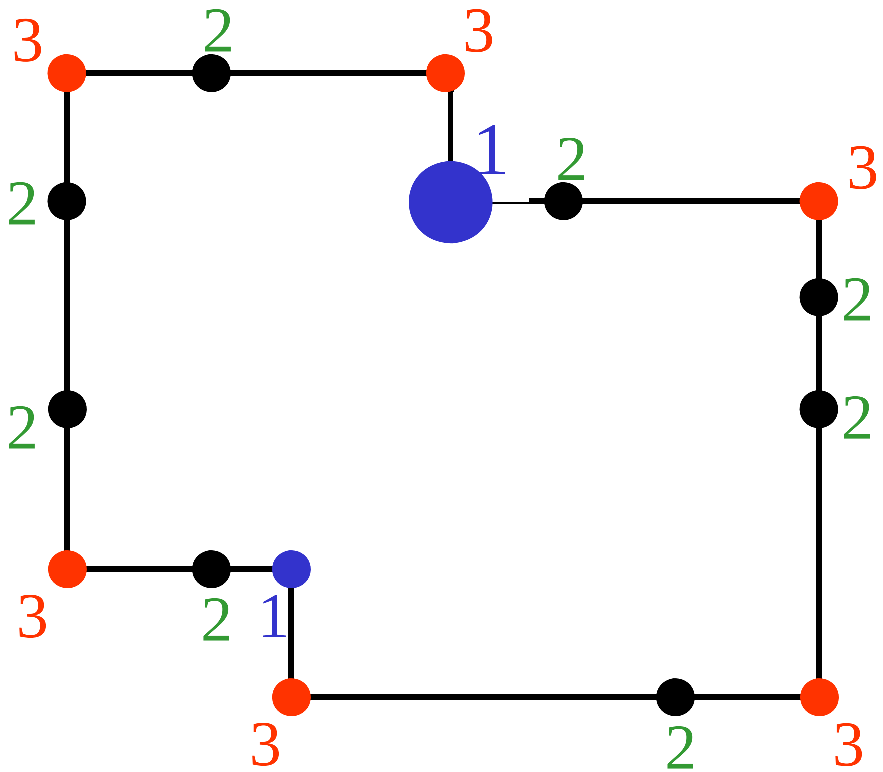

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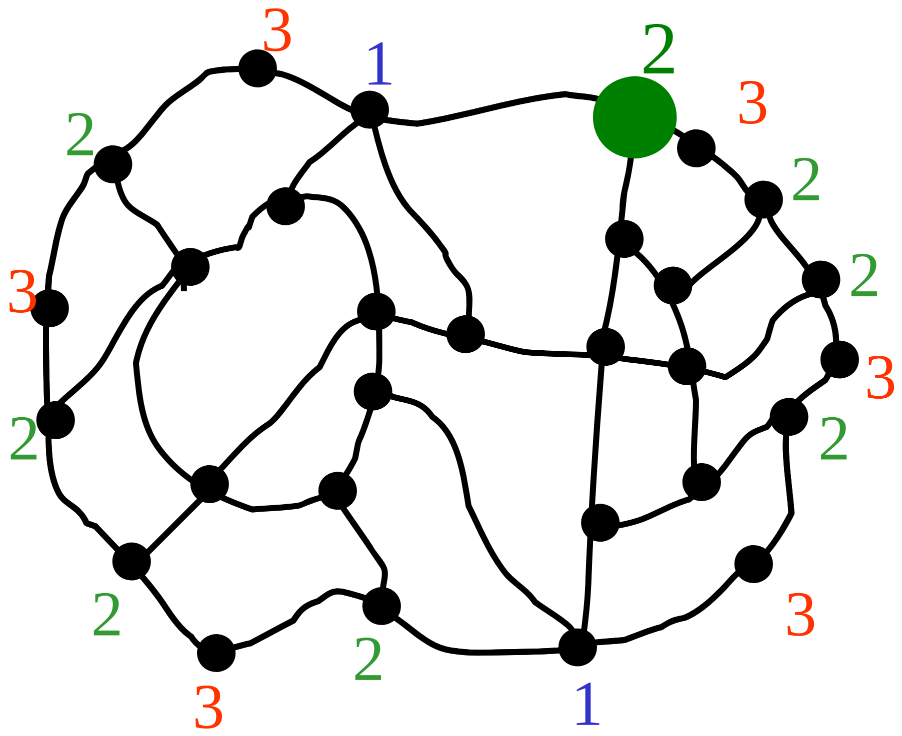

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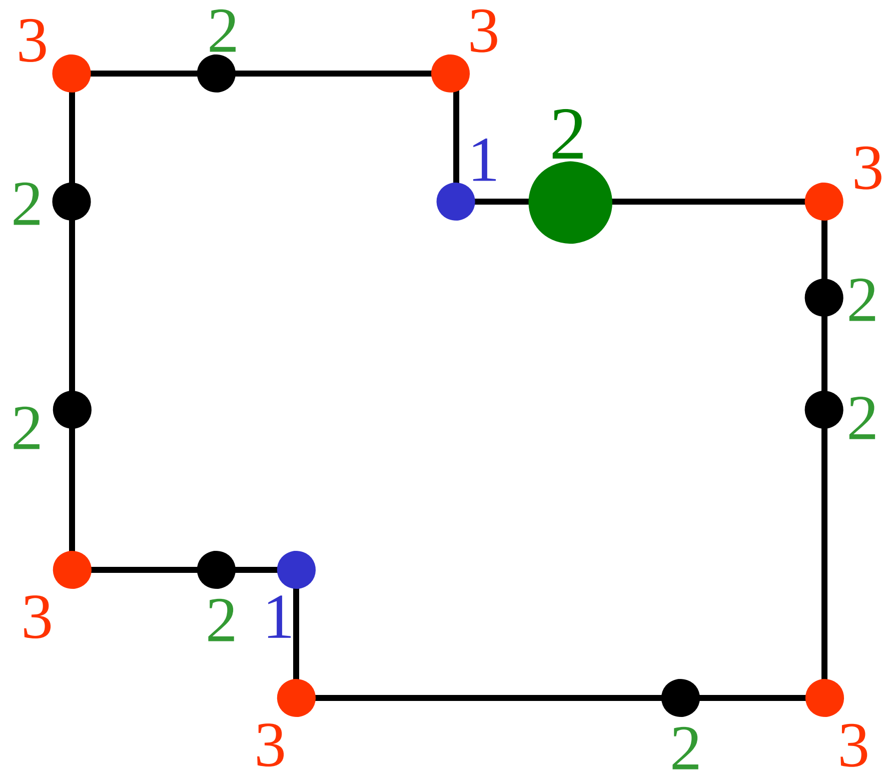

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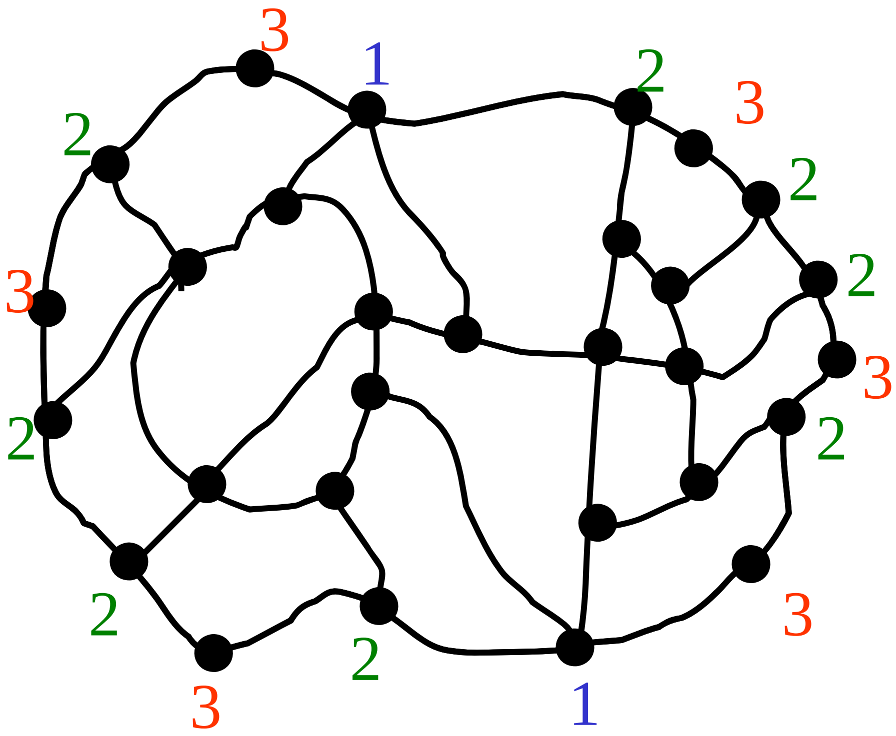
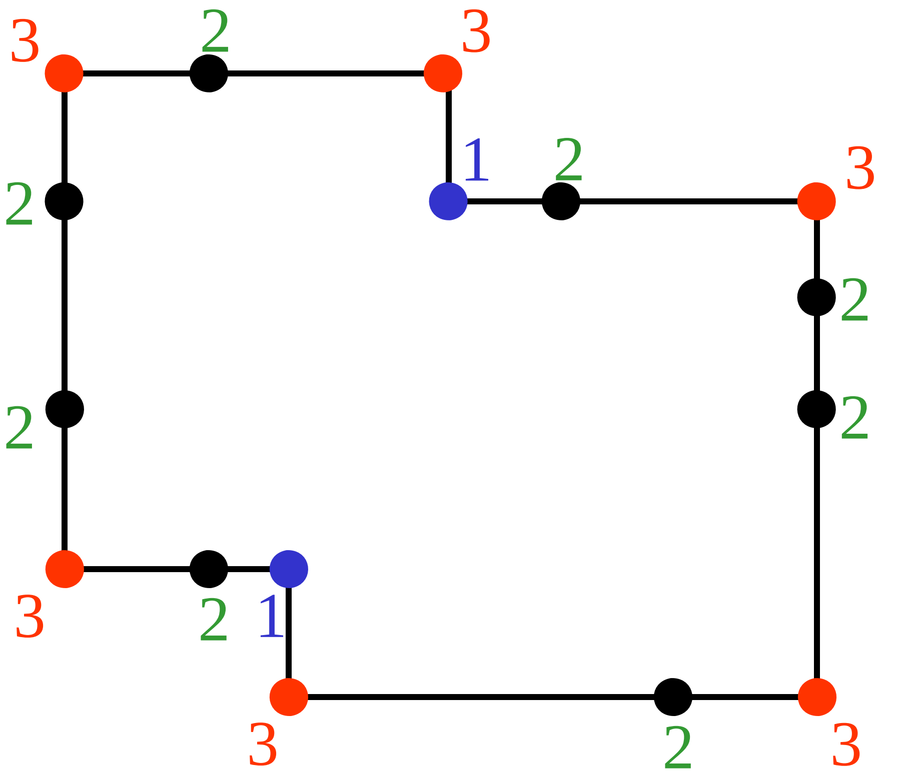
Suppose that a sketch of the outer face of G is prescribed, that is, all the outer angles of G are labeled with 1, 2 or 3

Inner rectangular drawing with sketched outer face



a plane graph *G*

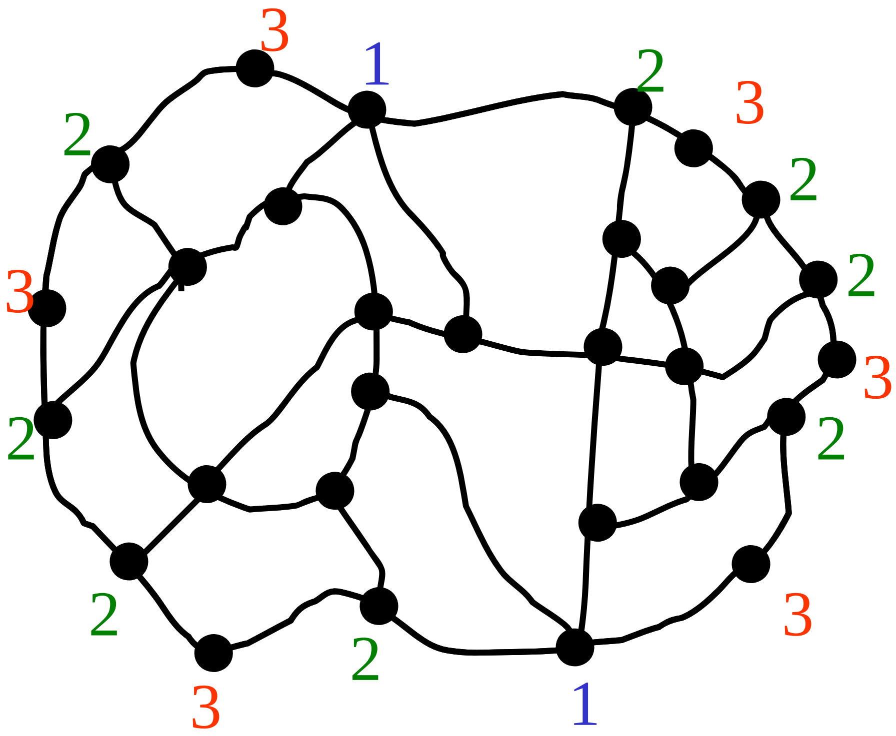


a sketch of the outer face of *G*

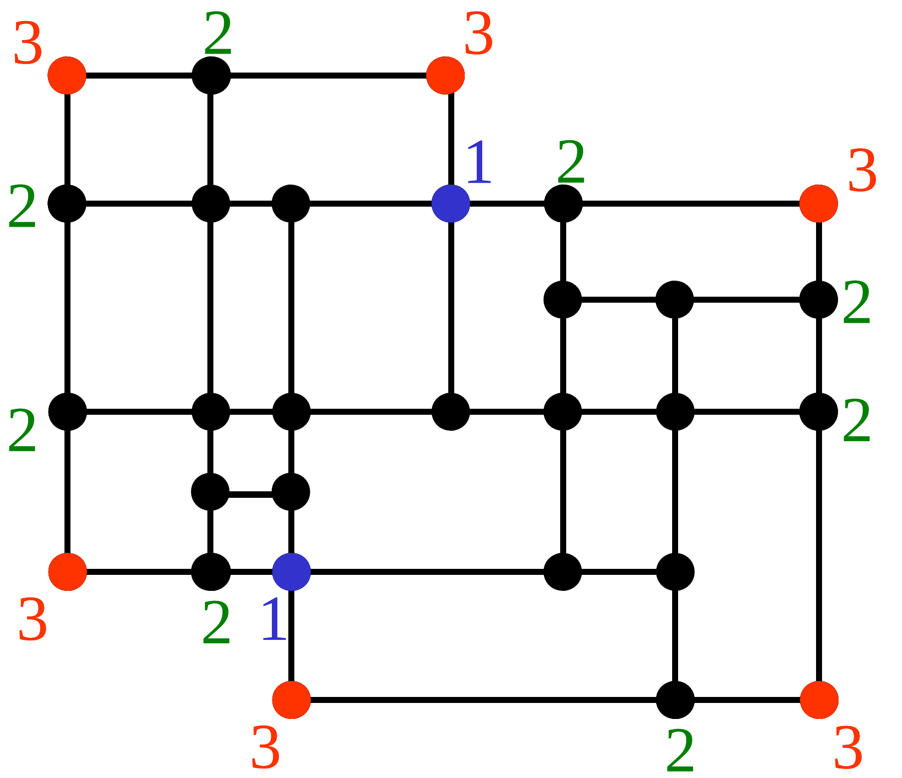
Suppose that a sketch of the outer face of *G* is prescribed, that is, all the outer angles of *G* are labeled with 1, 2 or 3

Find an inner rectangular drawing with a prescribed sketch of the outer face

Inner rectangular drawing with sketched outer face



a plane graph *G*



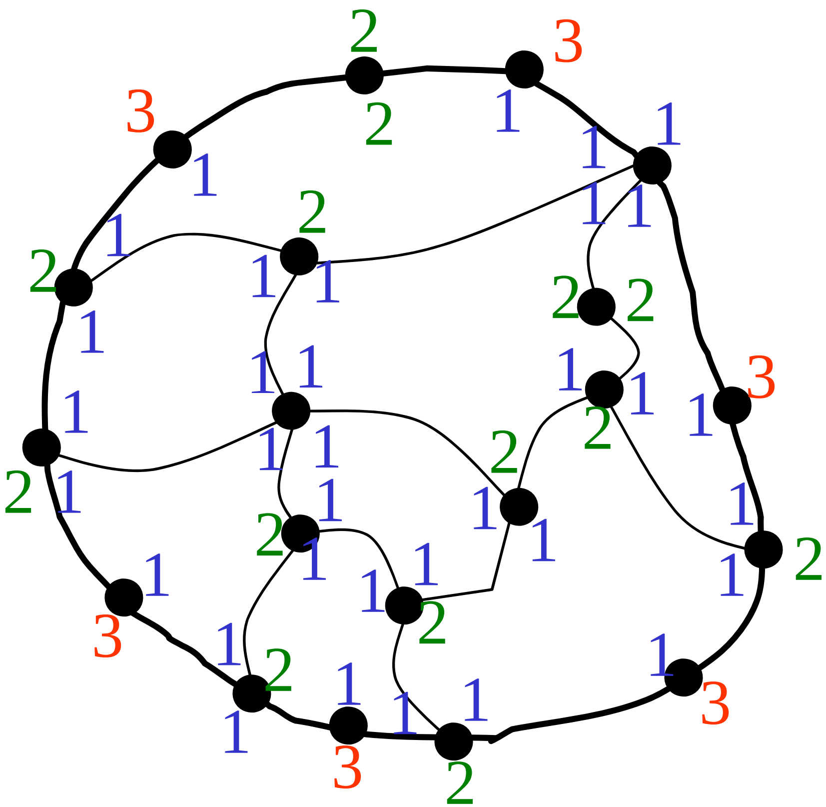
a sketch of the outer face of *G*

Suppose that a sketch of the outer face of *G* is prescribed, that is, all the outer angles of *G* are labeled with 1, 2 or 3

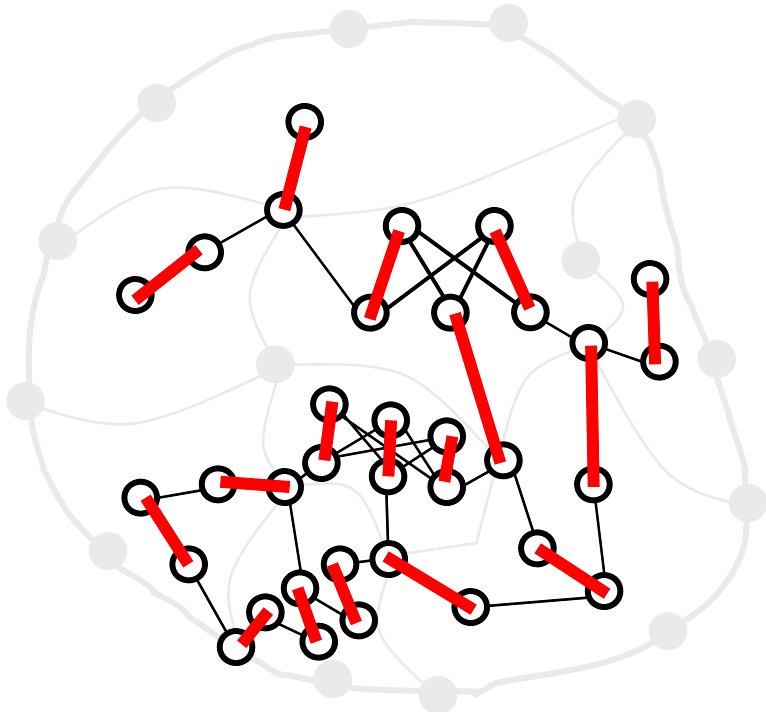
Find an inner rectangular drawing with a prescribed sketch of the outer face

A plane graph G has an inner rectangular drawing ⟺ G has a regular labeling

G has a regular labeling ⟺ G_d has a perfect matching



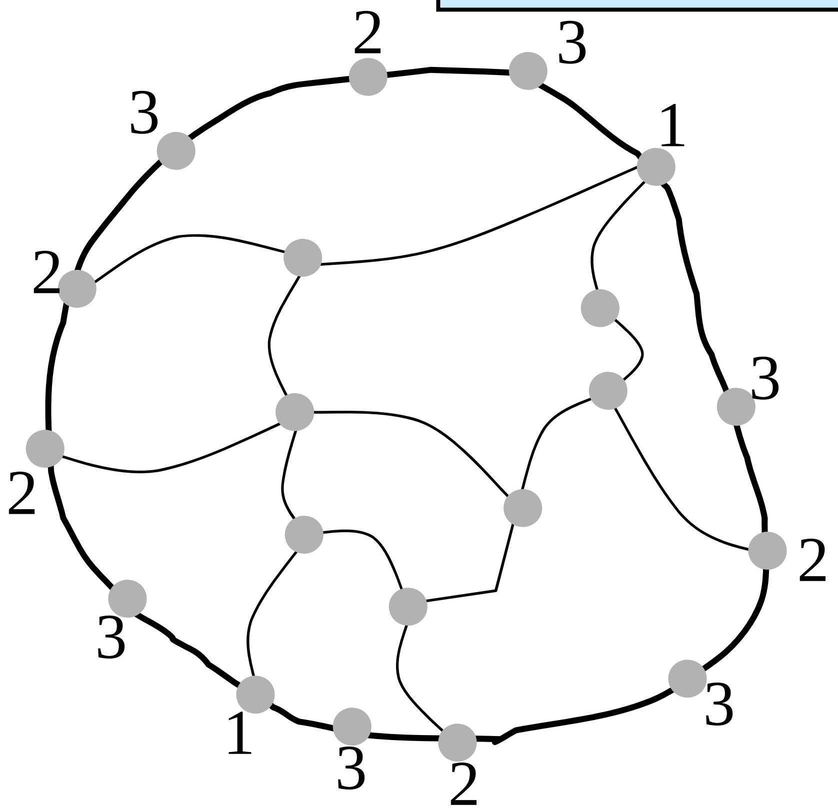
a plane graph G



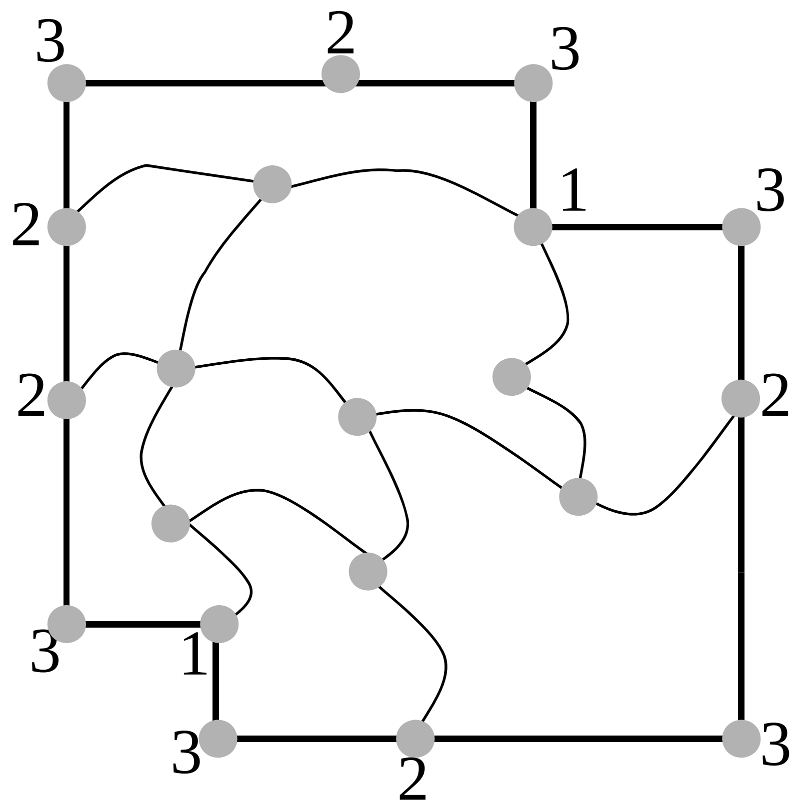
a decision graph G_d of G

Construct a decision graph G_d

Labels of some of the inner angles of *G* can be immediately determined

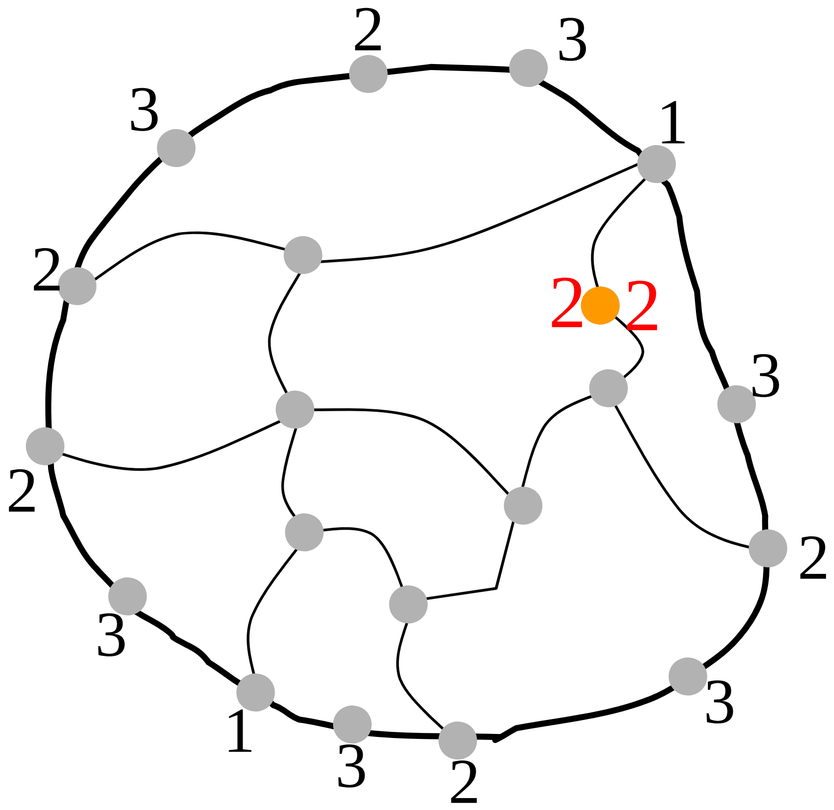


a plane graph *G*

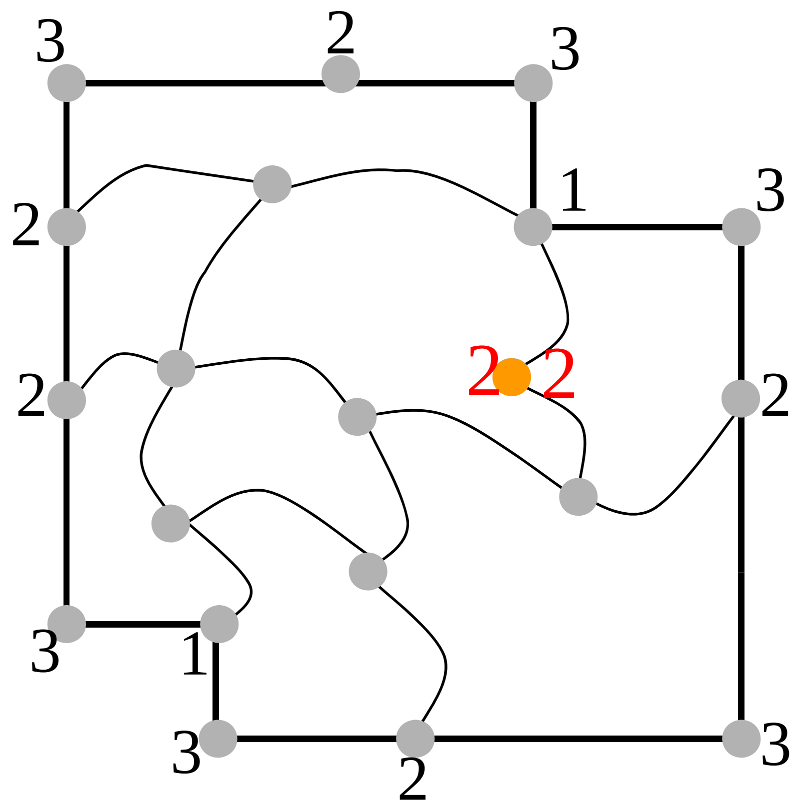


Construct a decision graph G_d

degree 2

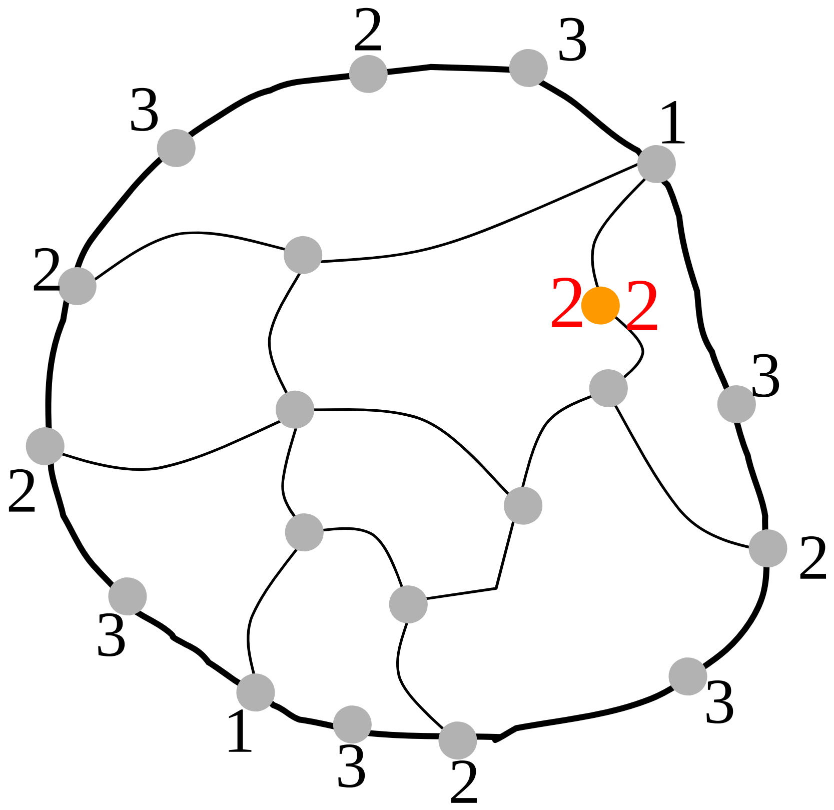


a plane graph *G*

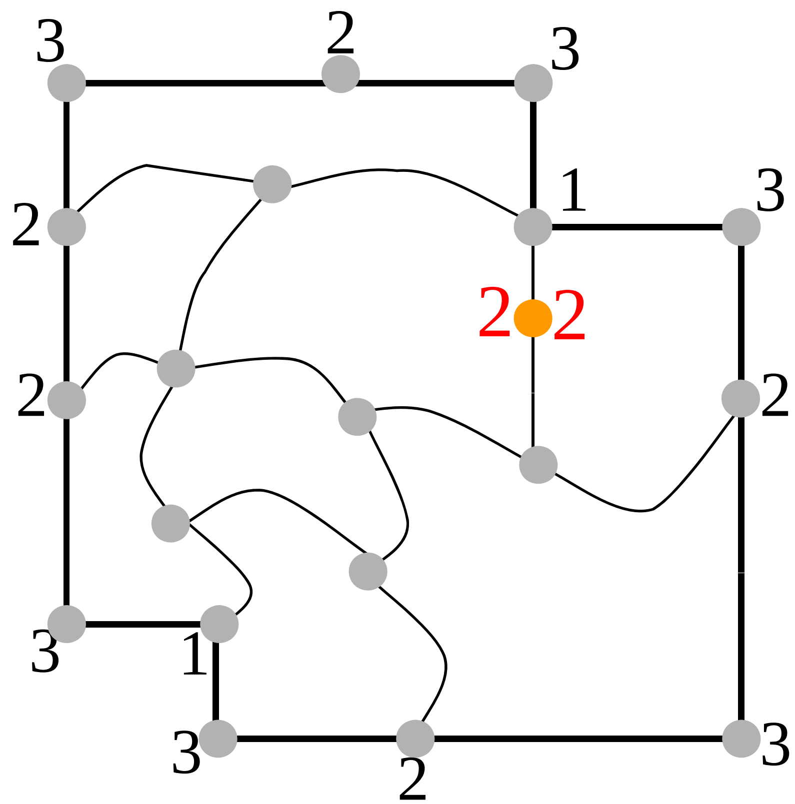


Construct a decision graph G_d

degree 2

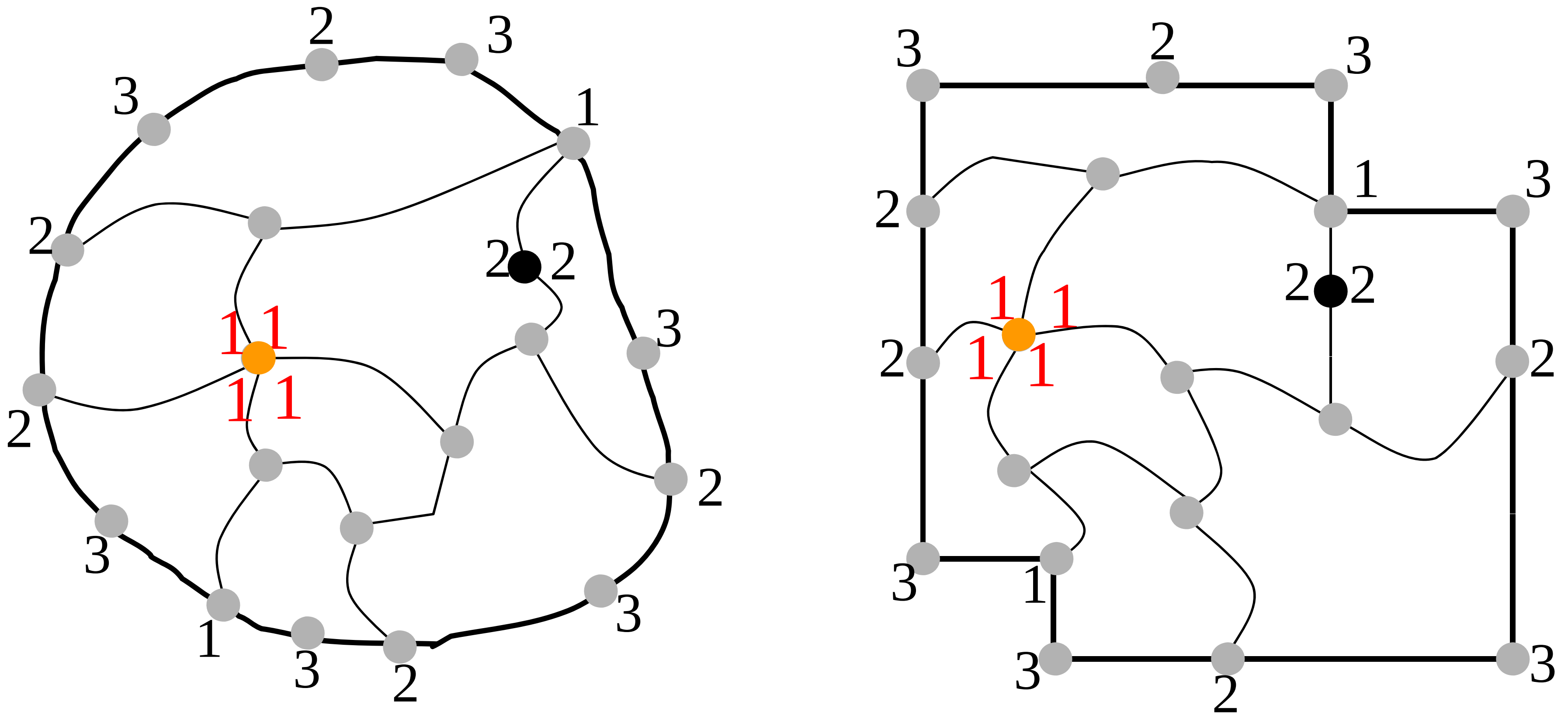


a plane graph *G*



Construct a decision graph G_d

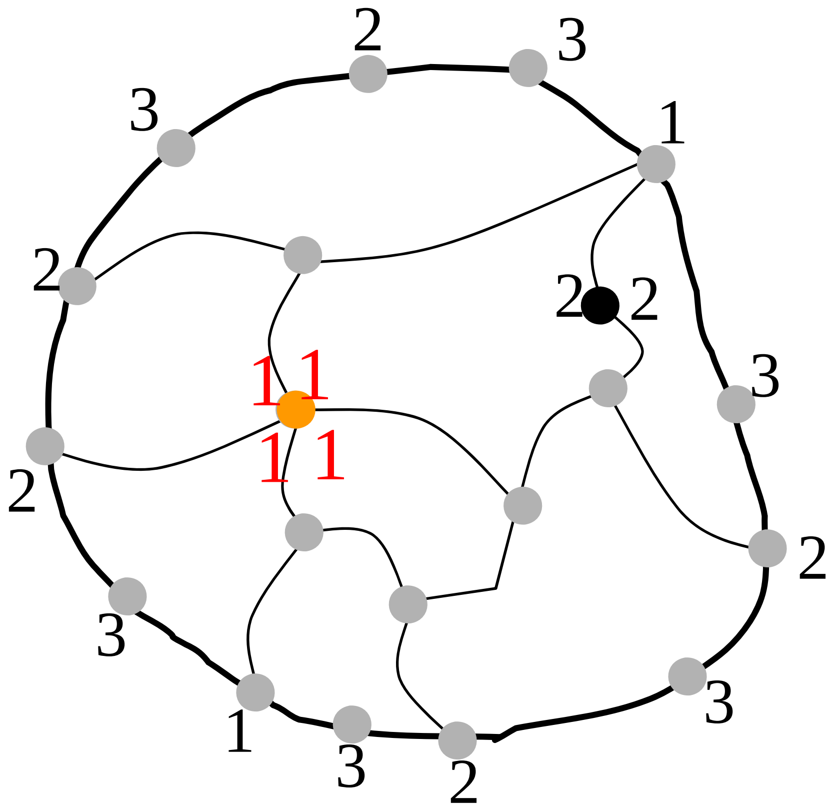
degree 4



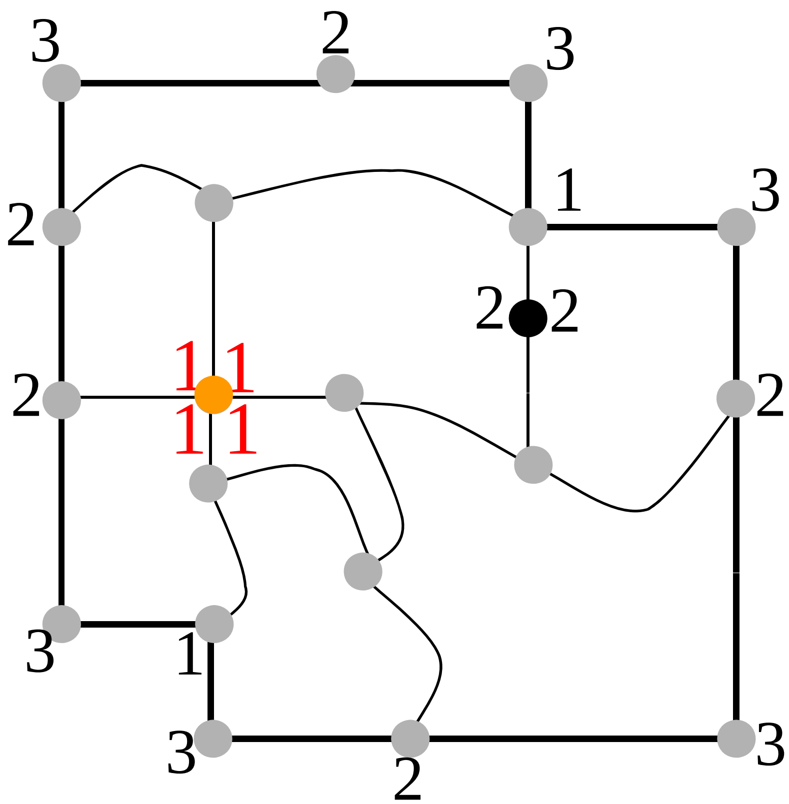
a plane graph G

Construct a decision graph G_d

degree 4

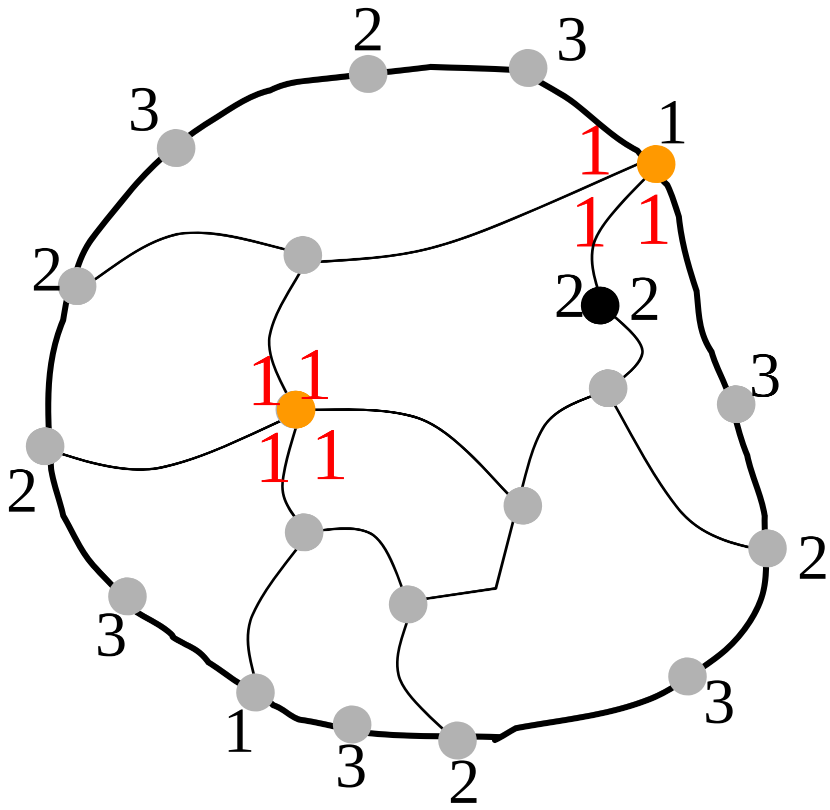


a plane graph *G*

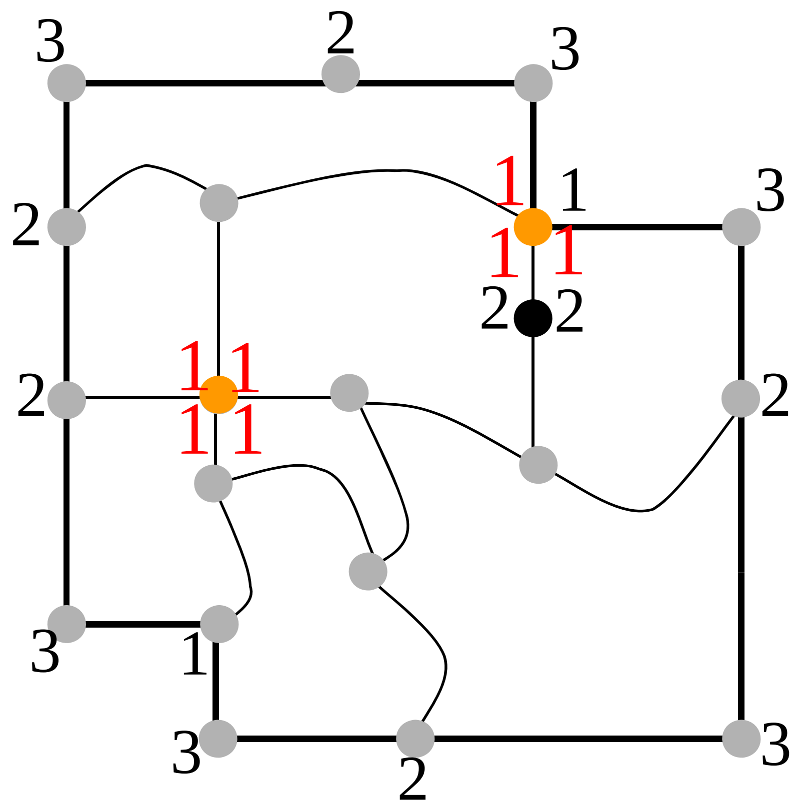


Construct a decision graph G_d

degree 4

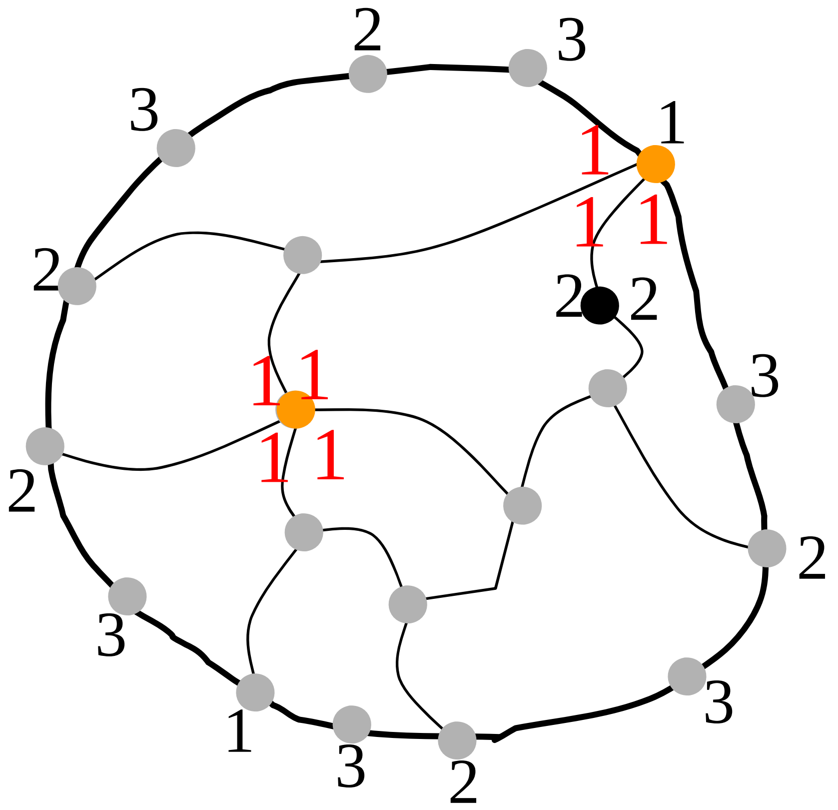


a plane graph G

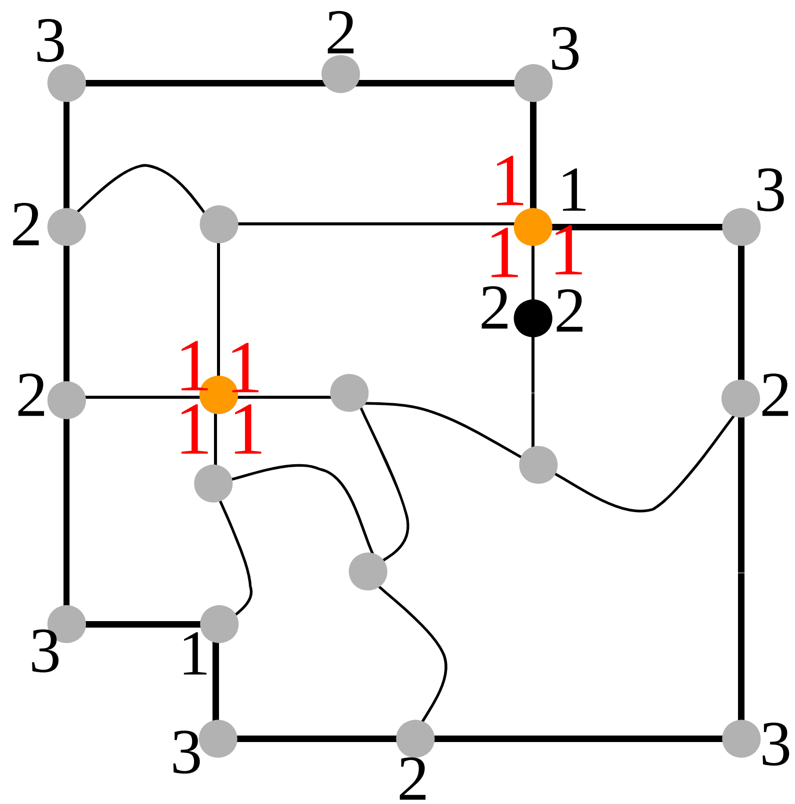


Construct a decision graph G_d

degree 4

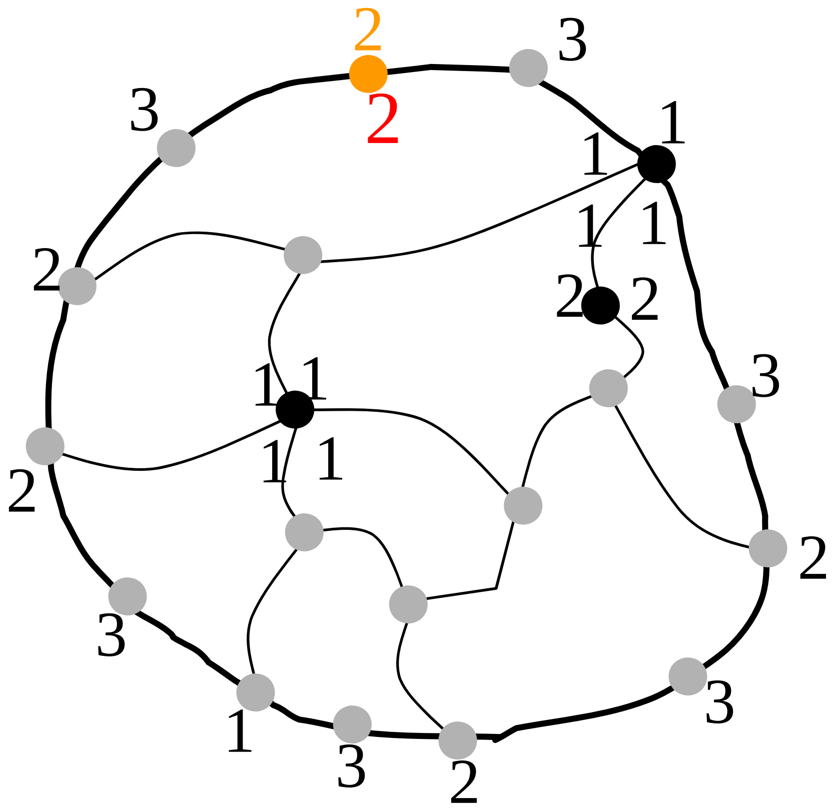


a plane graph G

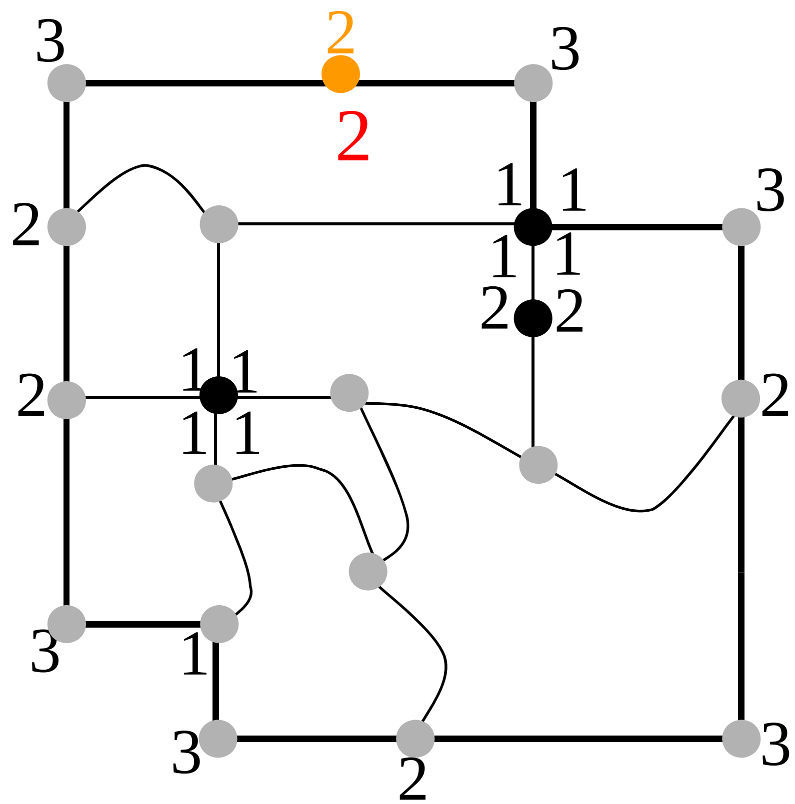


Construct a decision graph G_d

outer vertex of degree 2

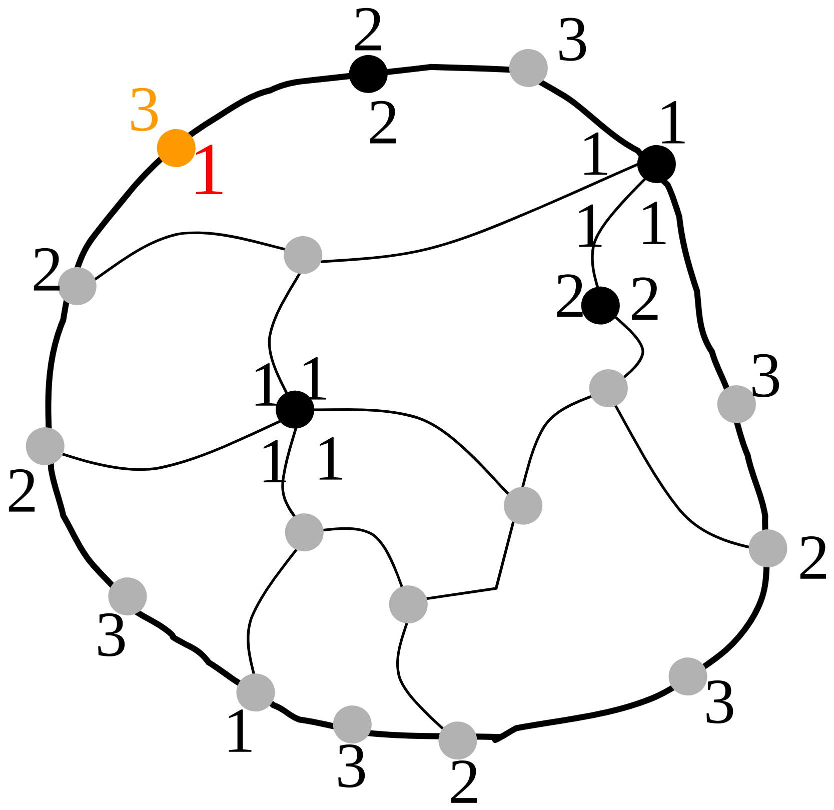


a plane graph G

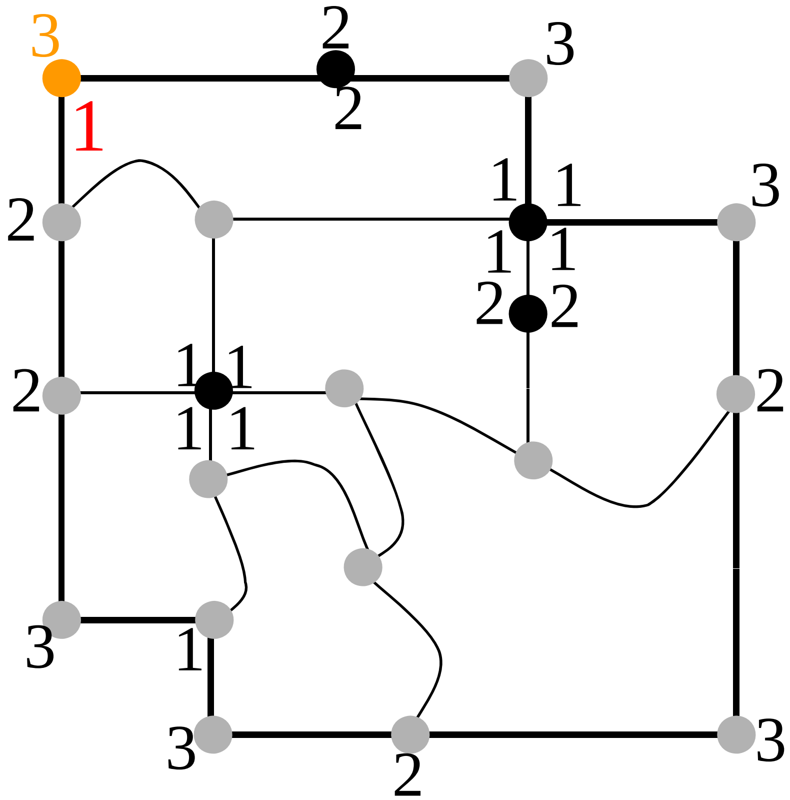


Construct a decision graph G_d

outer vertex of degree 2

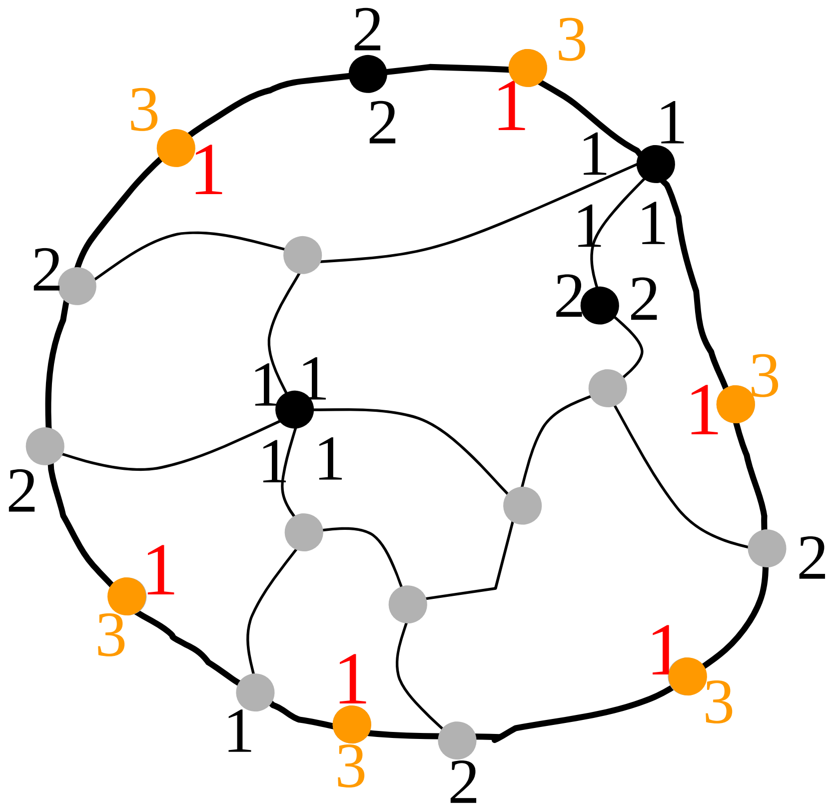


a plane graph G

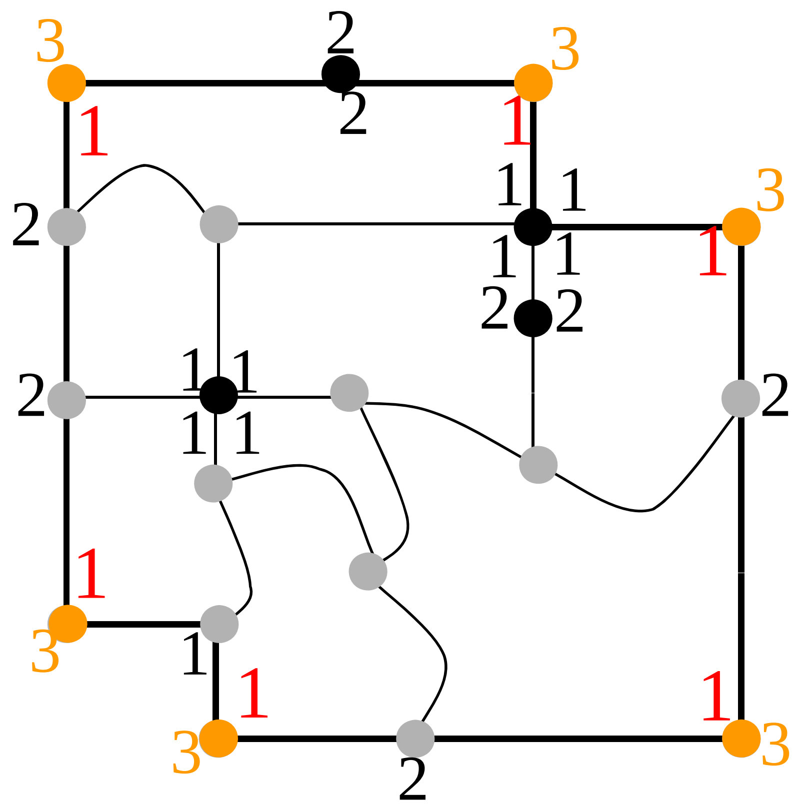


Construct a decision graph G_d

outer vertex of degree 2

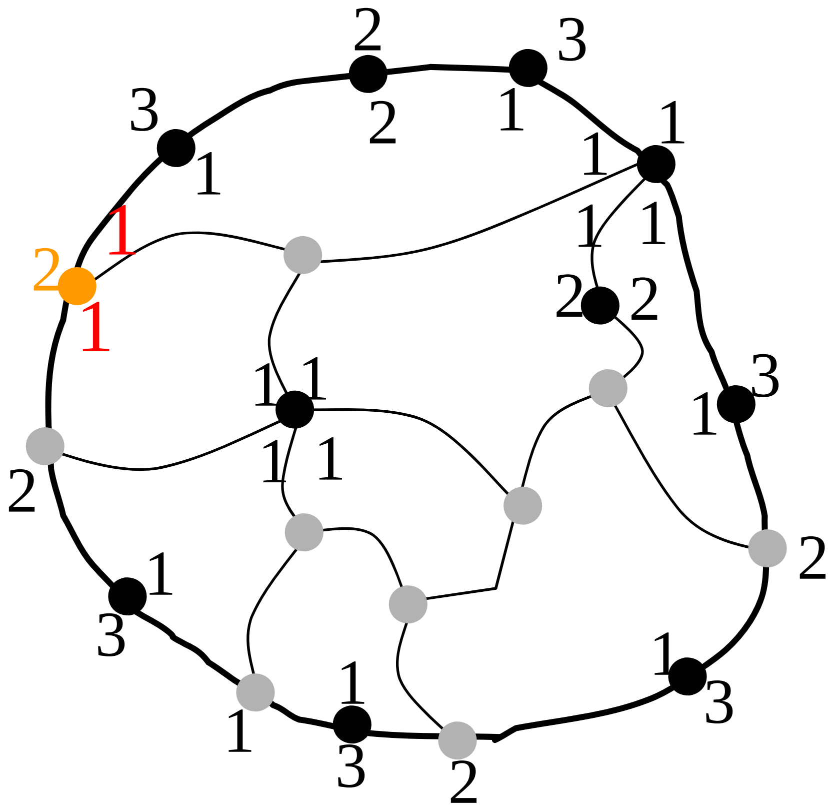


a plane graph *G*

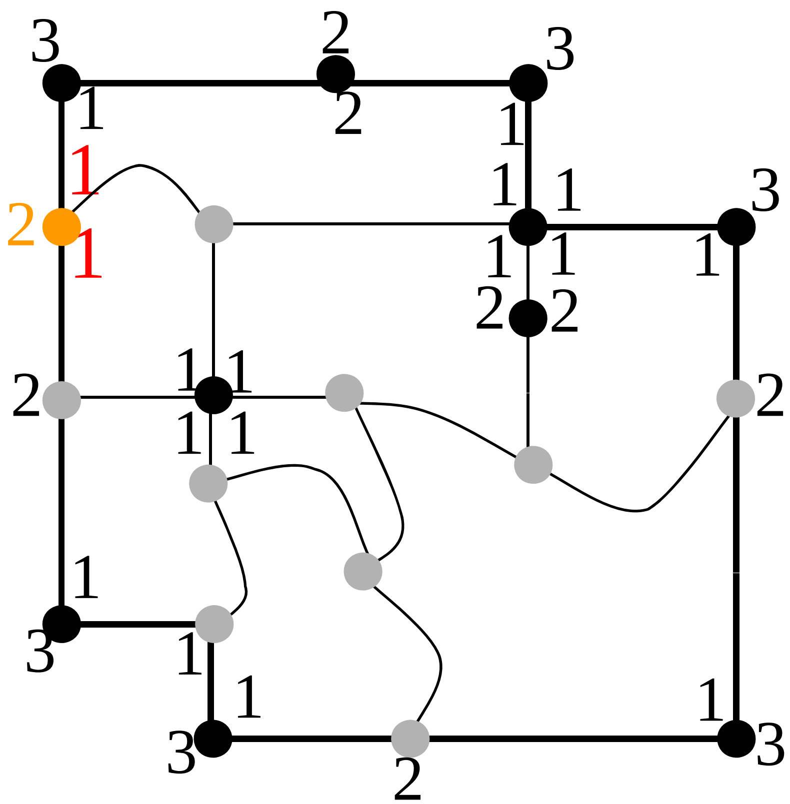


Construct a decision graph G_d

outer vertex of degree 3

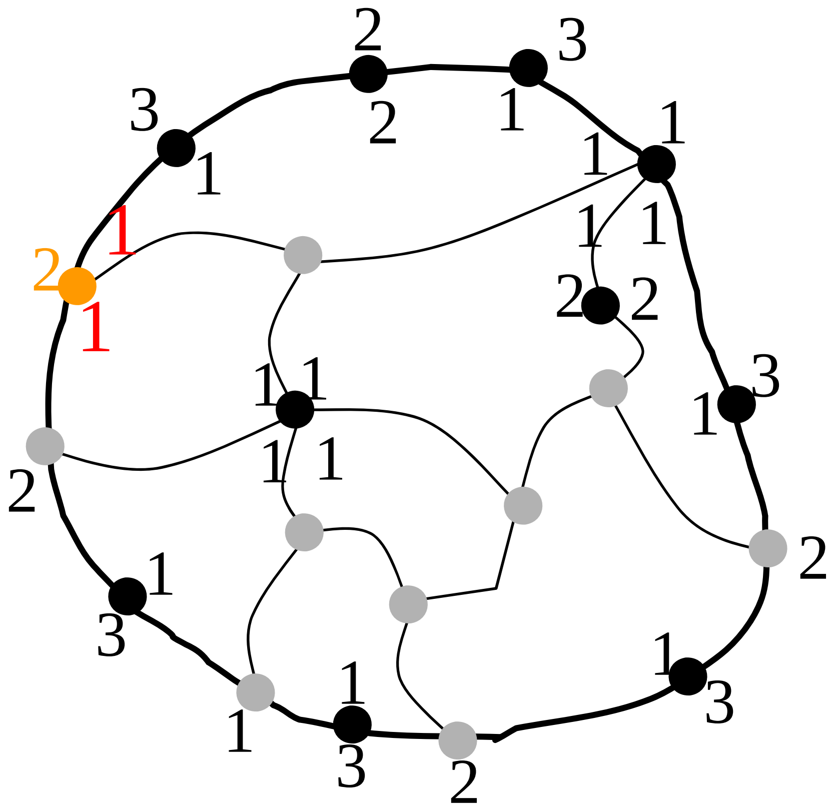


a plane graph *G*

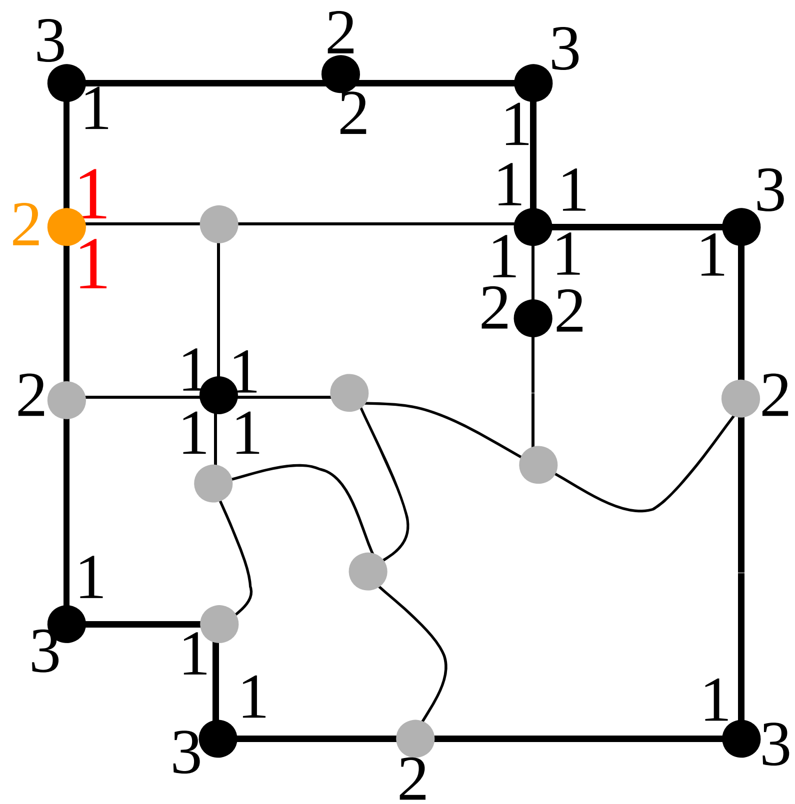


Construct a decision graph G_d

outer vertex of degree 3

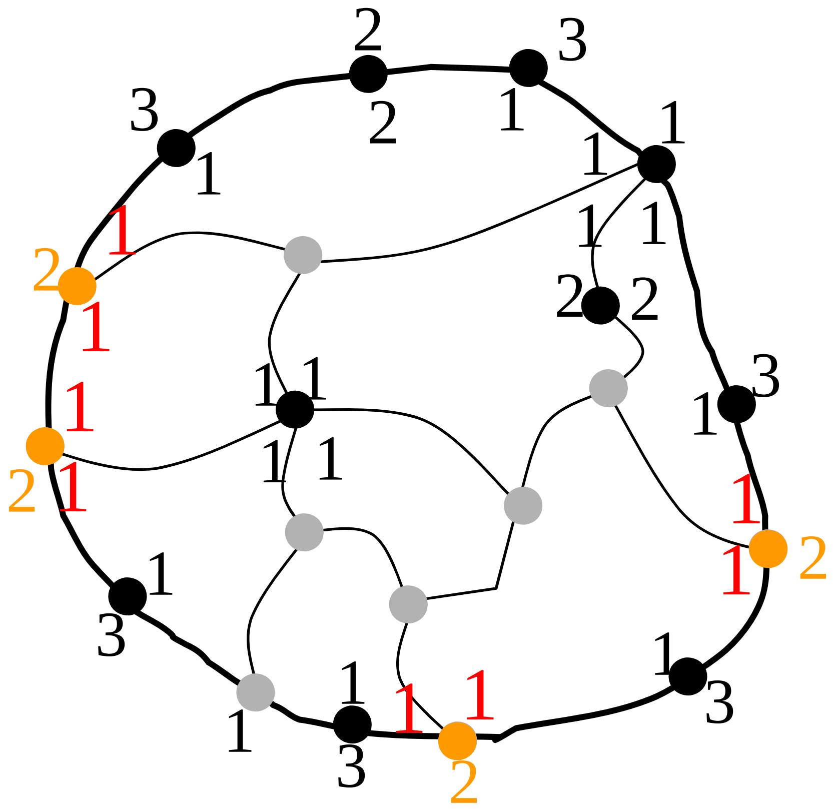


a plane graph G

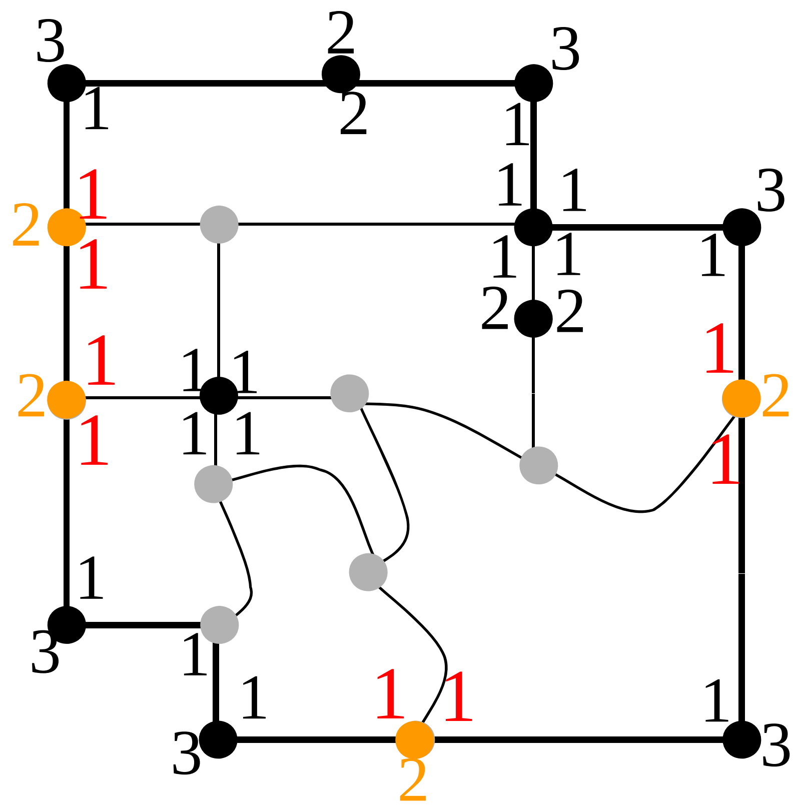


Construct a decision graph G_d

outer vertex of degree 3

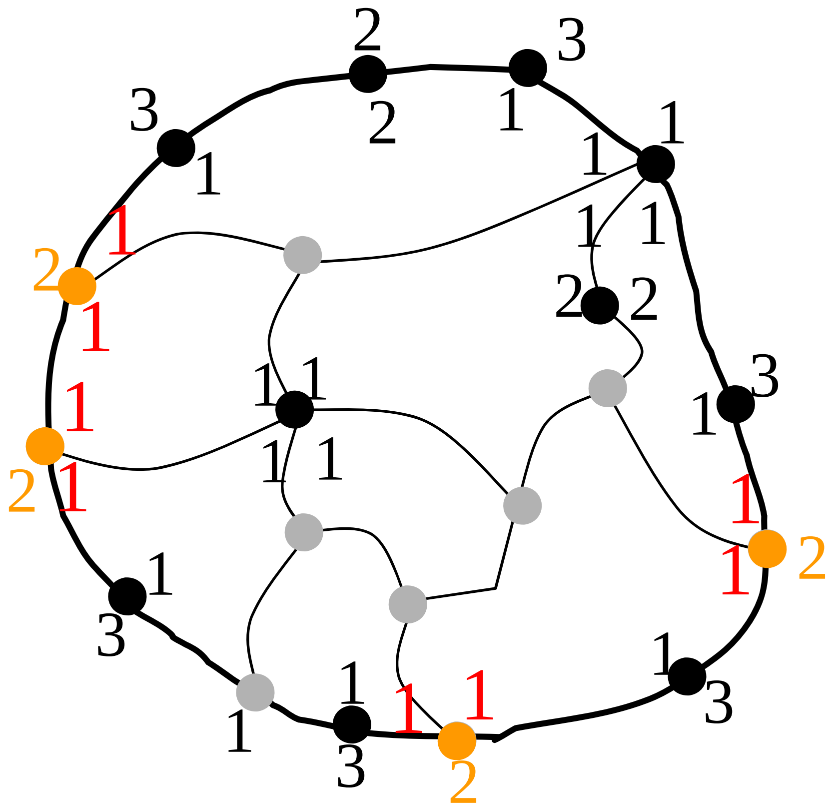


a plane graph *G*

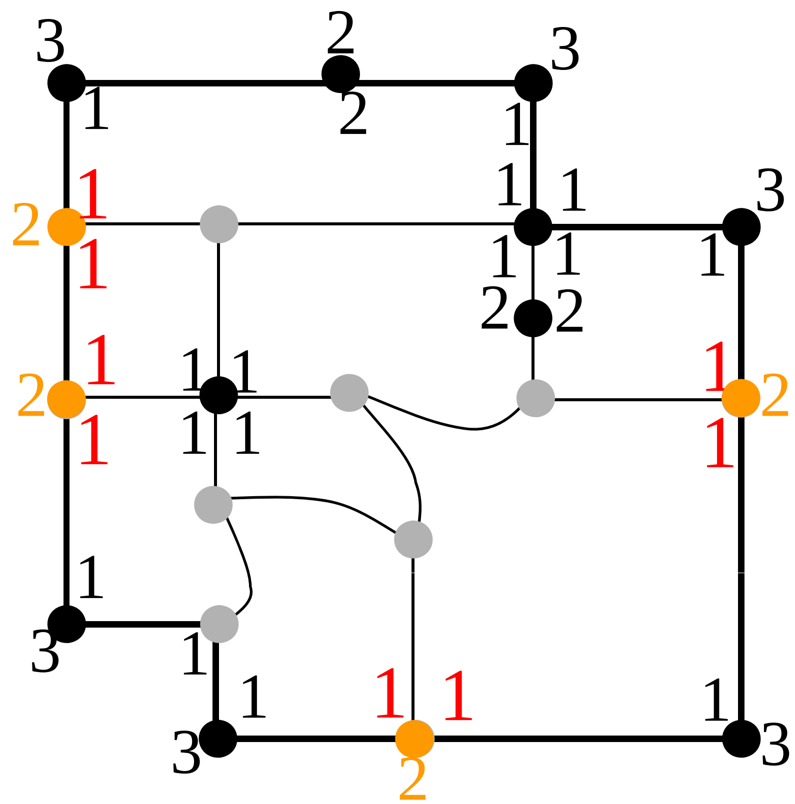


Construct a decision graph G_d

outer vertex of degree 3

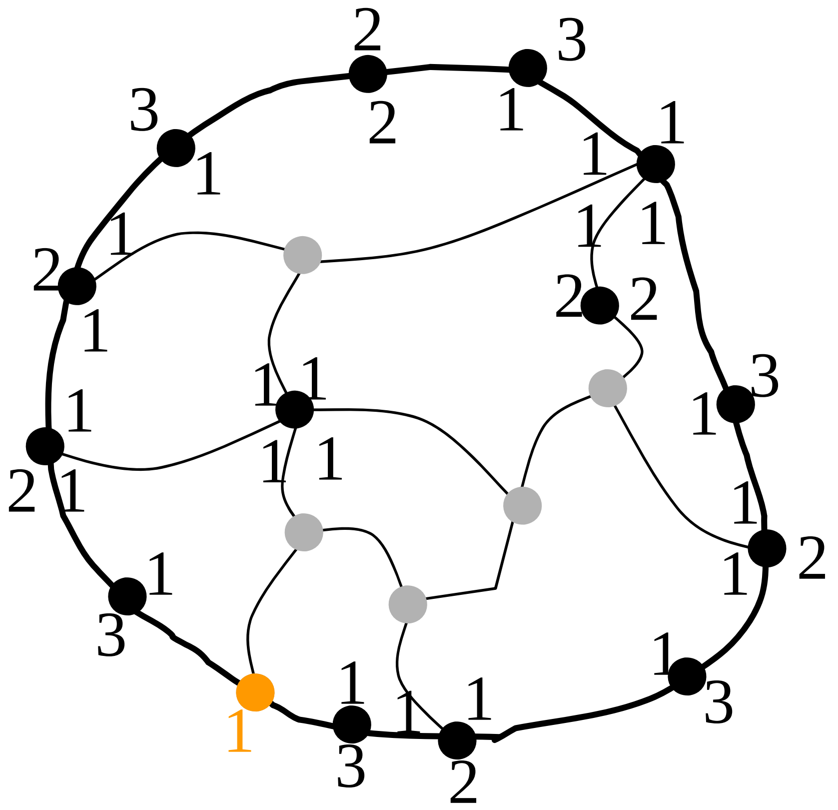


a plane graph G

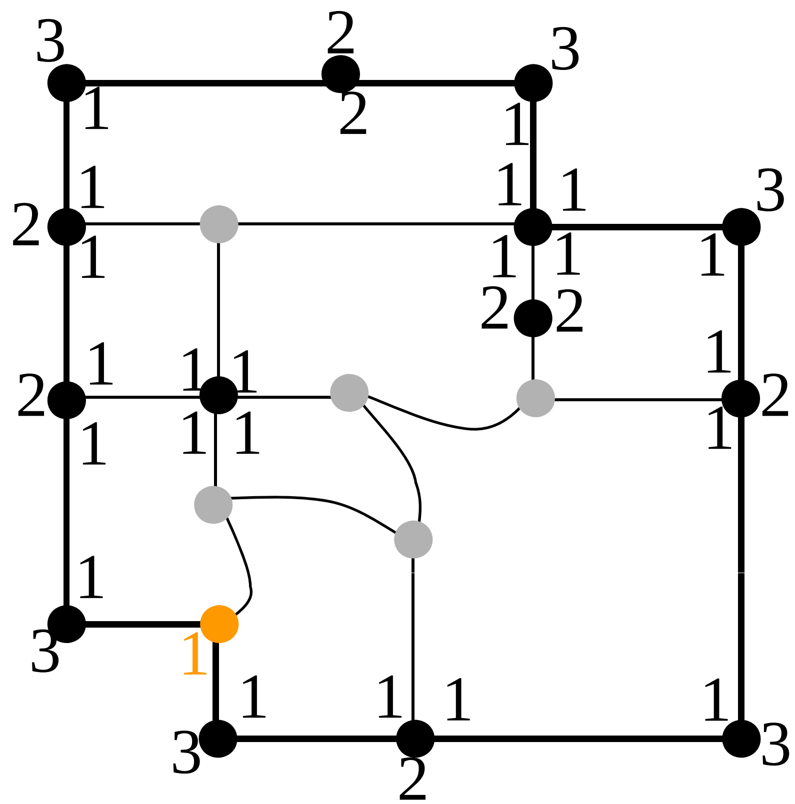


Construct a decision graph G_d

outer vertex of degree 3

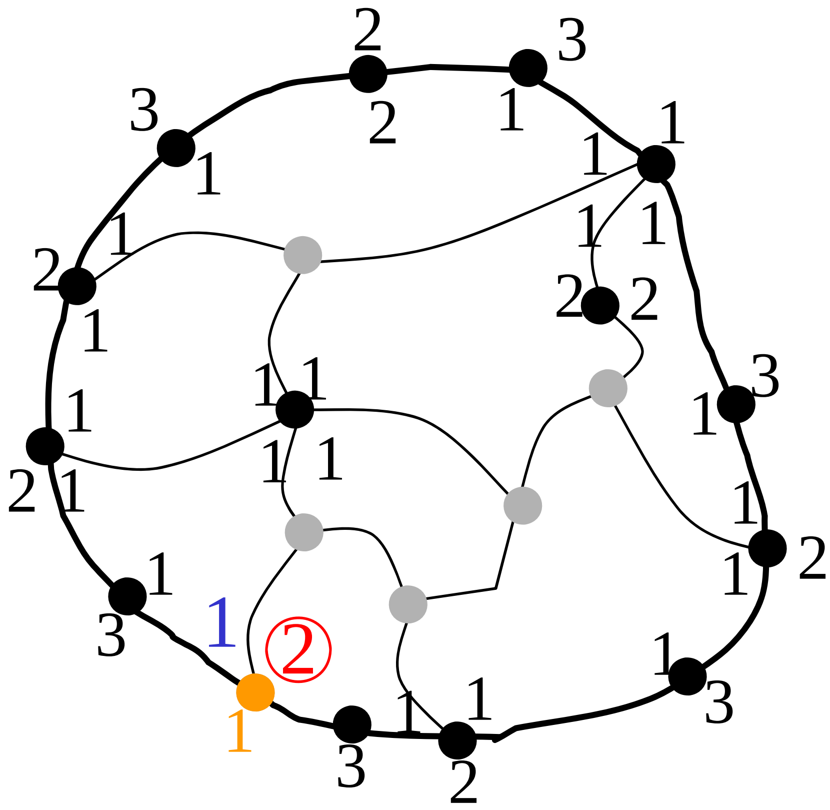


a plane graph *G*

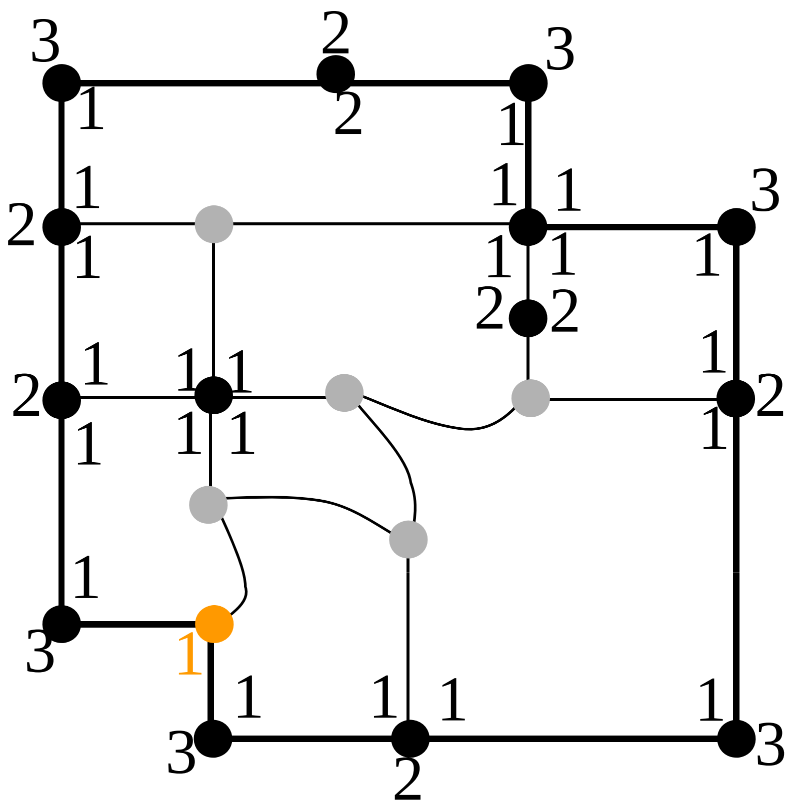


Construct a decision graph G_d

outer vertex of degree 3

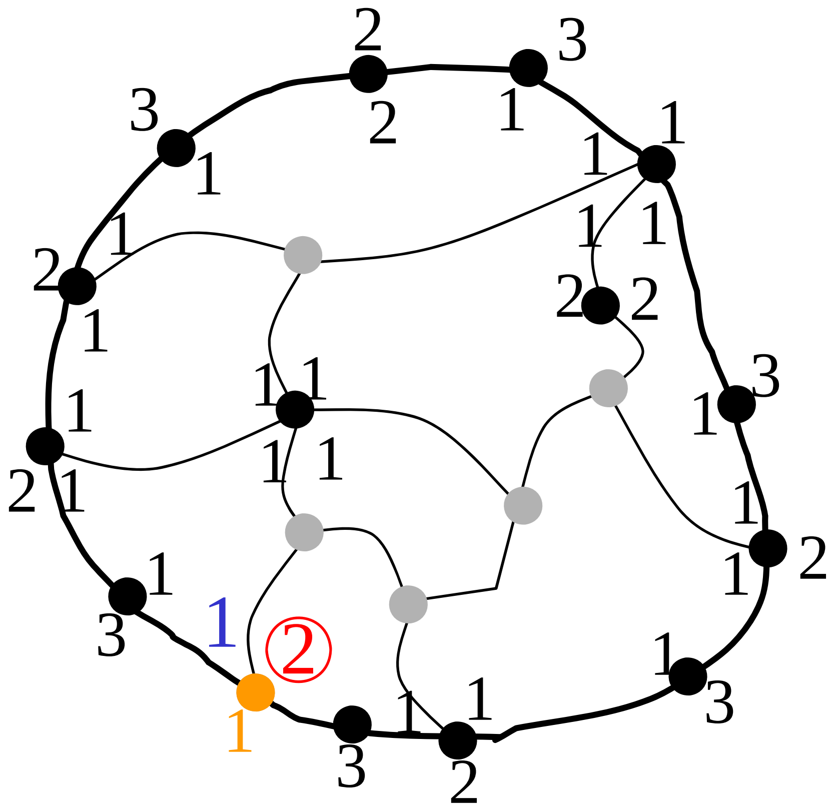


a plane graph *G*

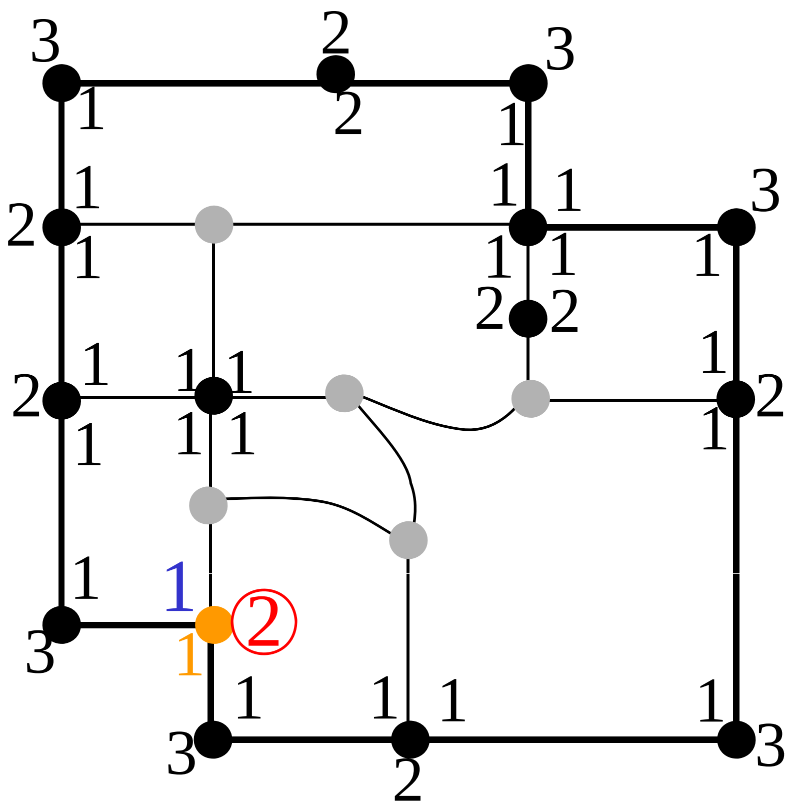


Construct a decision graph G_d

outer vertex of degree 3

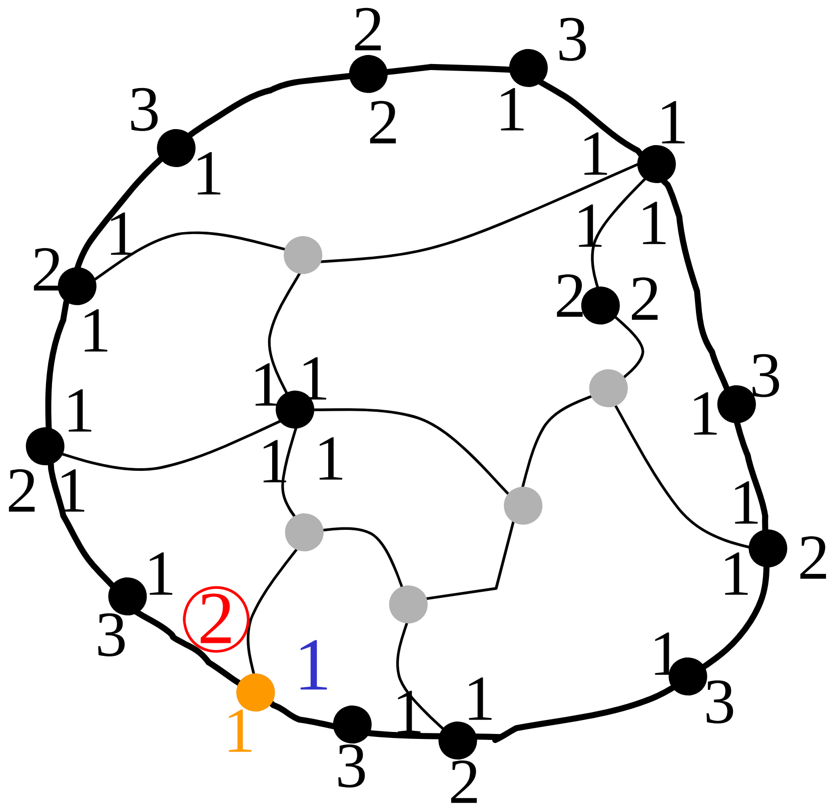


a plane graph G

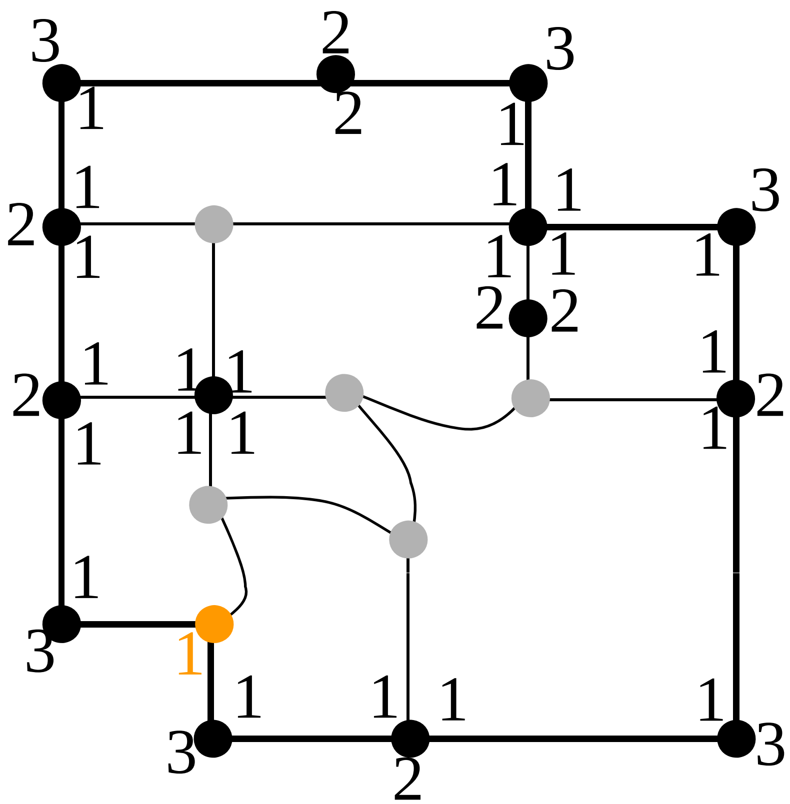


Construct a decision graph G_d

outer vertex of degree 3

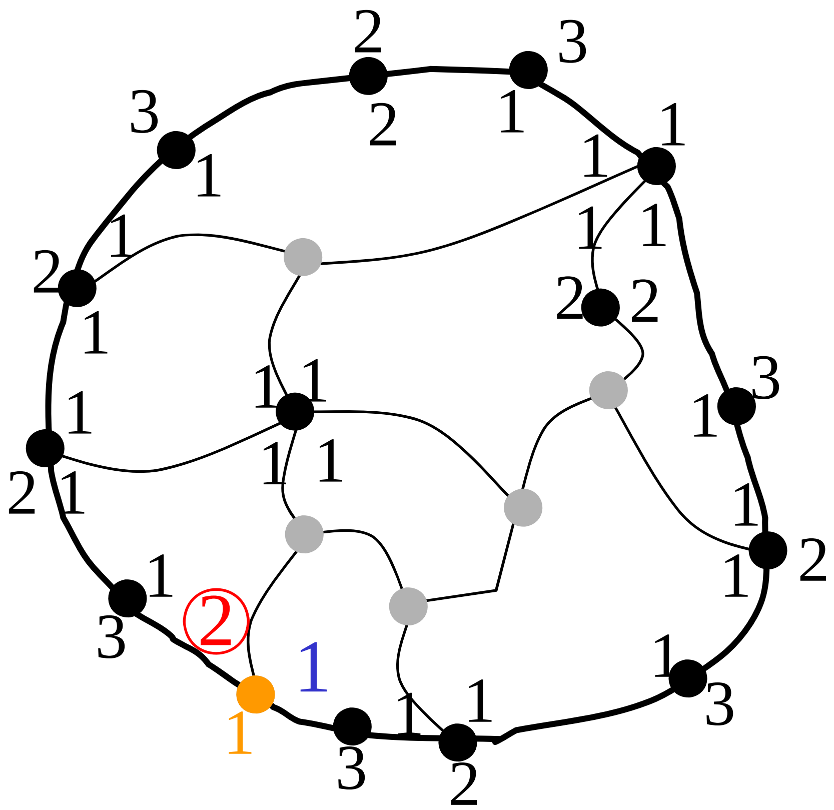


a plane graph *G*

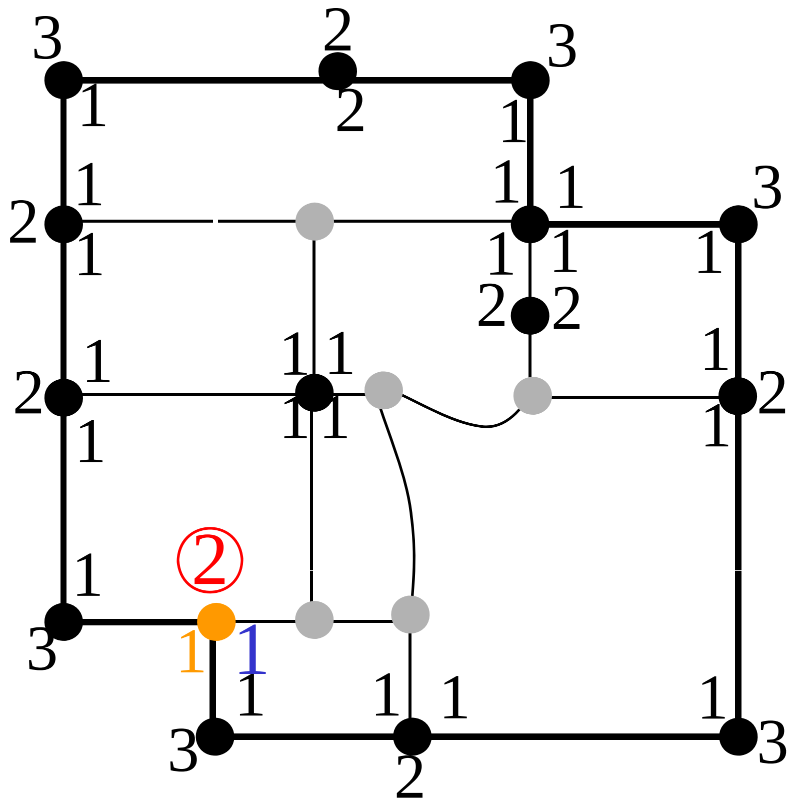


Construct a decision graph G_d

outer vertex of degree 3

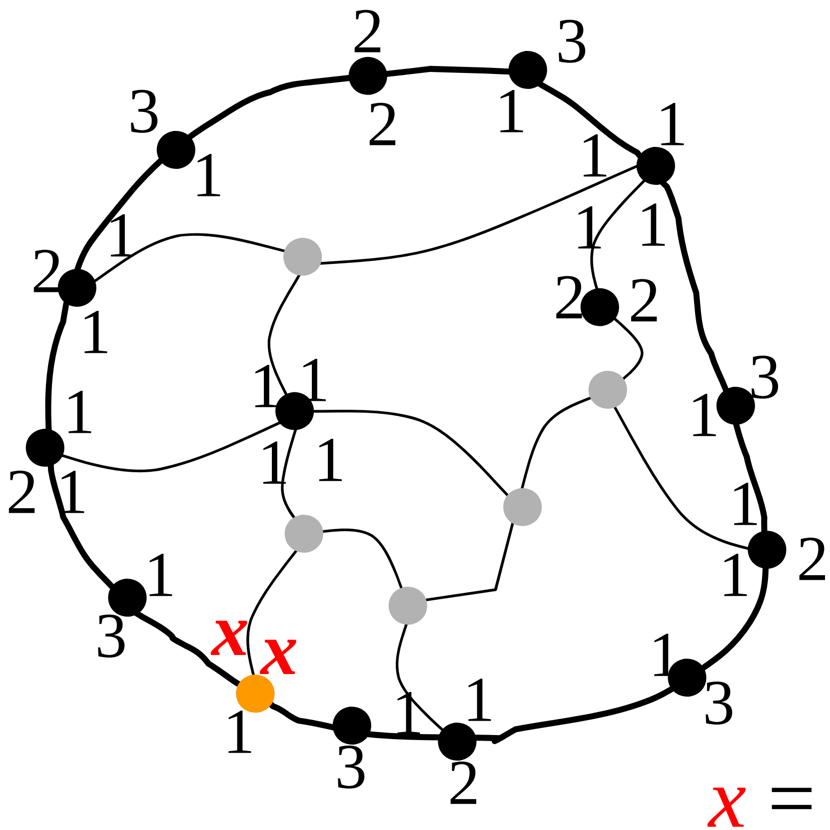


a plane graph *G*

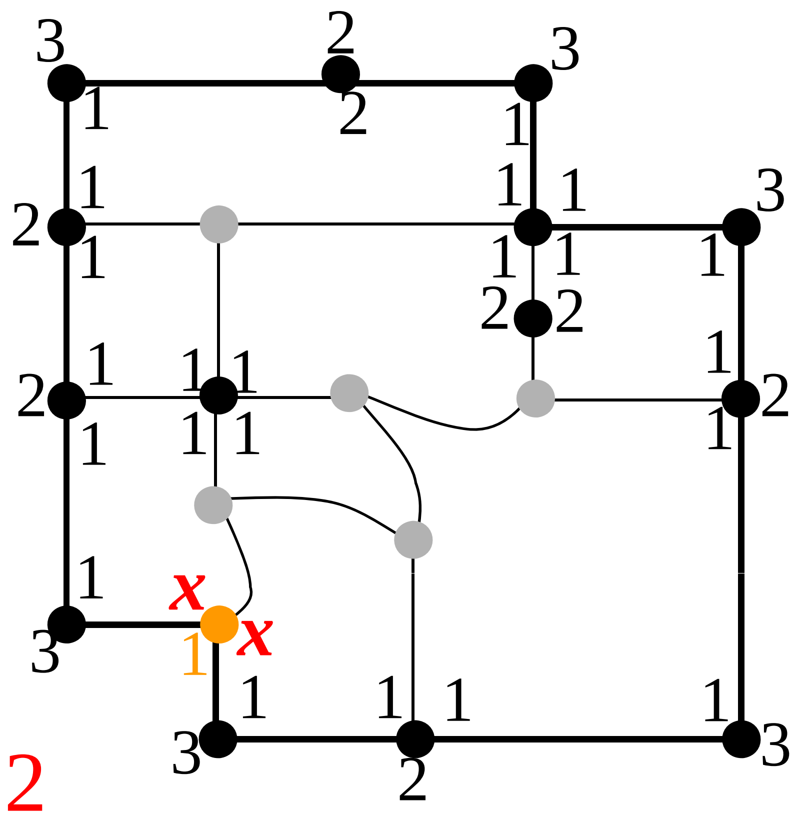


Construct a decision graph G_d

outer vertex of degree 3



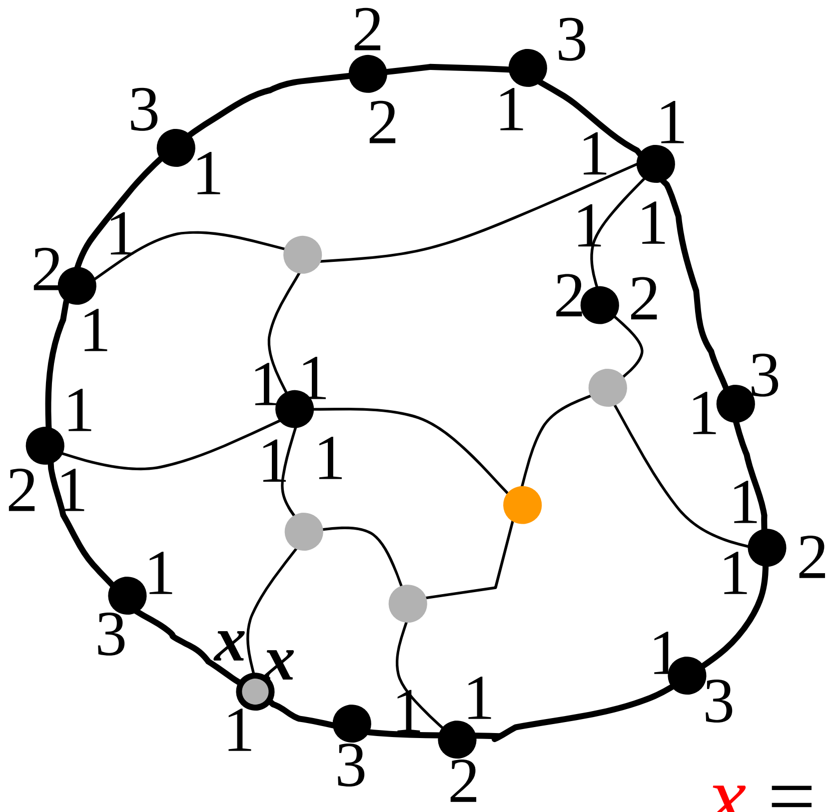
a plane graph G



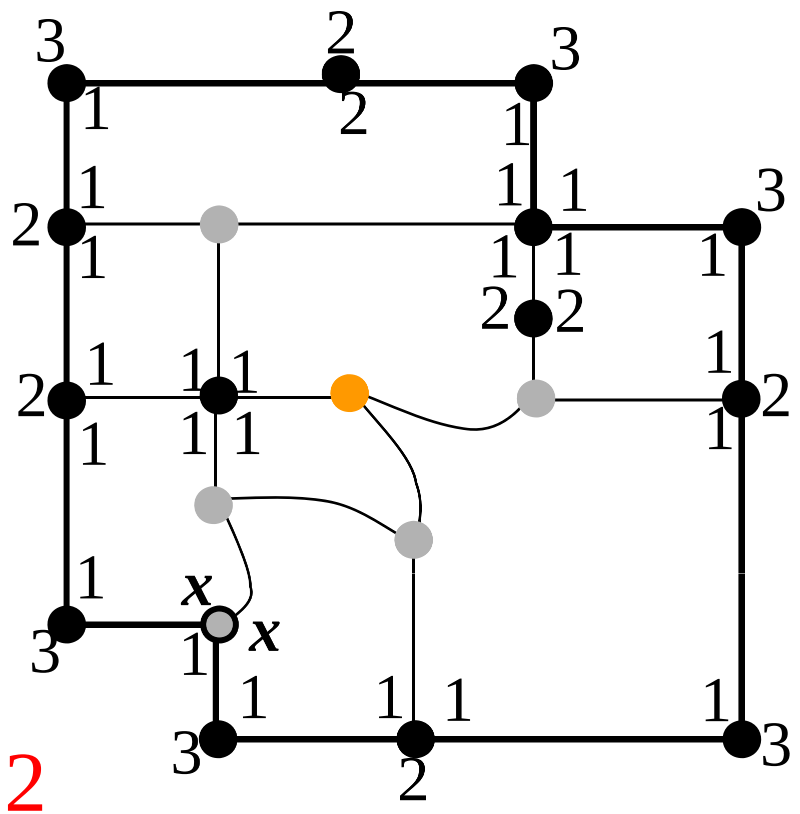
x = 1 or 2

Construct a decision graph G_d

degree 3



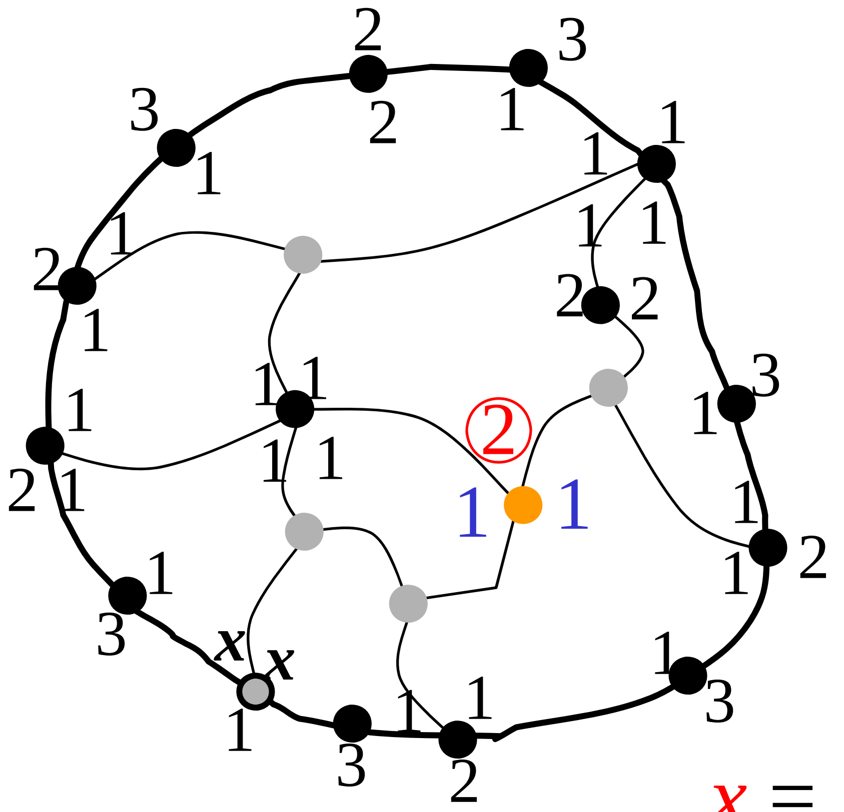
a plane graph *G*



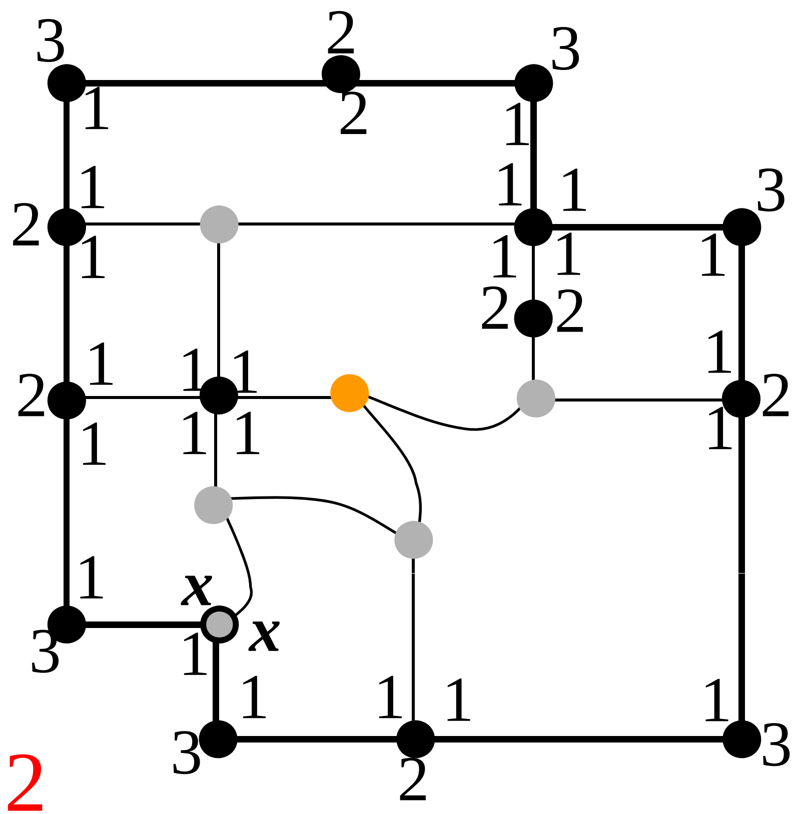
$x = 1$ or 2

Construct a decision graph G_d

degree 3

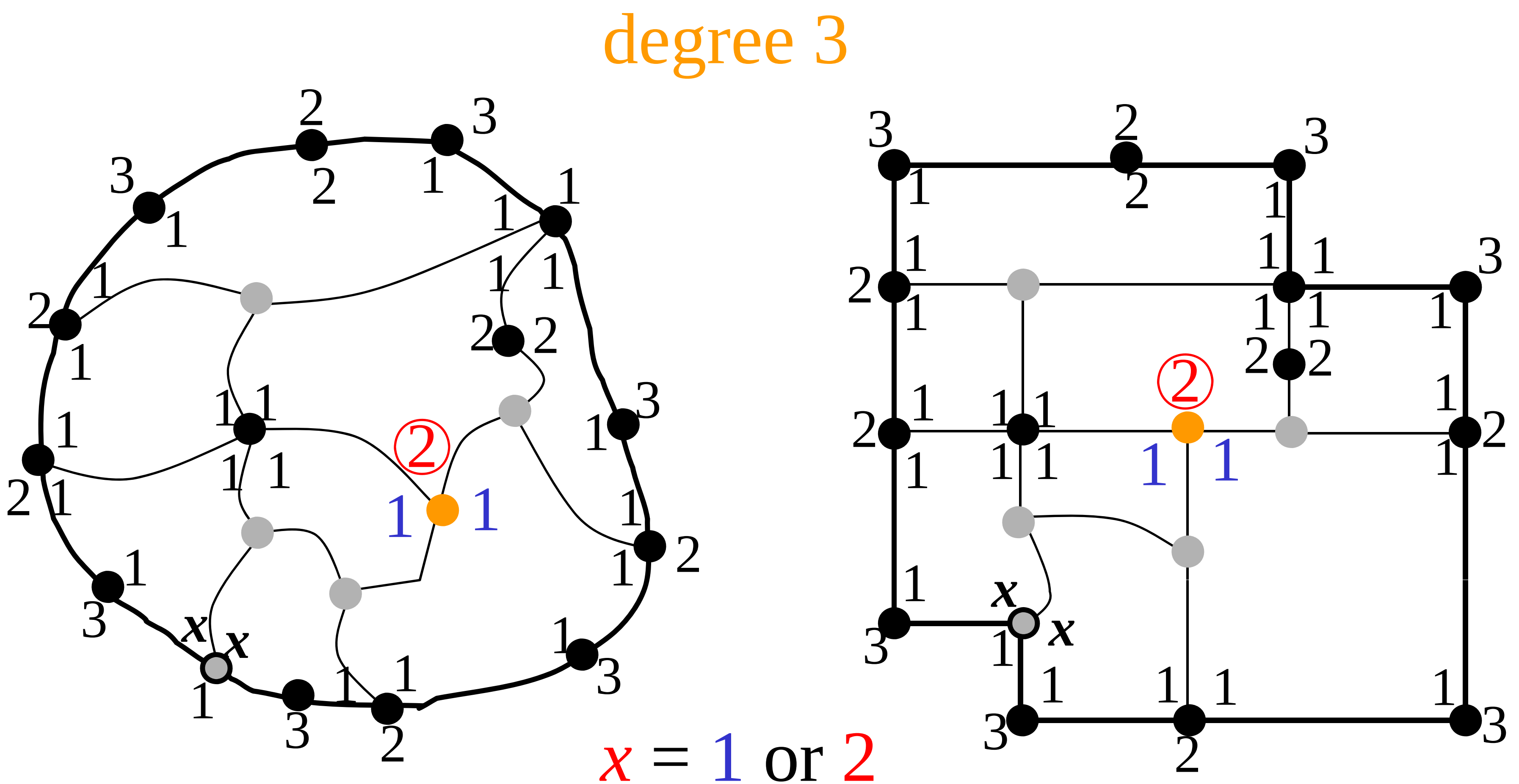


x = 1 or 2



a plane graph G

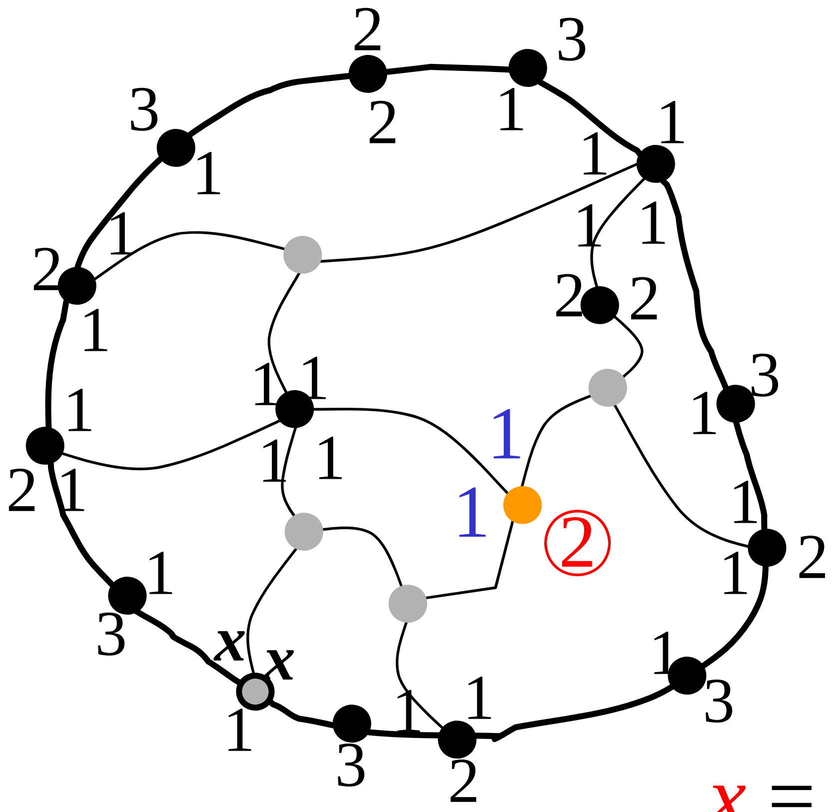
Construct a decision graph G_d



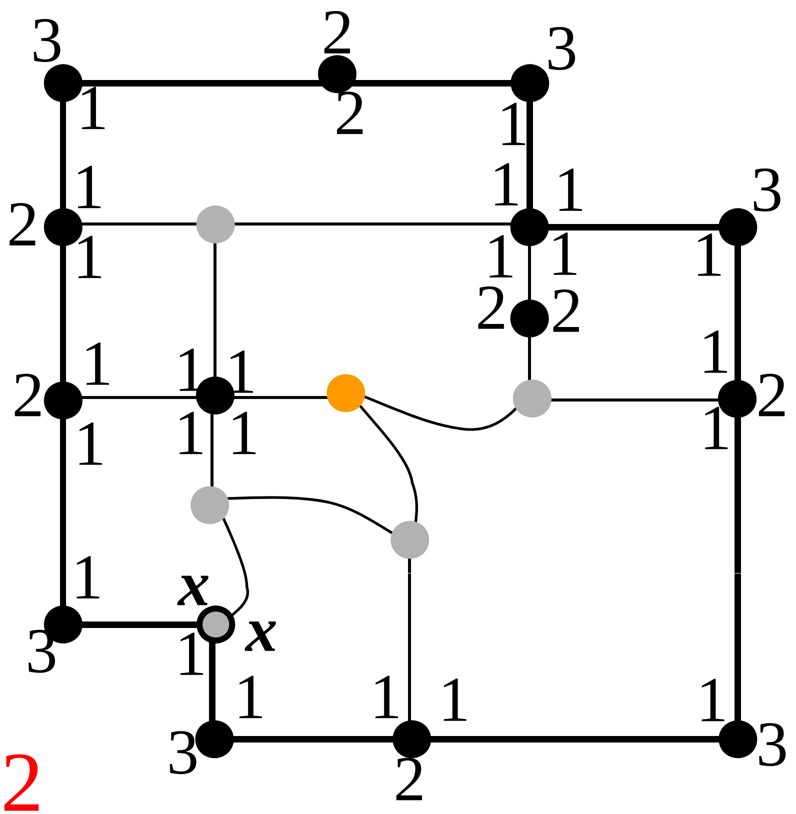
a plane graph G

Construct a decision graph G_d

degree 3



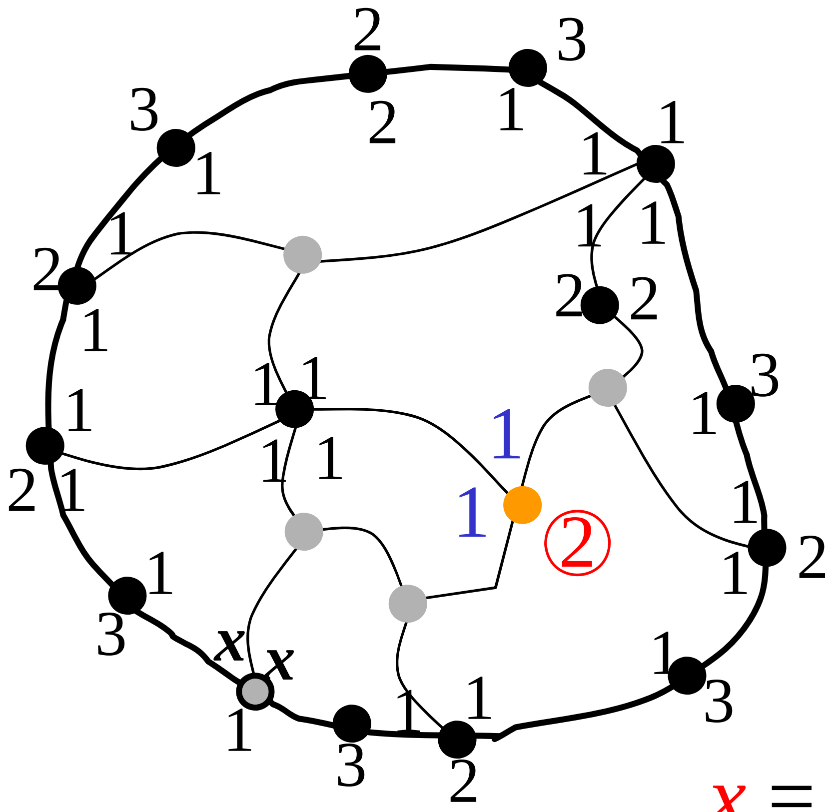
x = 1 or 2



a plane graph G

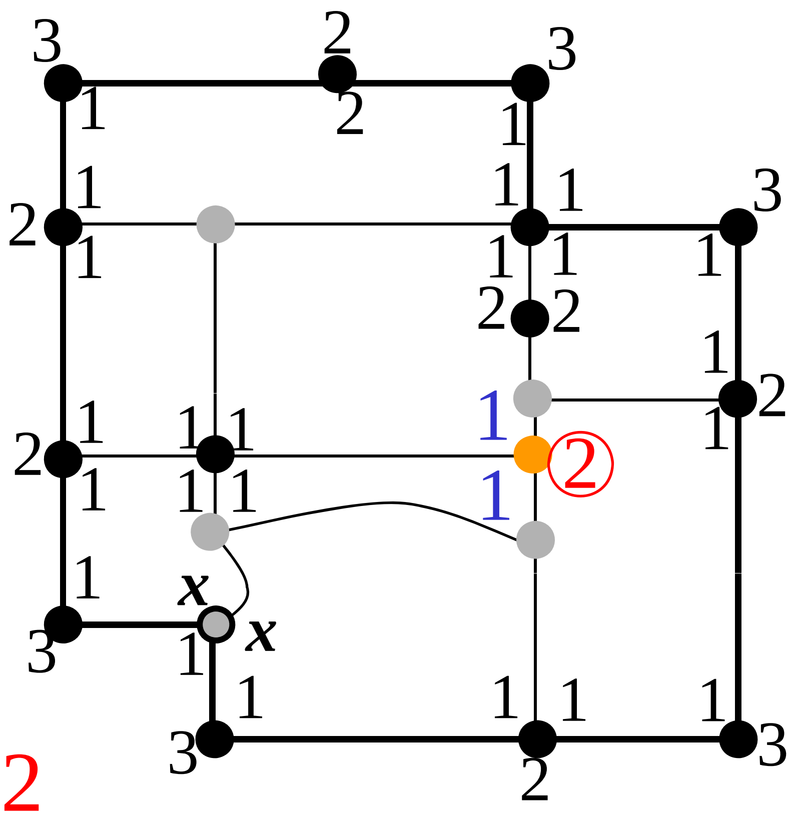
Construct a decision graph G_d

degree 3



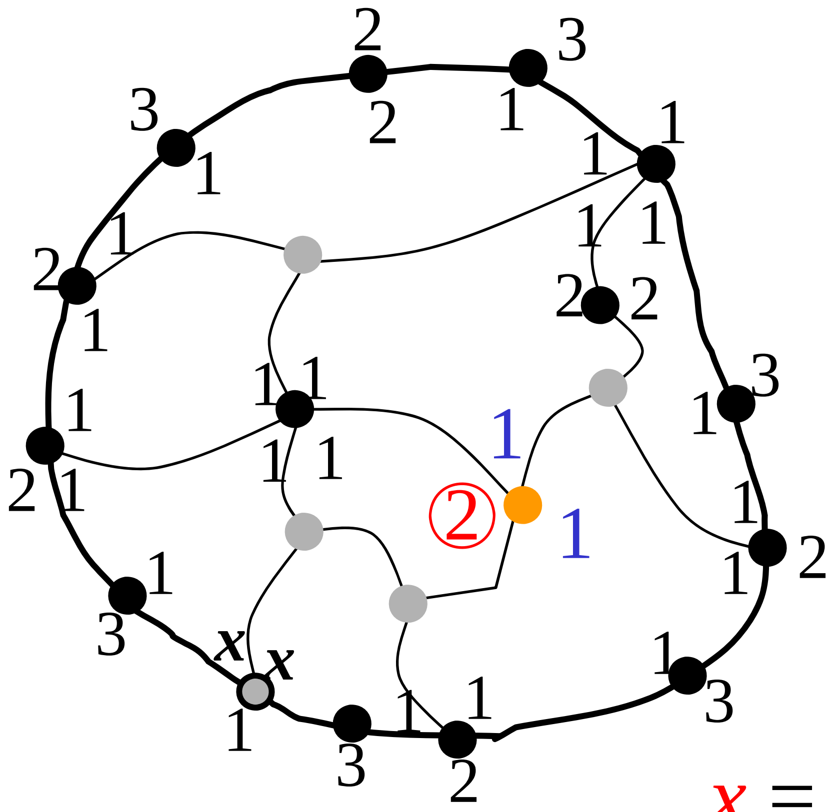
a plane graph G

x = 1 or 2

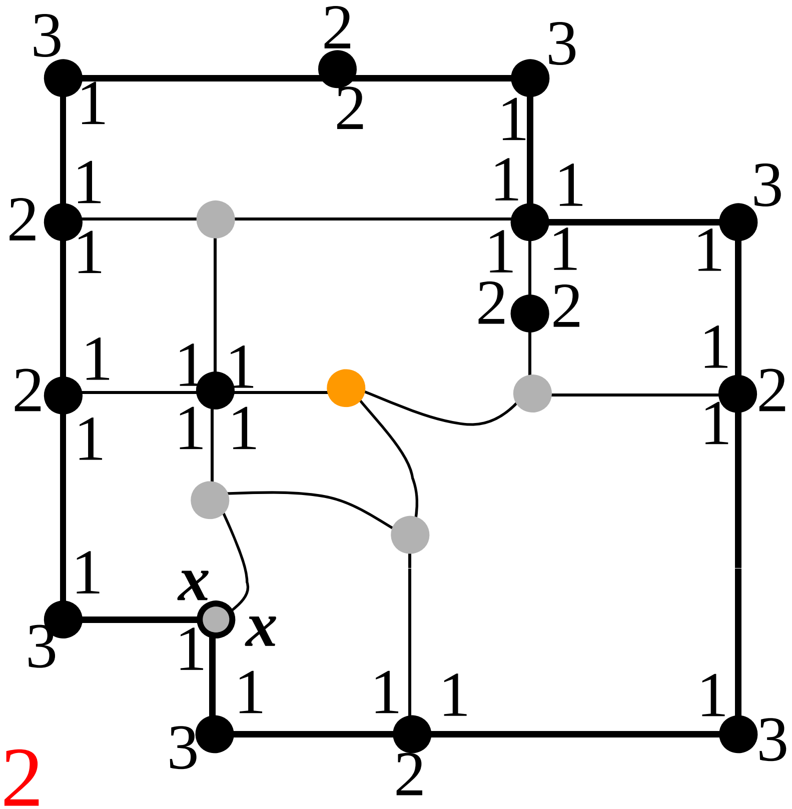


Construct a decision graph G_d

degree 3



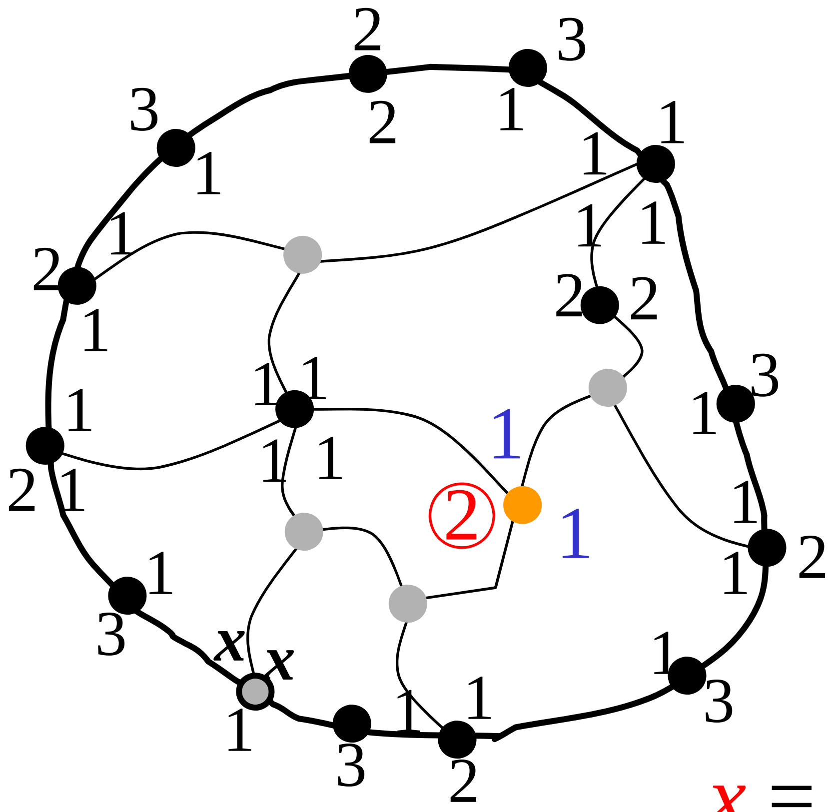
x = 1 or 2



a plane graph G

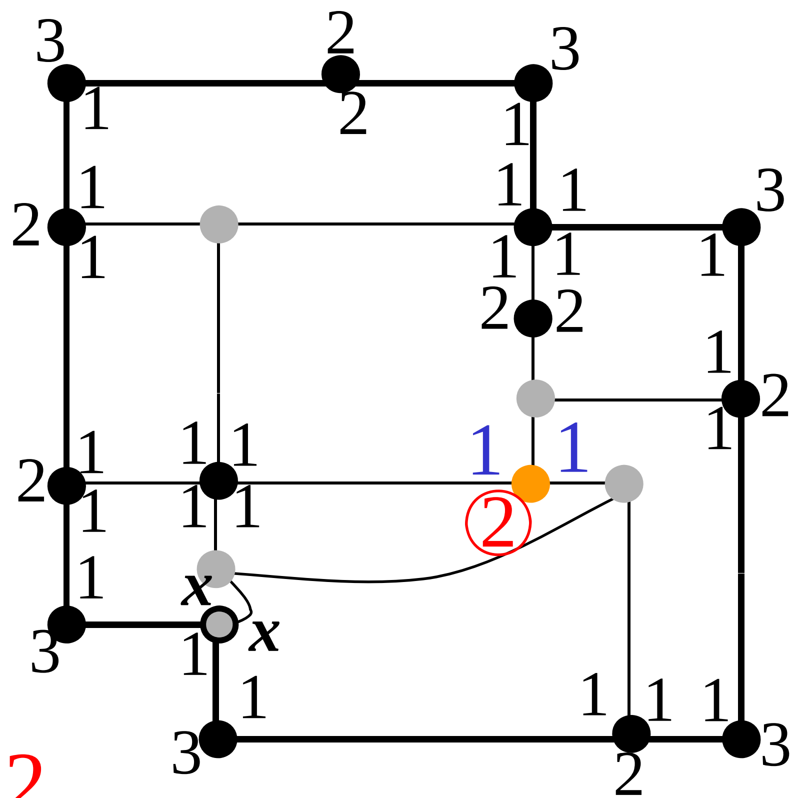
Construct a decision graph G_d

degree 3

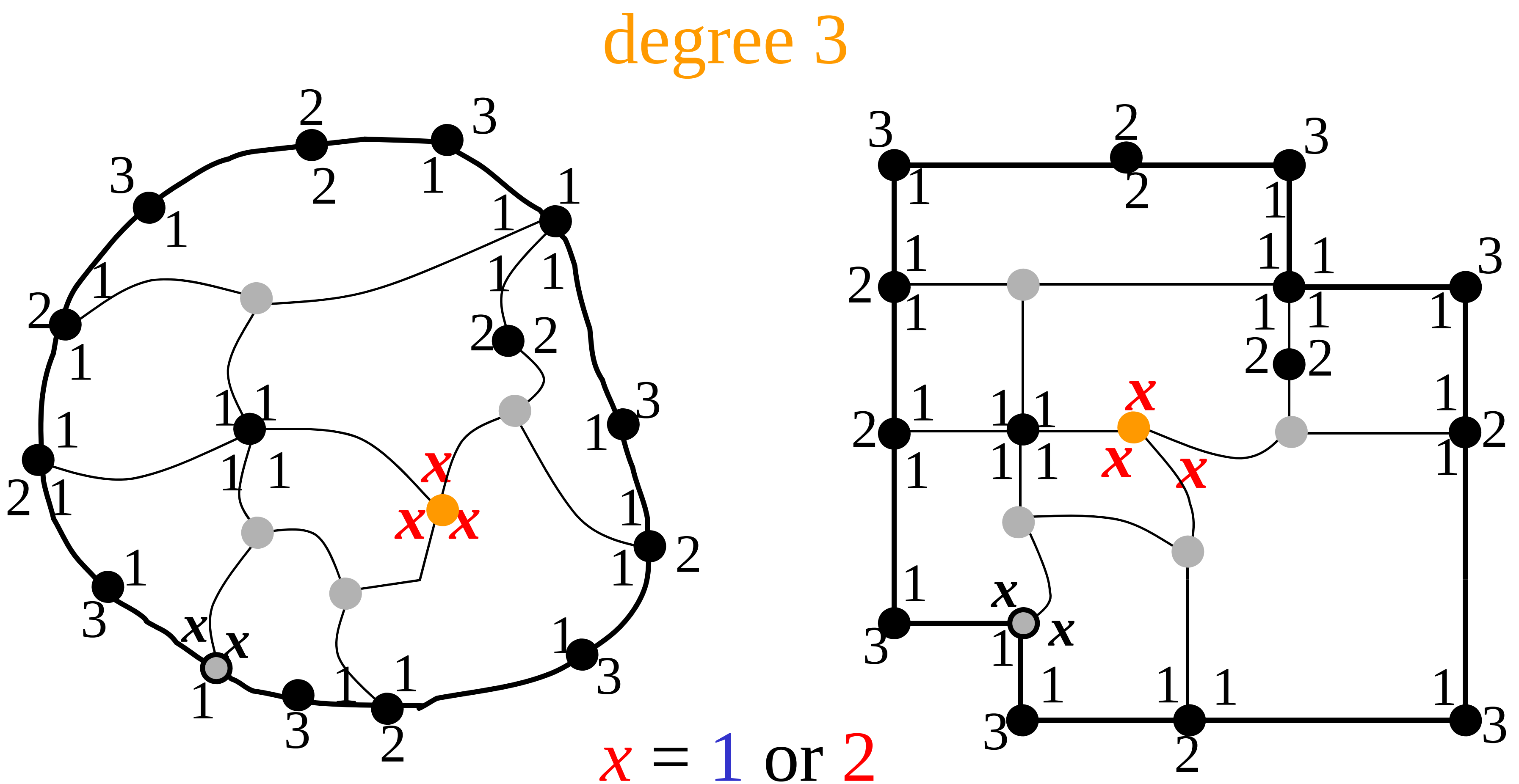


a plane graph *G*

x = 1 or 2



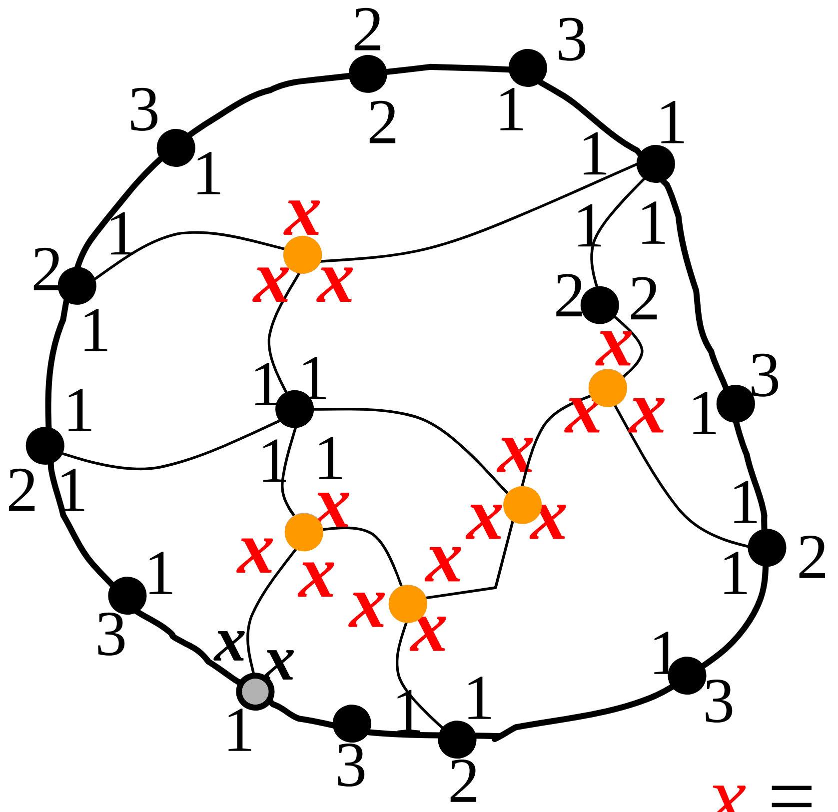
Construct a decision graph G_d



a plane graph G

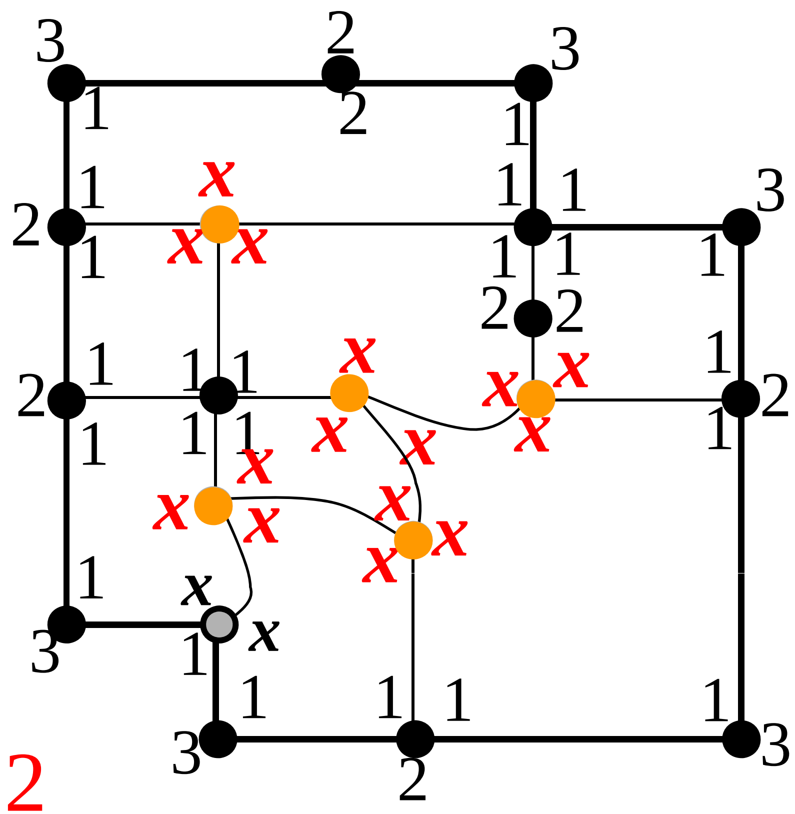
Construct a decision graph G_d

degree 3

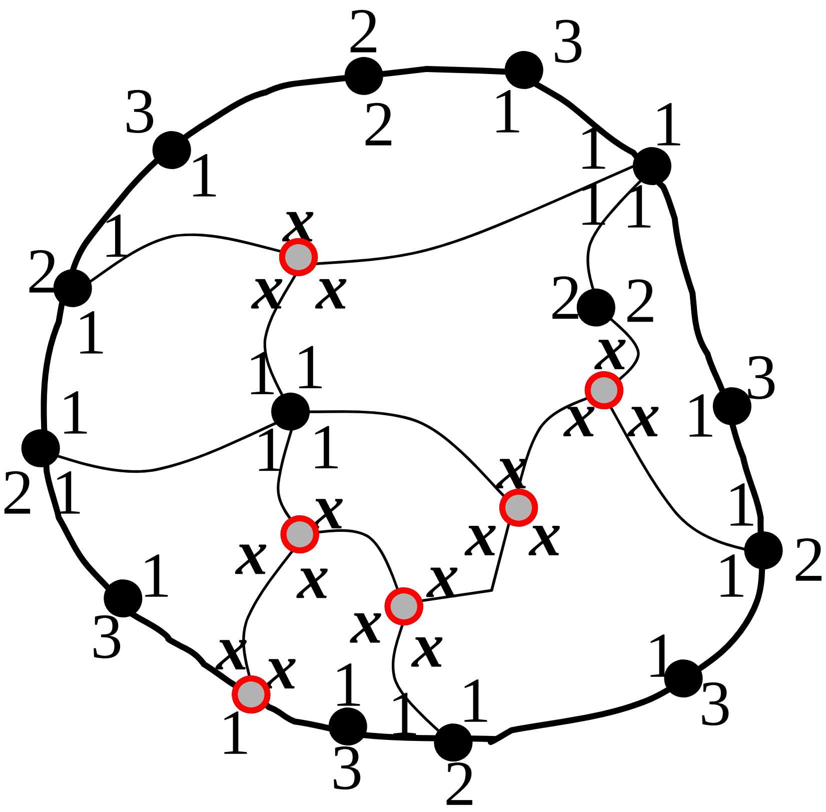


a plane graph G

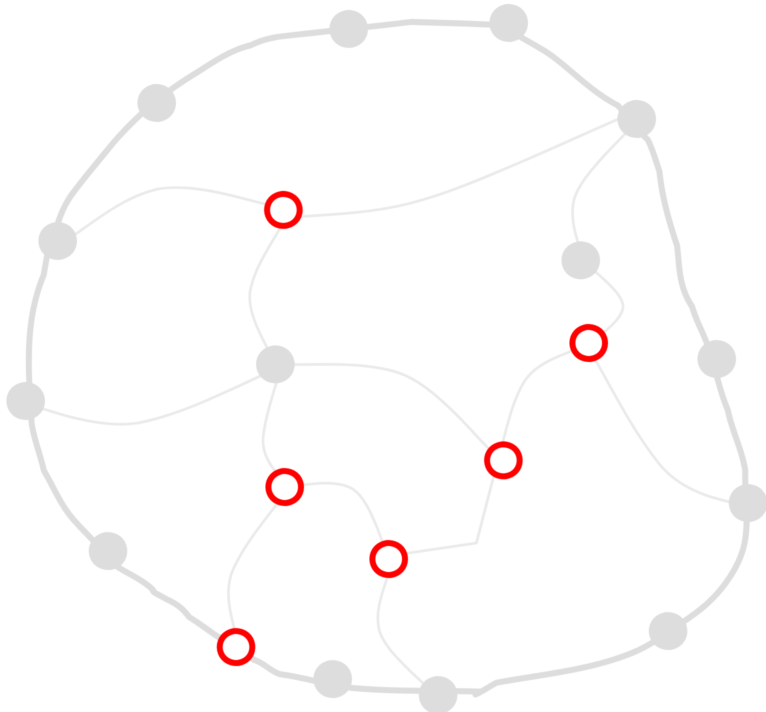
x = 1 or 2



Construct a decision graph G_d

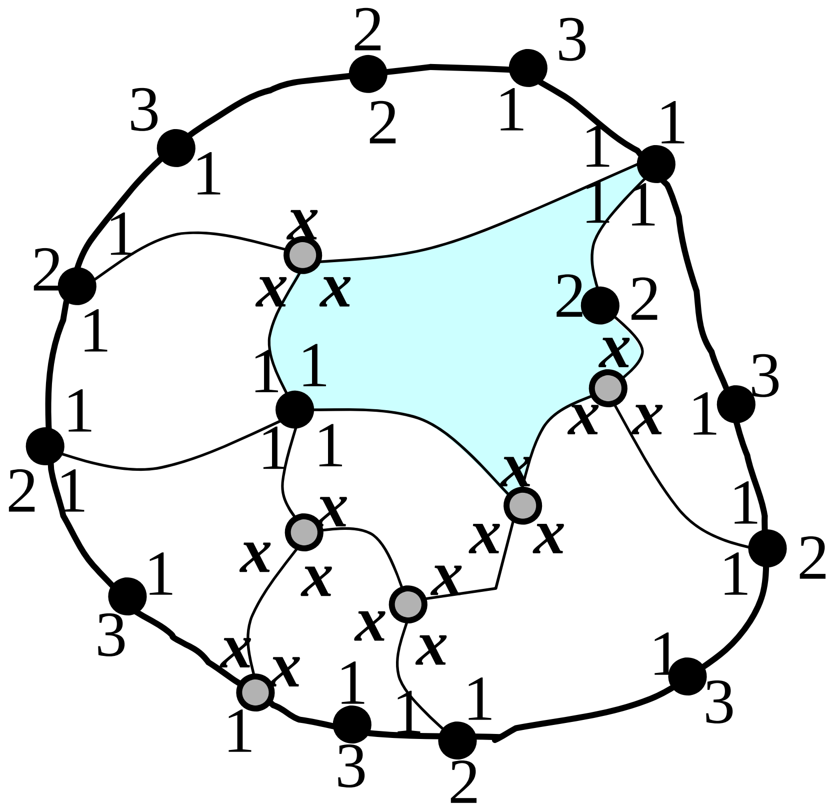


a plane graph *G*

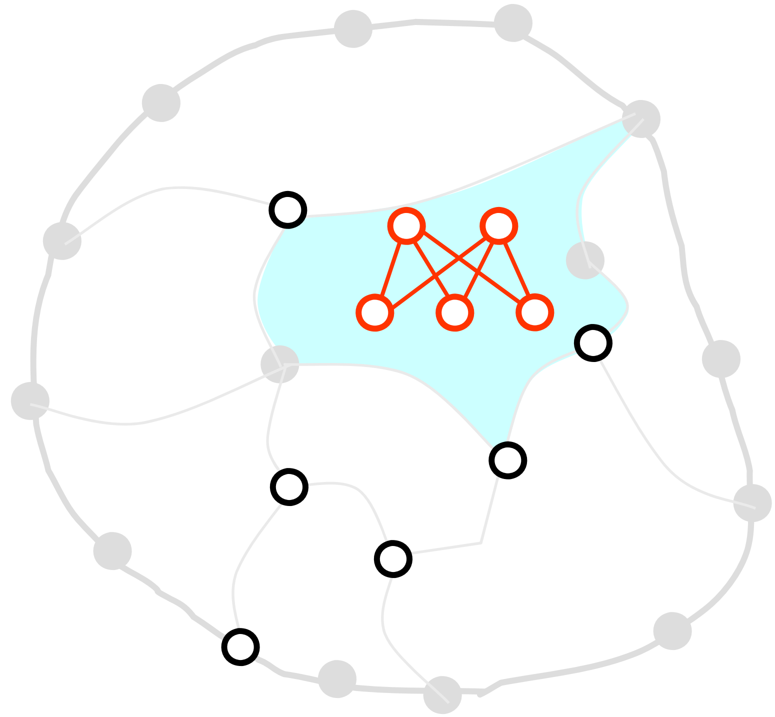


a decision graph G_d of *G*

Construct a decision graph G_d

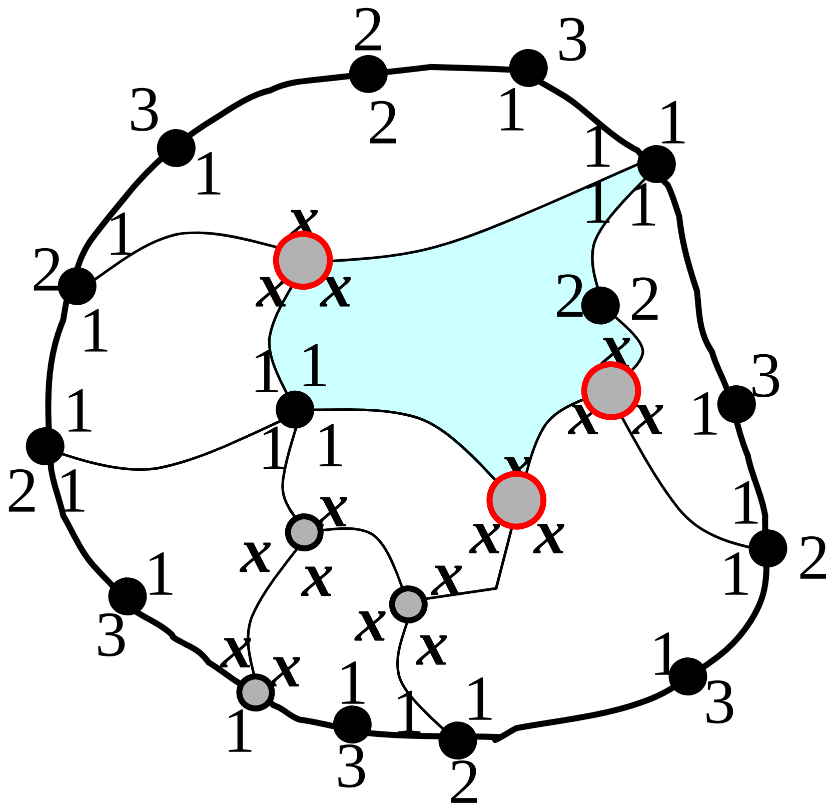


a plane graph G

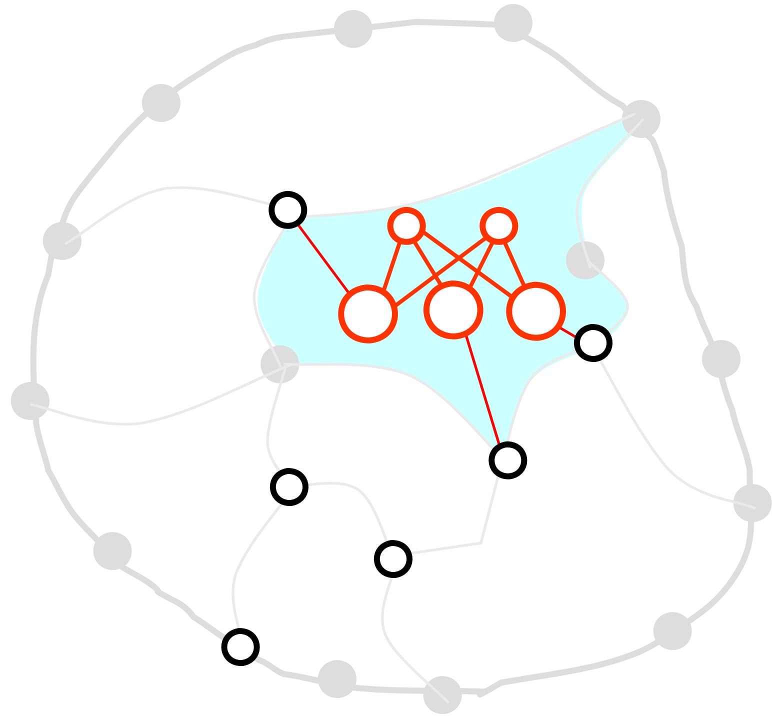


a decision graph G_d of G

Construct a decision graph G_d



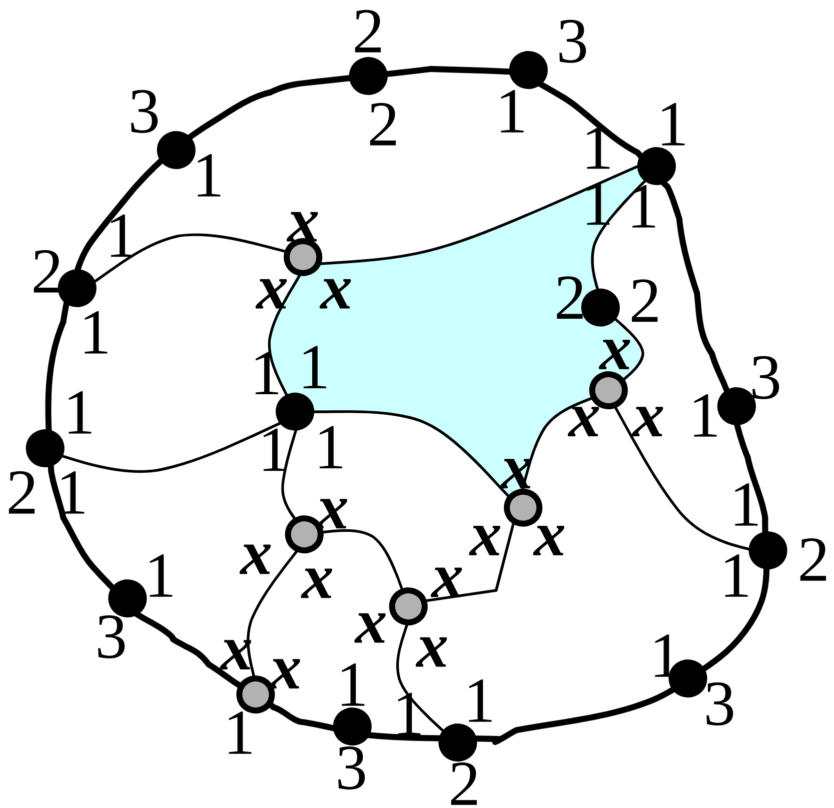
a plane graph G



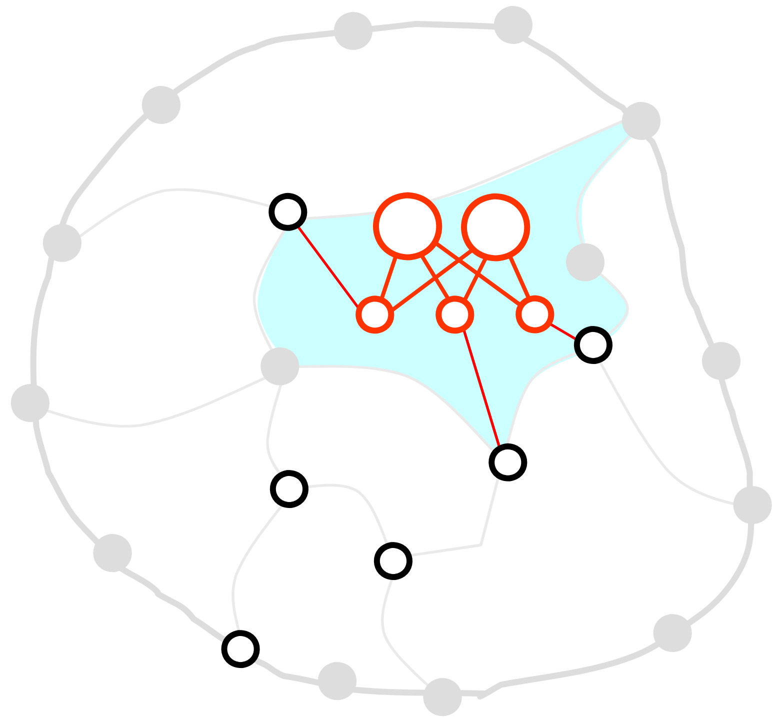
a decision graph G_d of G

Construct a decision graph G_d

2 of *x*'s must be 1's.

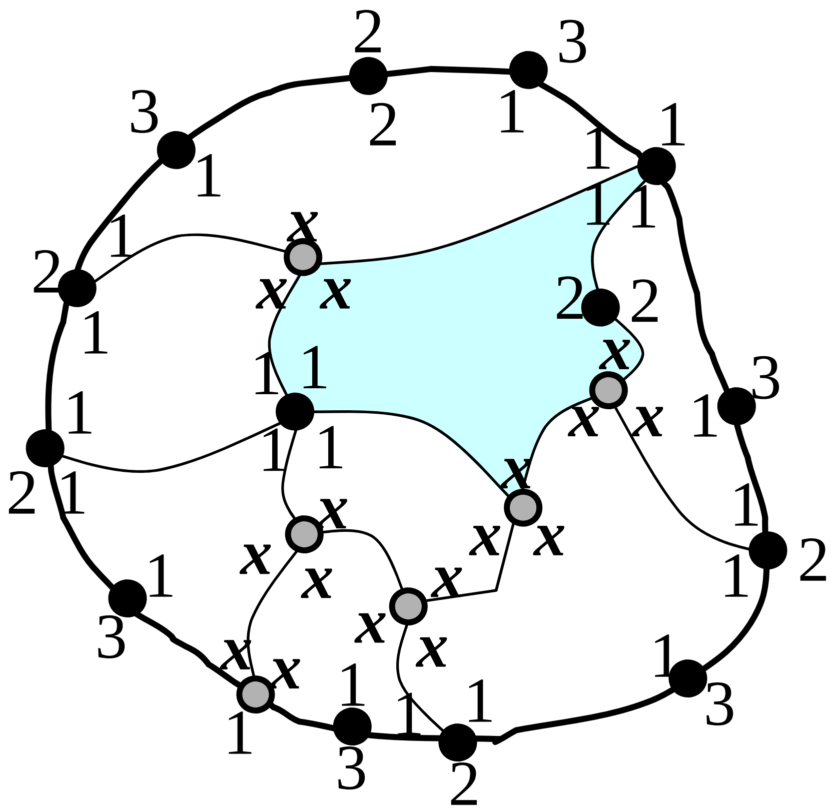


a plane graph *G*

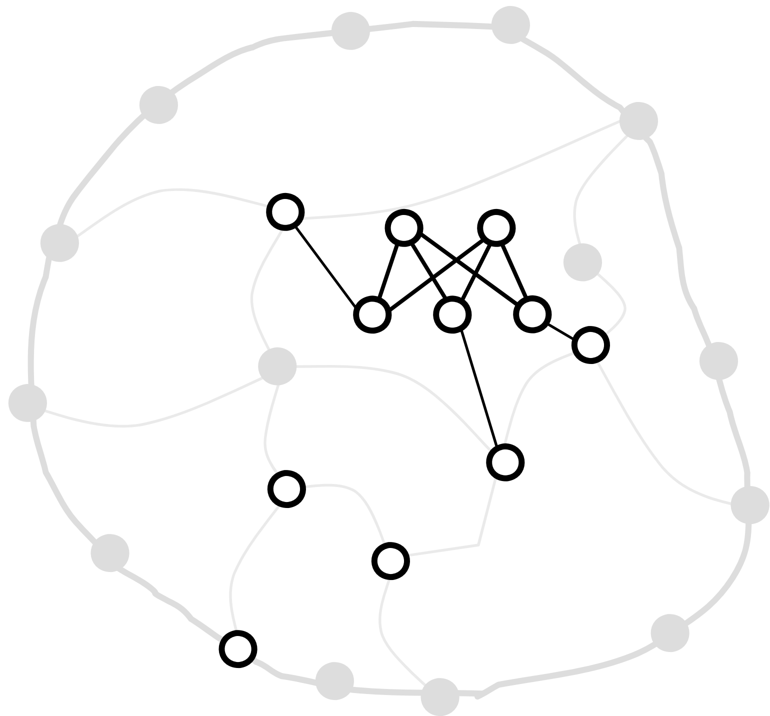


a decision graph G_d of *G*

Construct a decision graph G_d

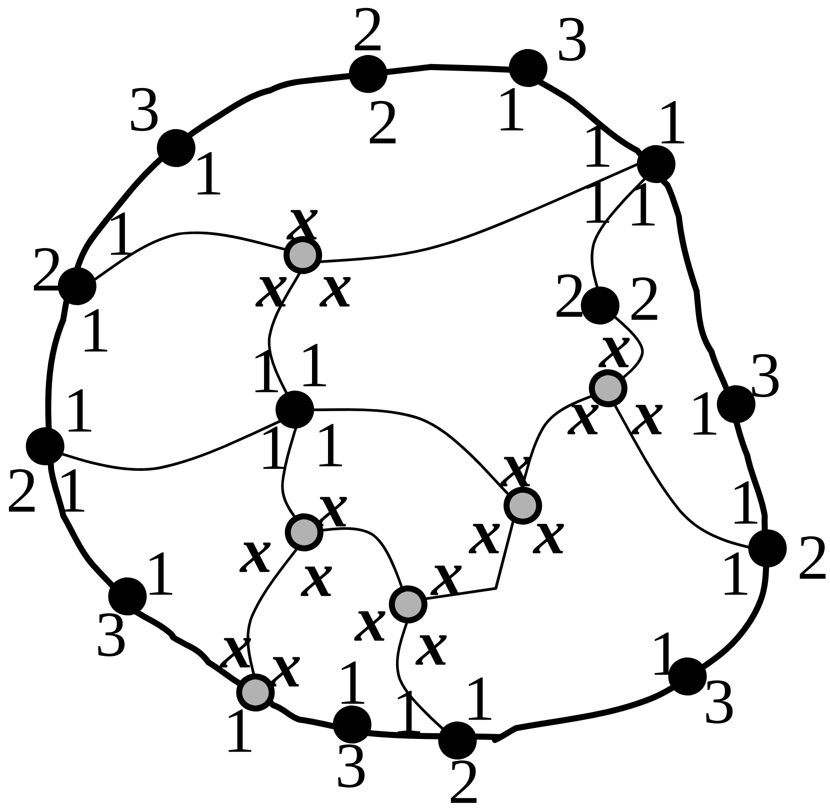


a plane graph G

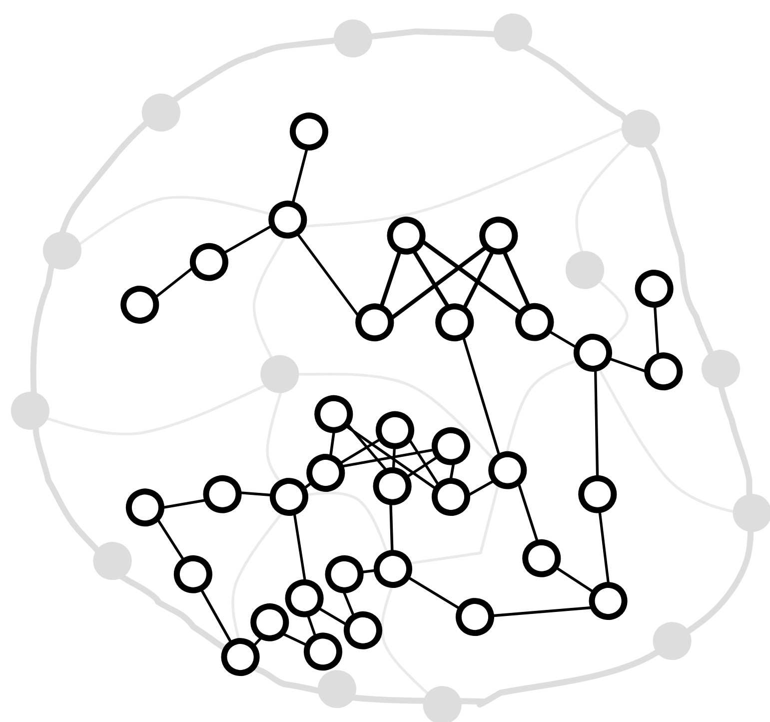


a decision graph G_d of G

Construct a decision graph G_{d}



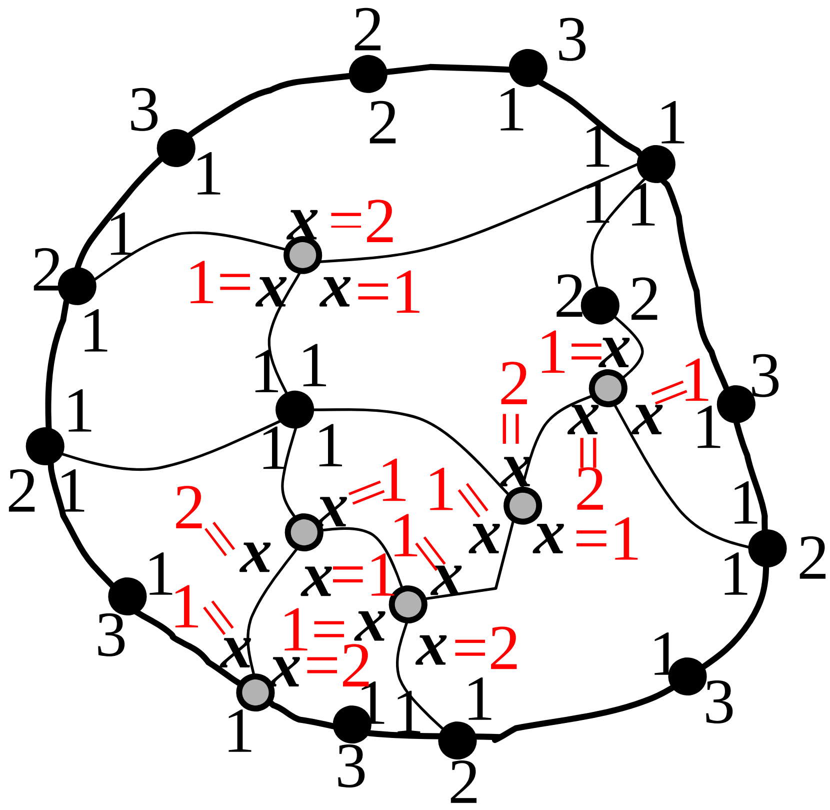
a plane graph G



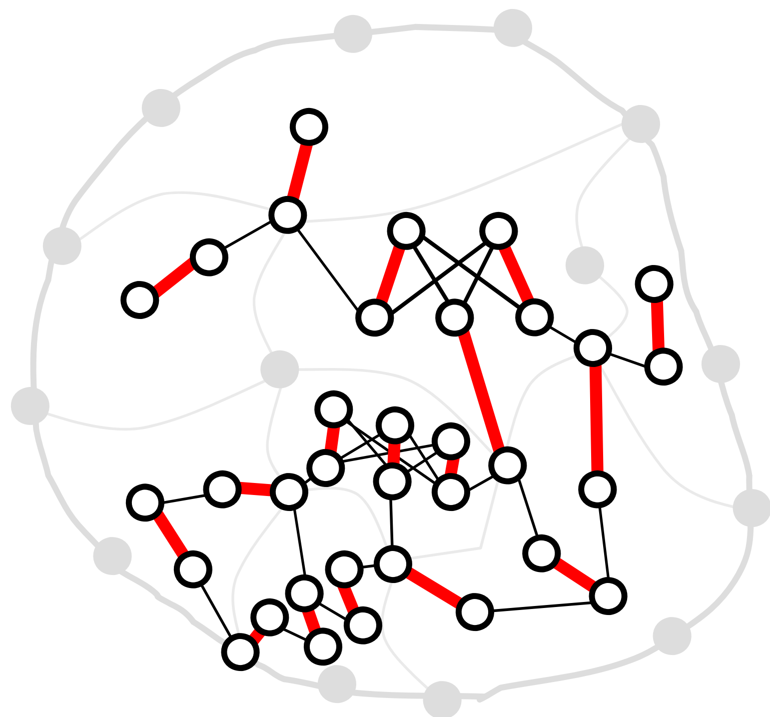
a decision graph G_{d} of G

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇔ G_d has a perfect matching



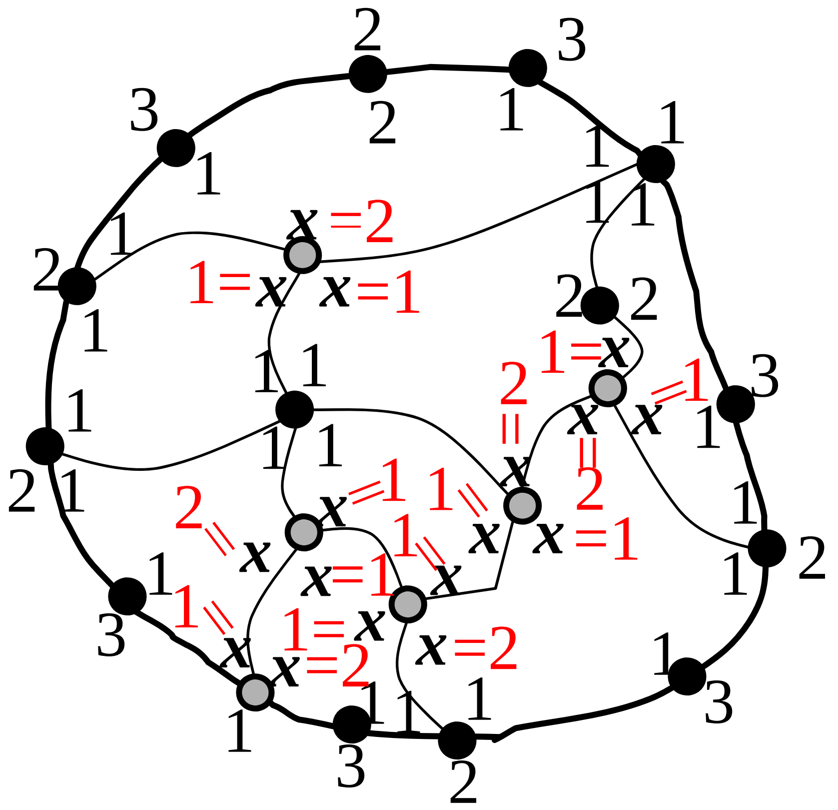
a plane graph G



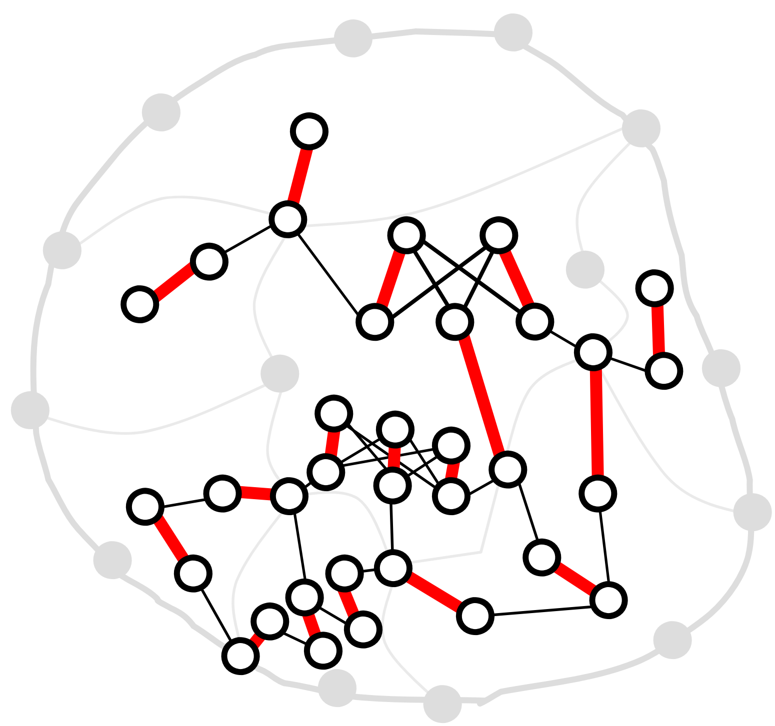
a decision graph G_d of G

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇨ G_d has a perfect matching



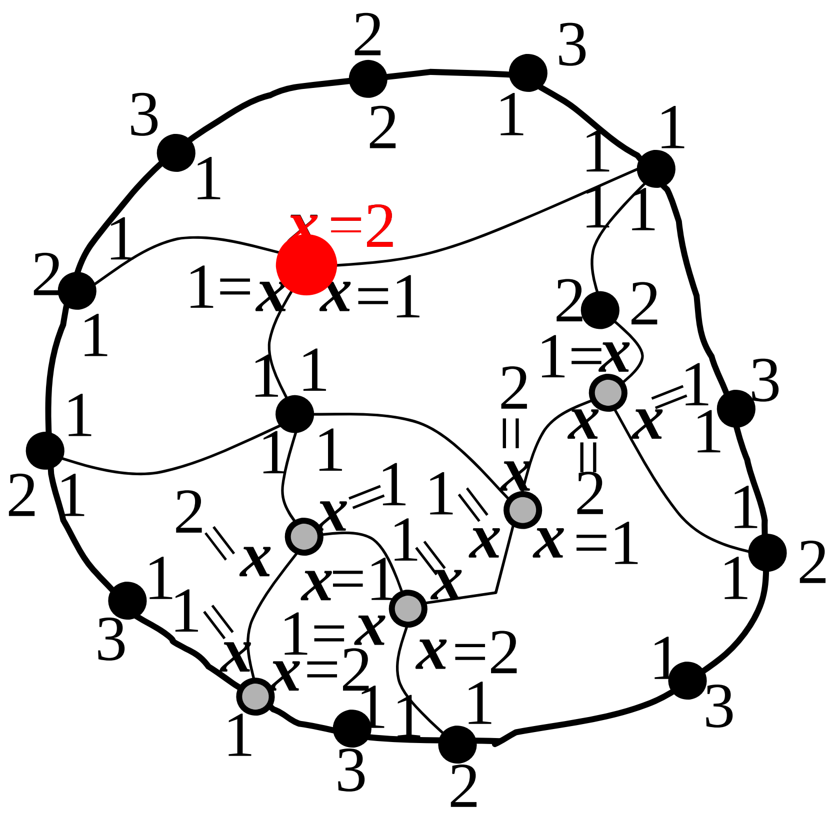
a plane graph *G*



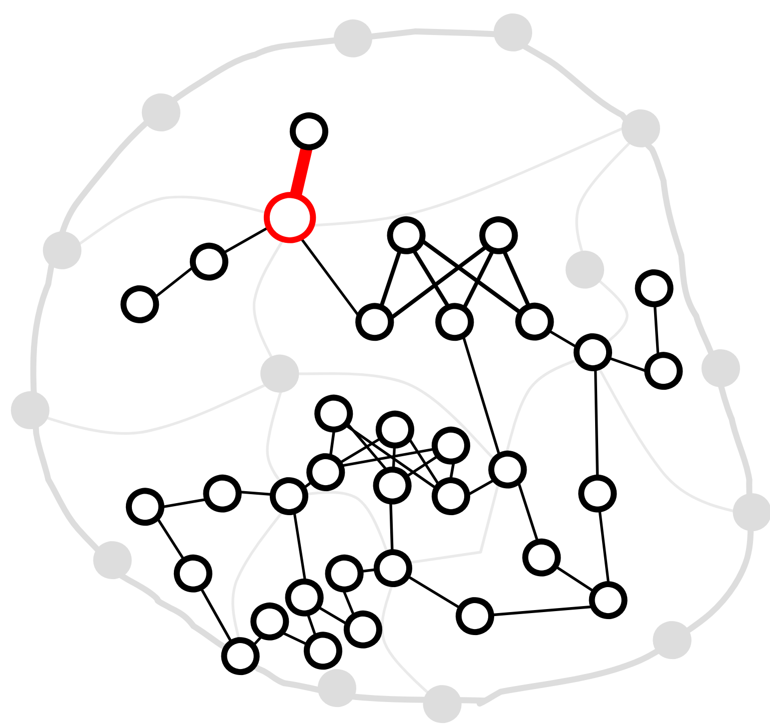
a decision graph G_d of *G*

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇒ G_d has a perfect matching



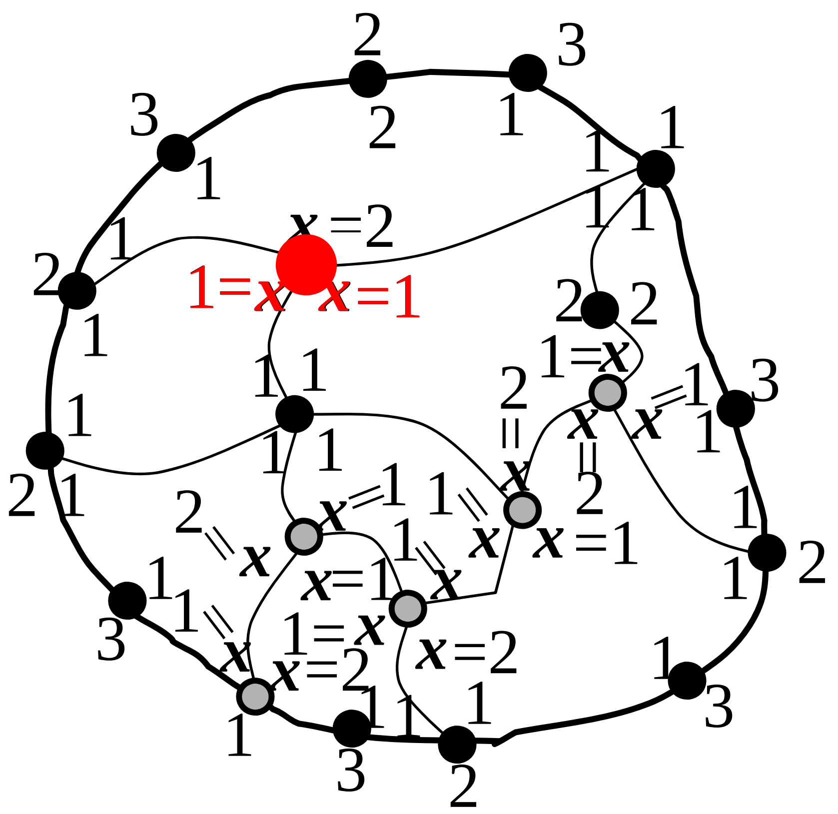
a plane graph *G*



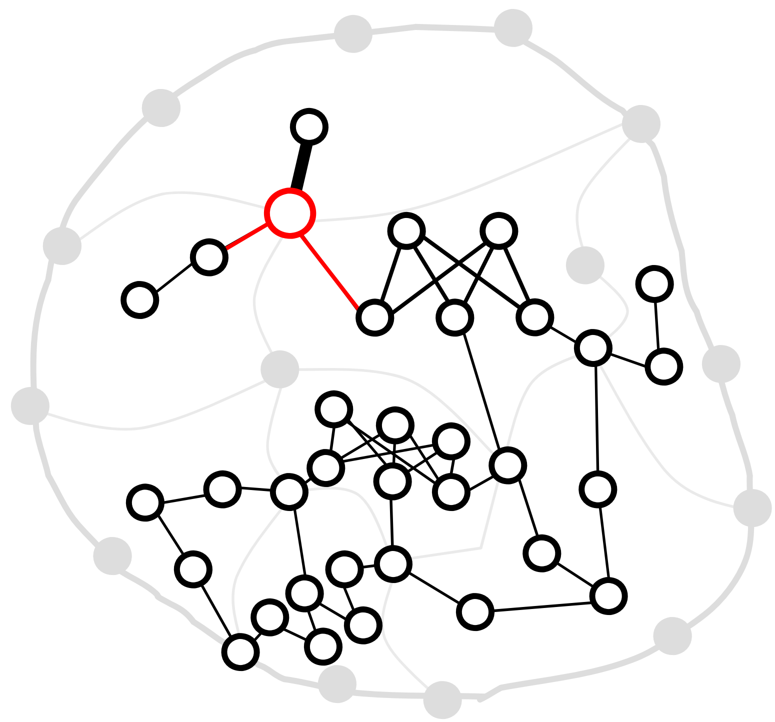
a decision graph G_d of *G*

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇒ G_d has a perfect matching



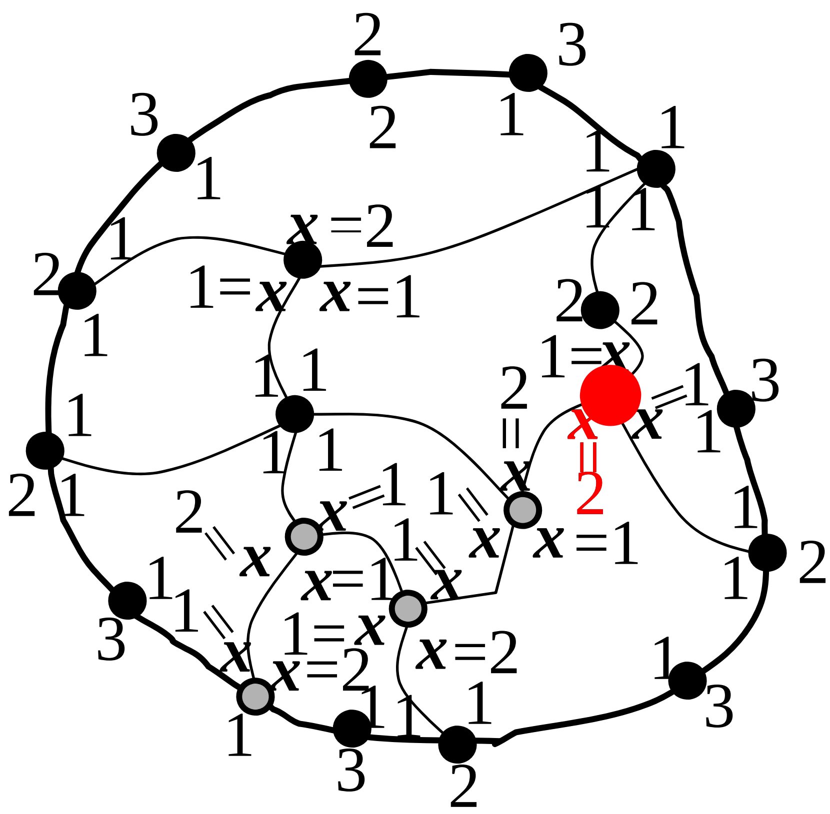
a plane graph G



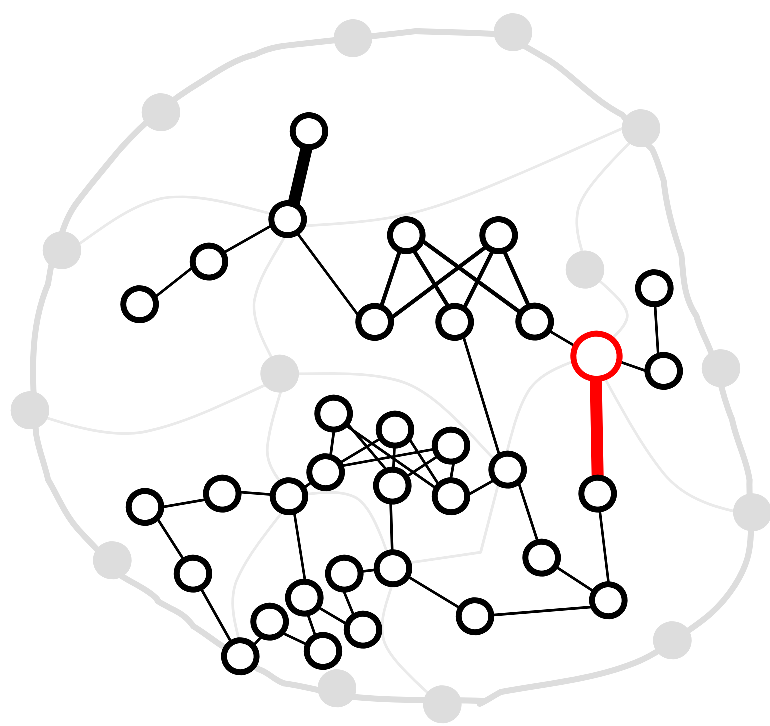
a decision graph G_d of G

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇒ G_d has a perfect matching



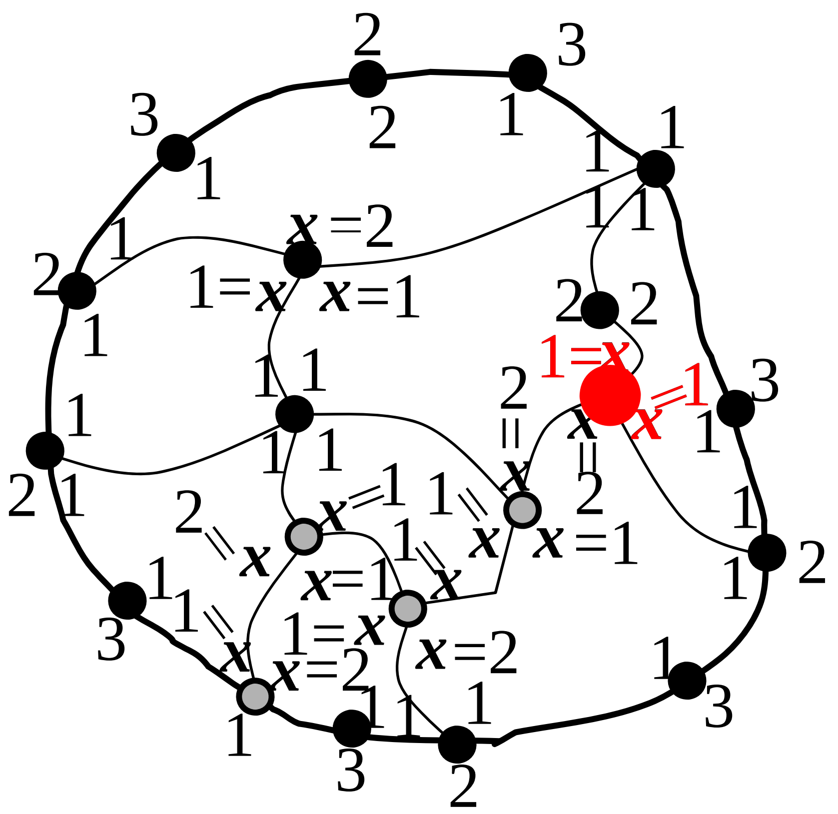
a plane graph *G*



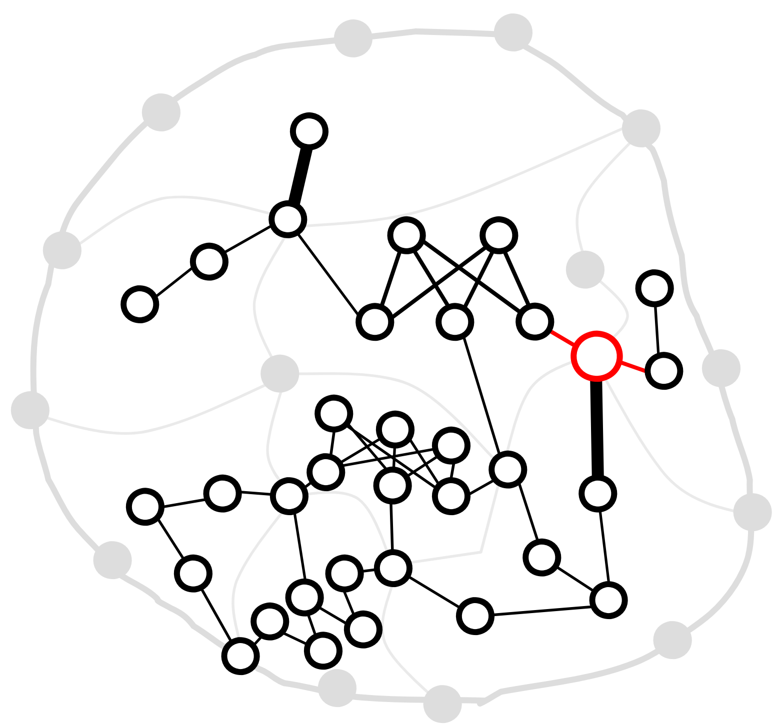
a decision graph G_d of *G*

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇨ G_d has a perfect matching



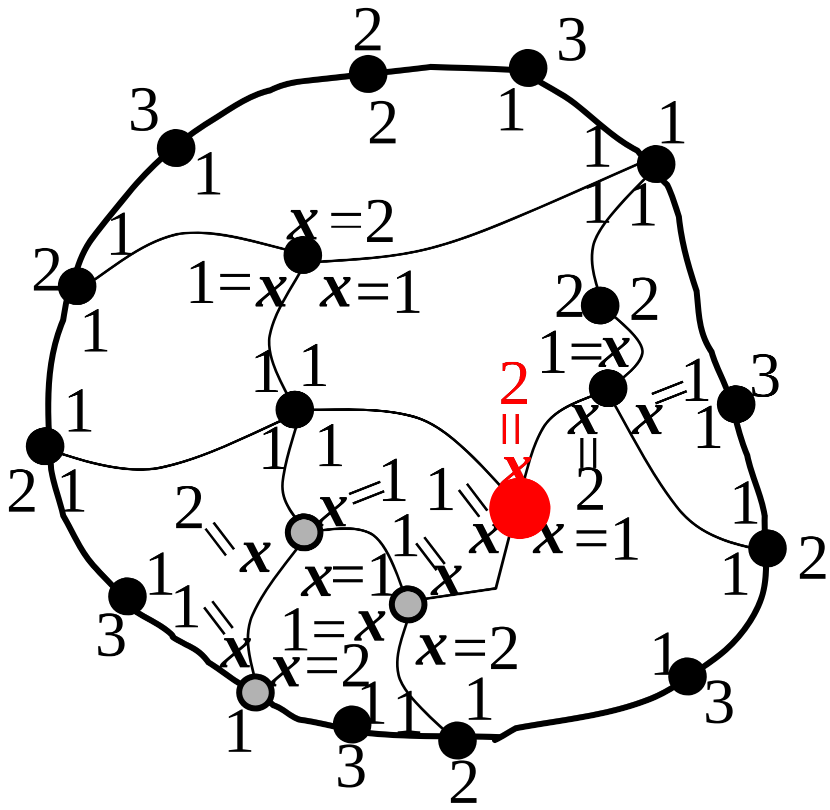
a plane graph G



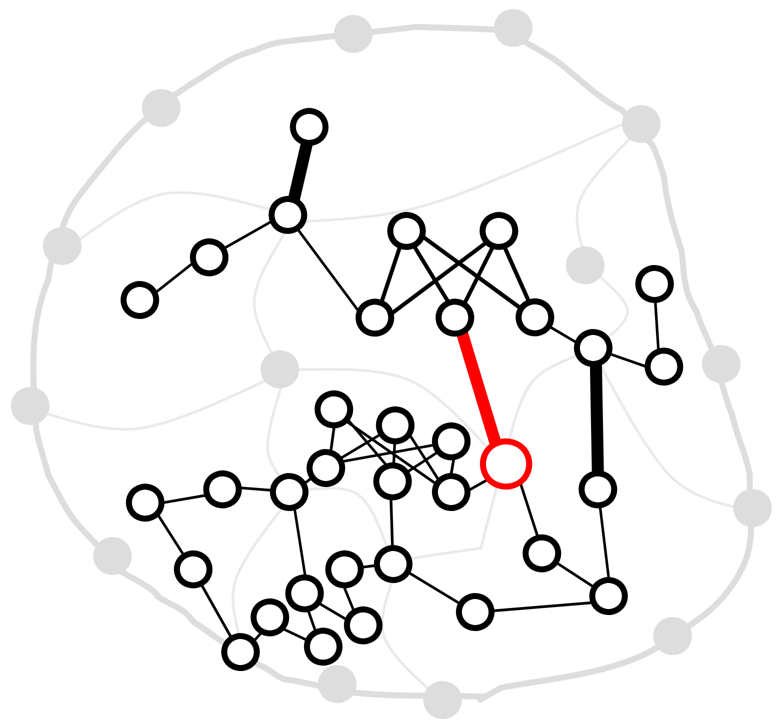
a decision graph G_d of G

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇒ G_d has a perfect matching



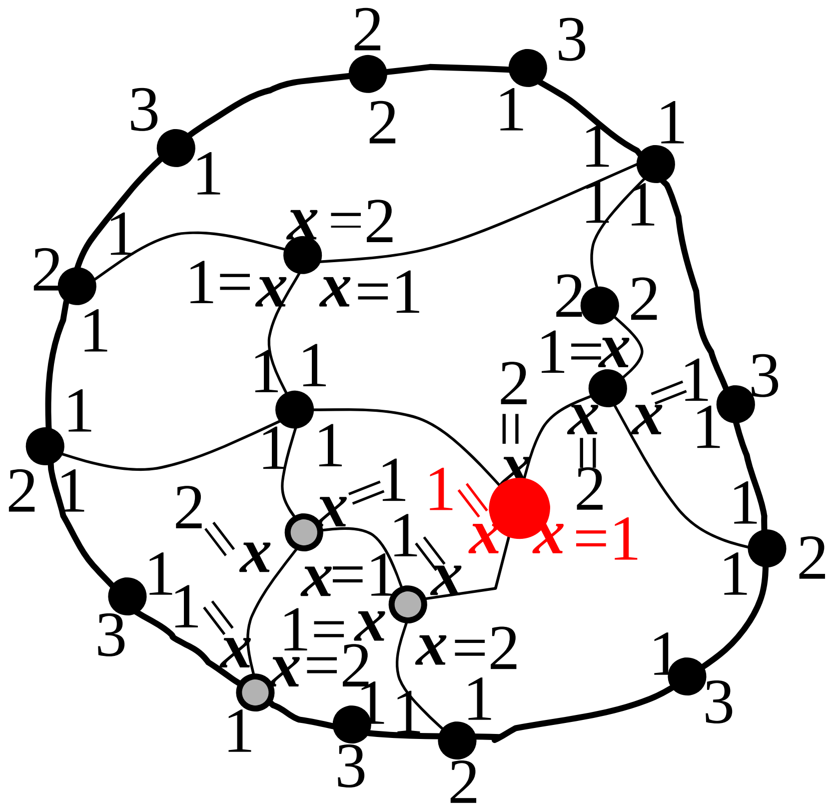
a plane graph *G*



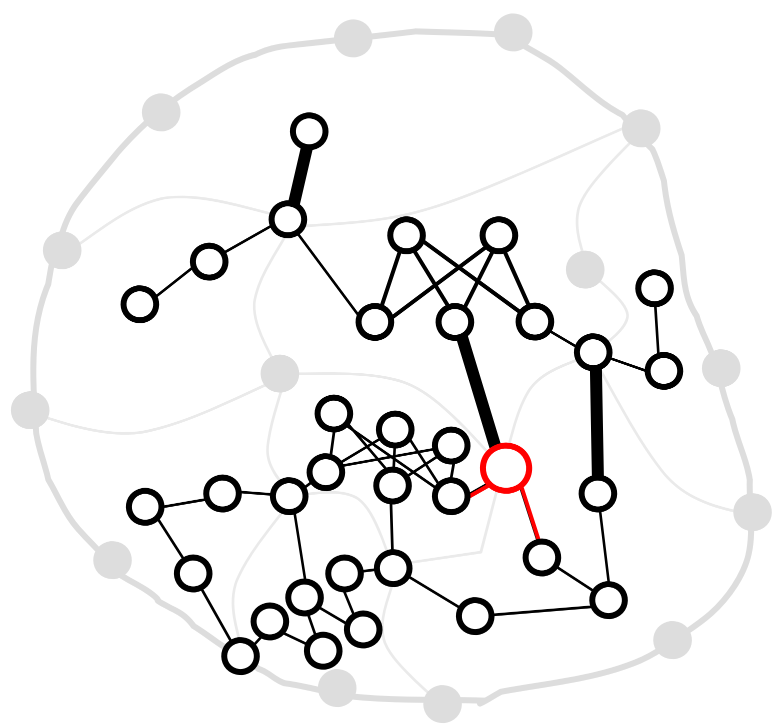
a decision graph G_d of *G*

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇒ G_d has a perfect matching



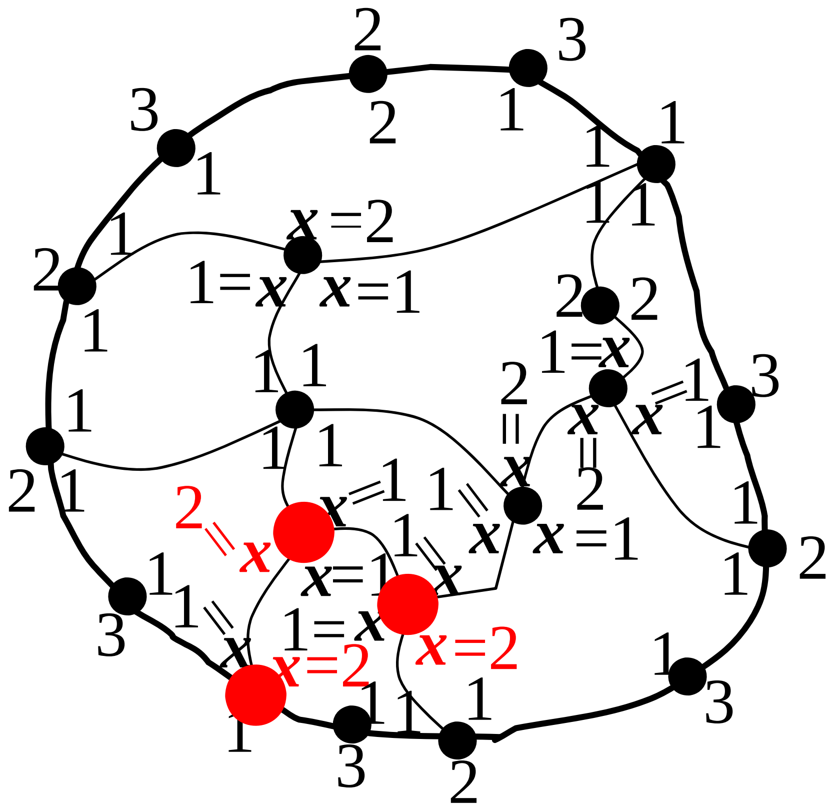
a plane graph G



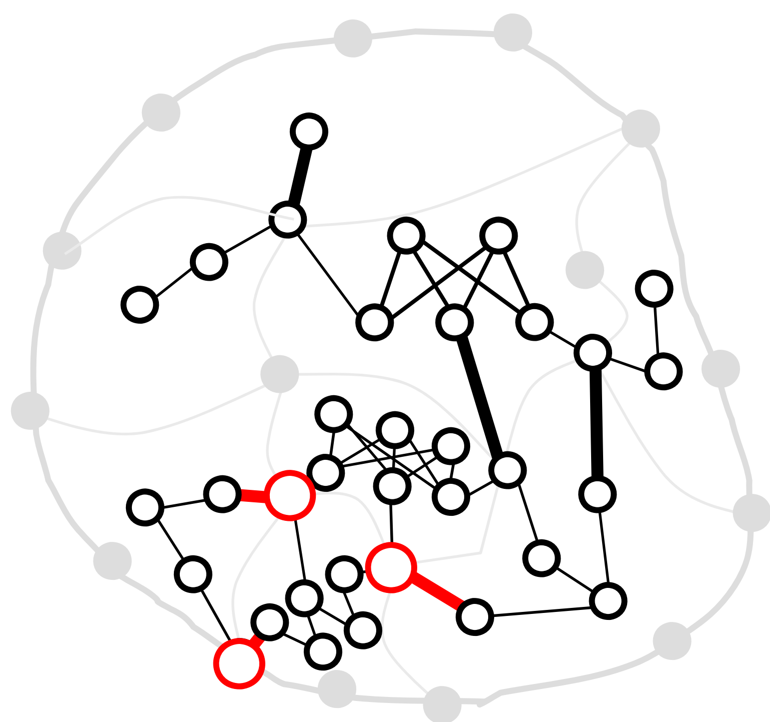
a decision graph G_d of G

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇨ G_d has a perfect matching



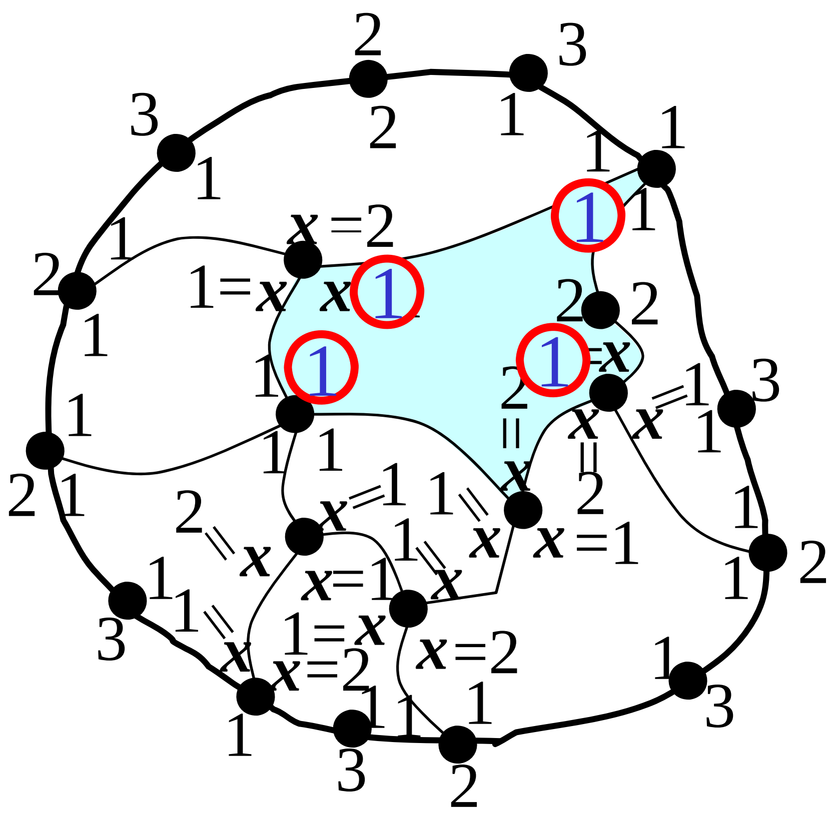
a plane graph G



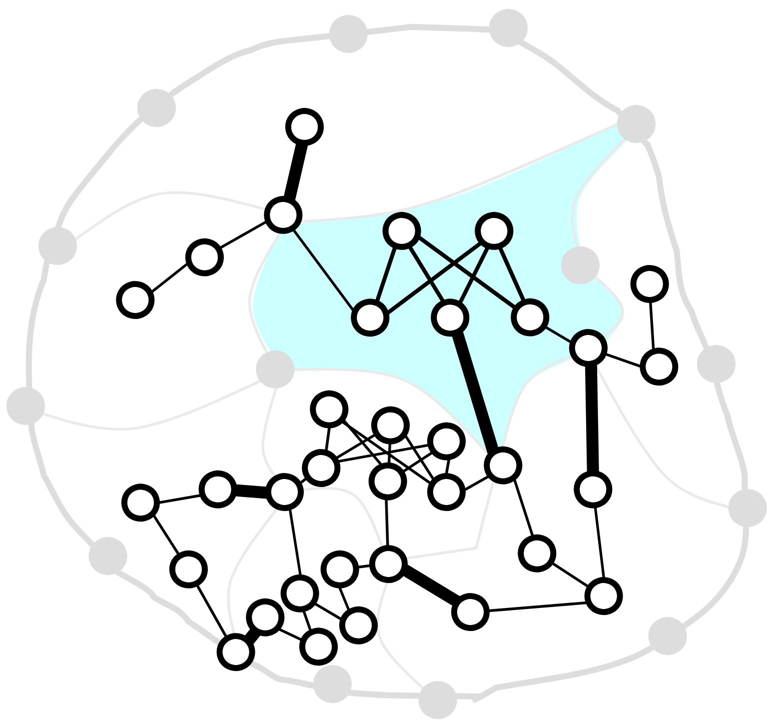
a decision graph G_d of G

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇒ G_d has a perfect matching



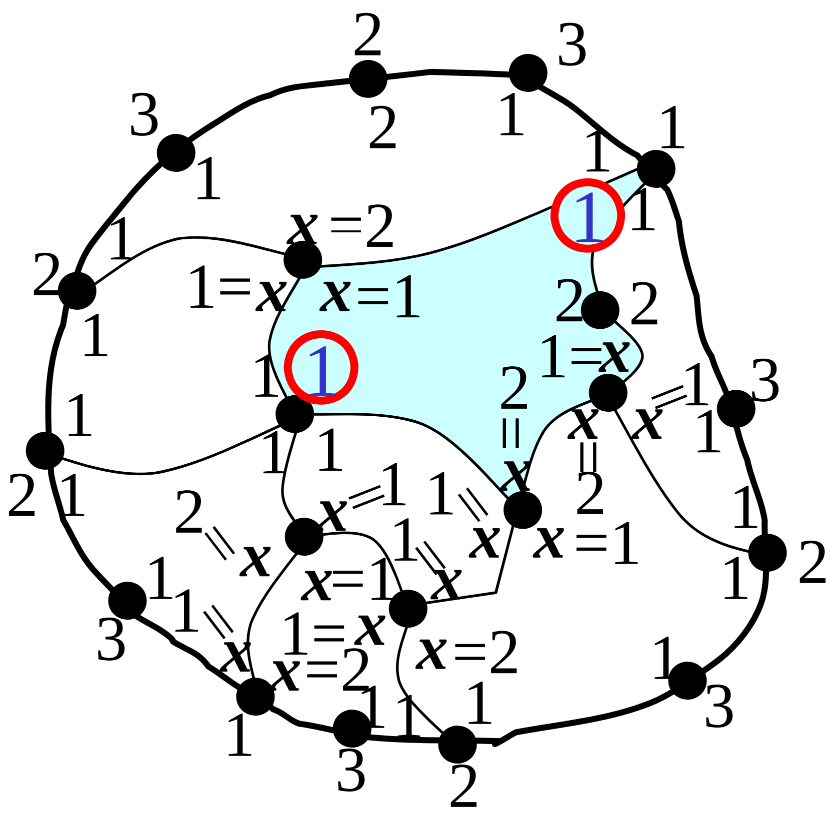
a plane graph G



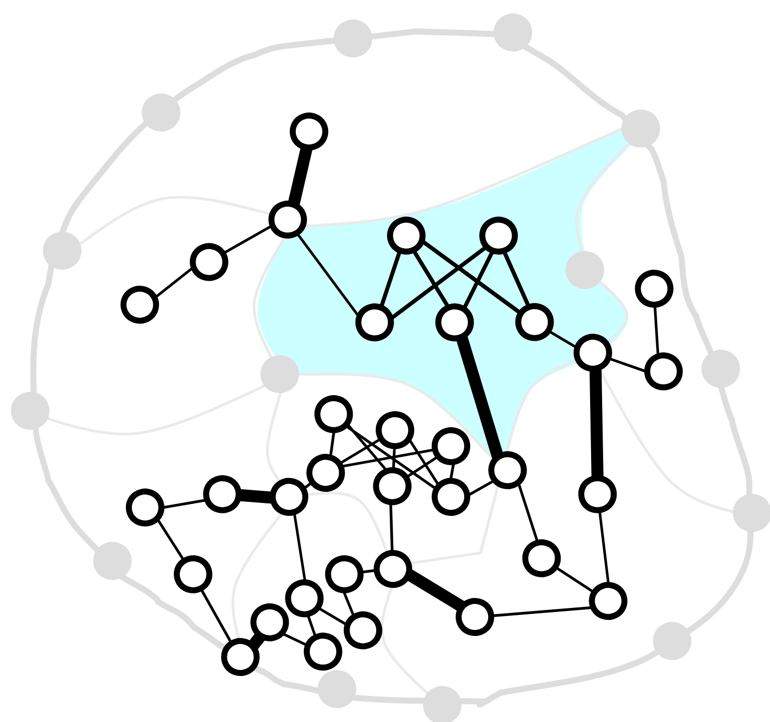
a decision graph G_d of G

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⟹ G_d has a perfect matching



a plane graph G

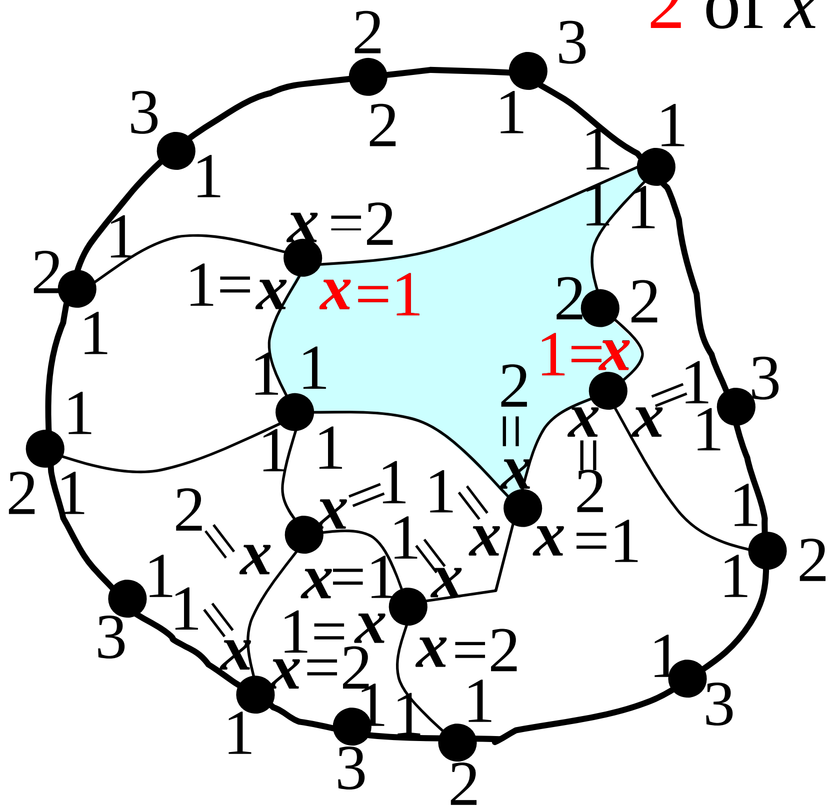


a decision graph G_d of G

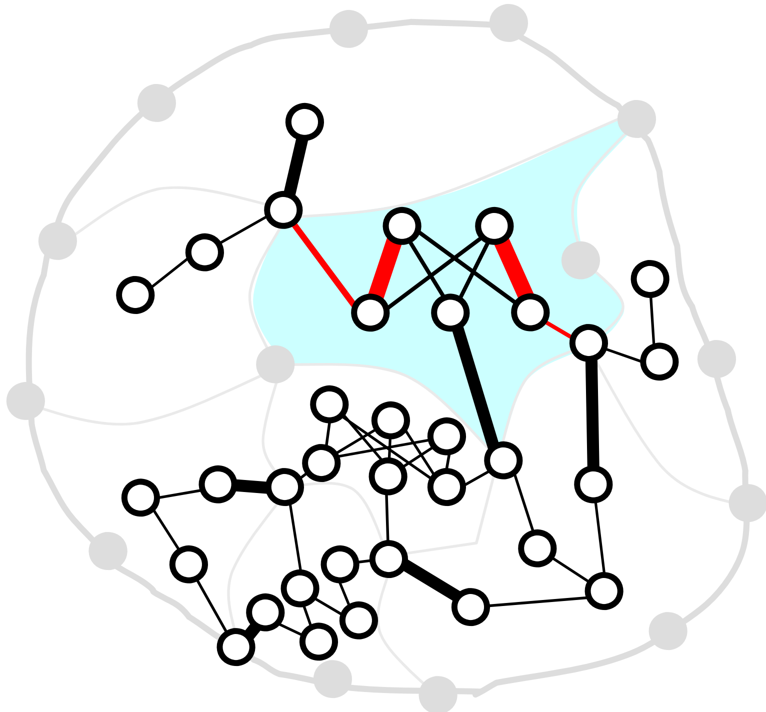
A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇒ G_d has a perfect matching

2 of x's are 1's.



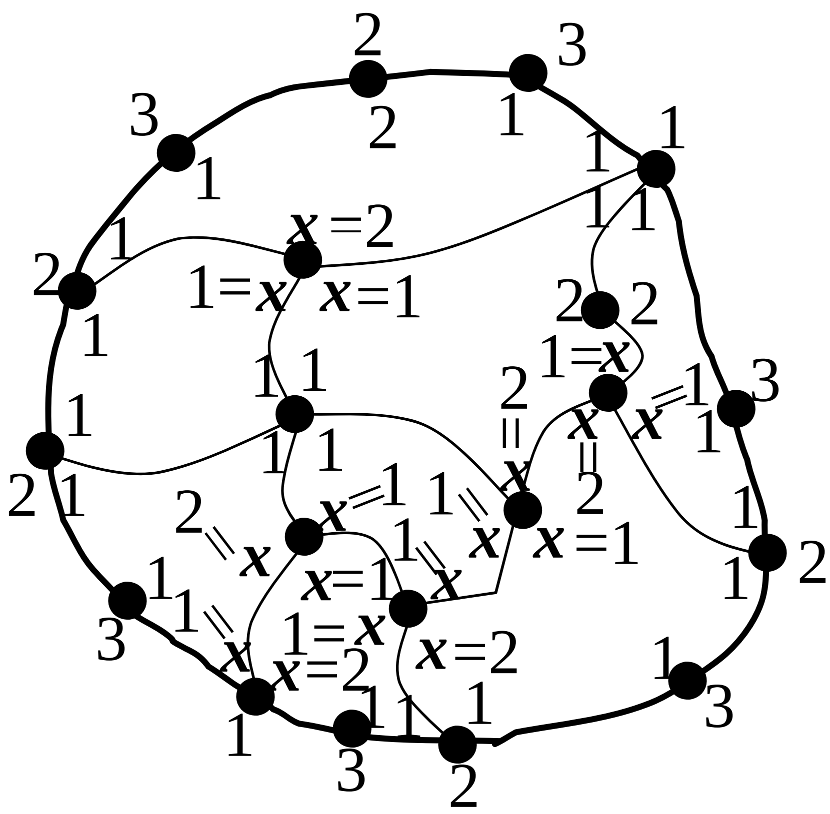
a plane graph G



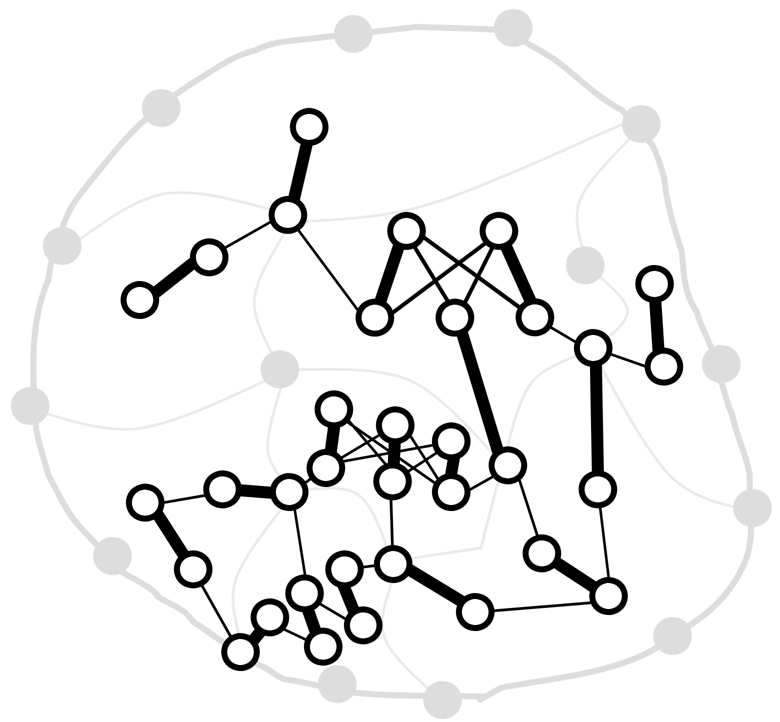
a decision graph G_d of G

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇒ G_d has a perfect matching



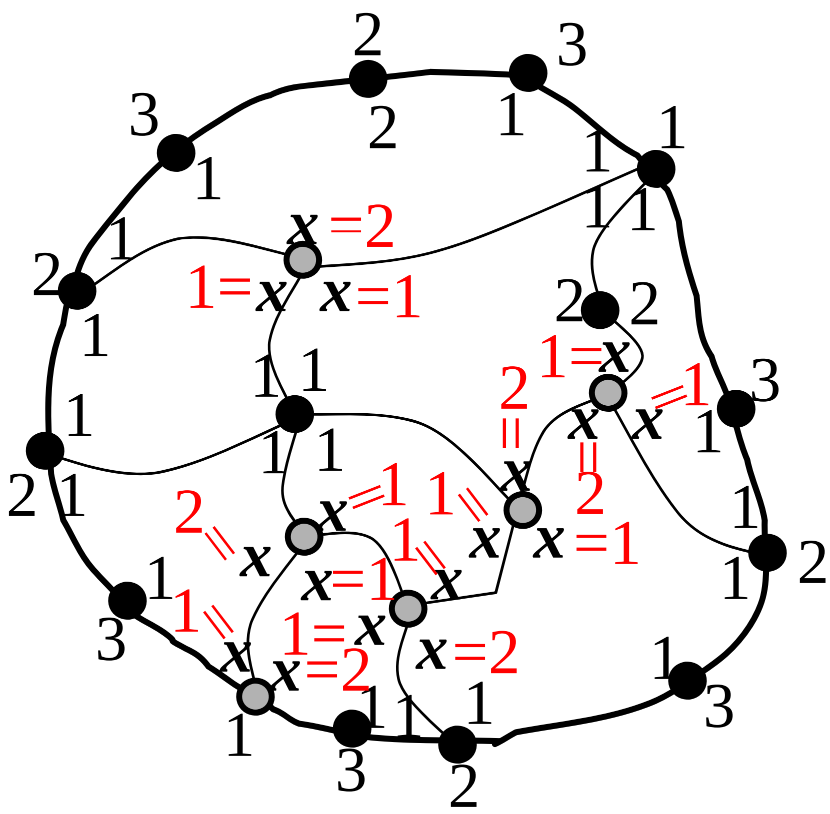
a plane graph *G*



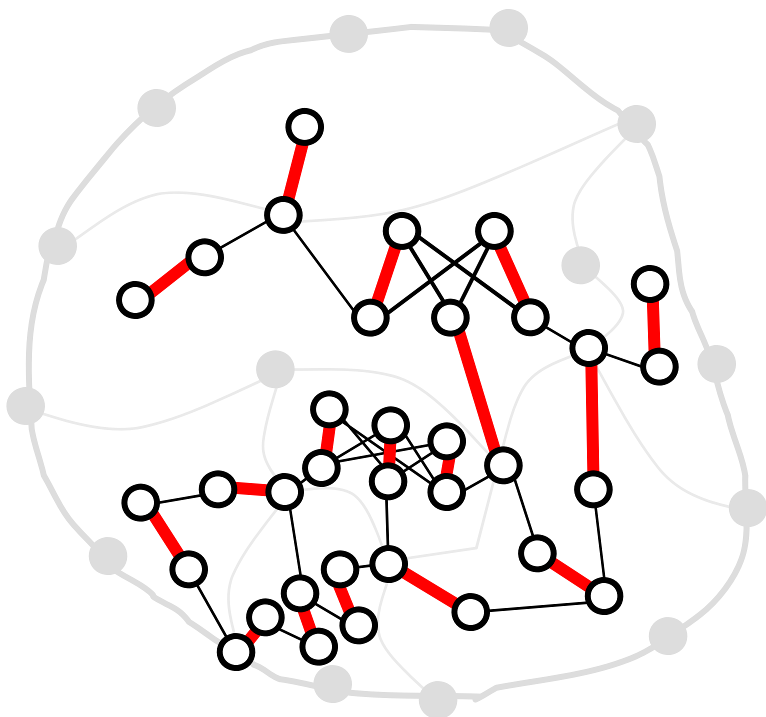
a decision graph G_d of *G*

A necessary and sufficient condition for the existence of a regular labeling

G has a regular labeling ⇐ G_d has a perfect matching



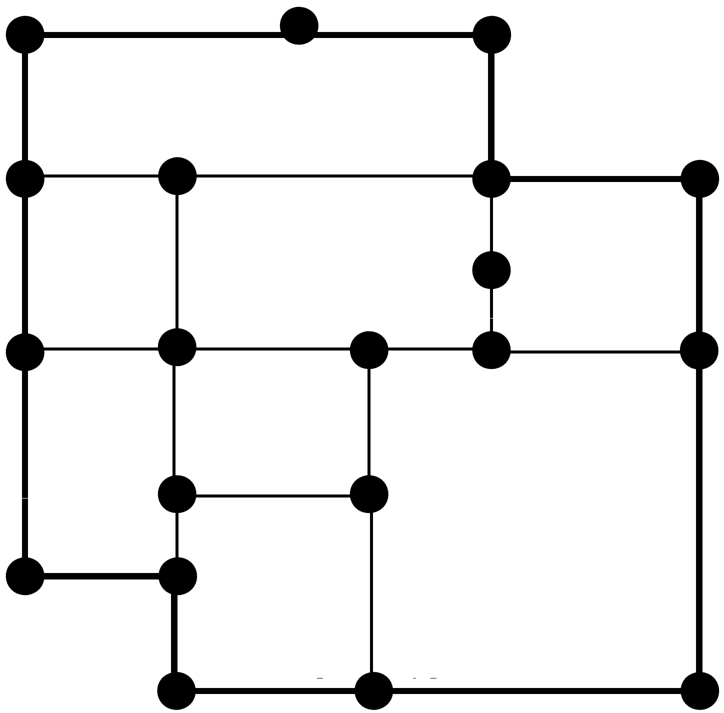
a plane graph *G*



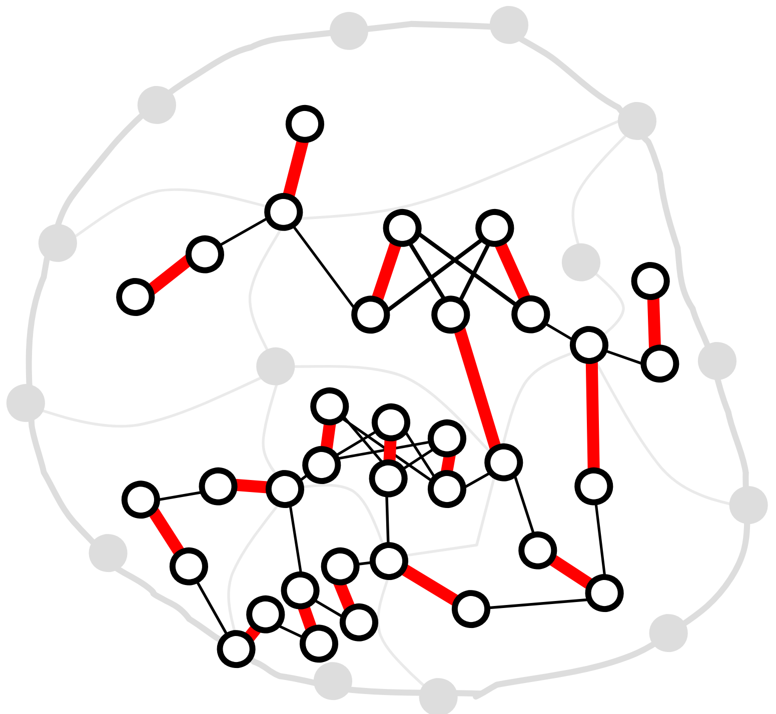
a decision graph G_d of *G*

A necessary and sufficient condition for the existence of an inner rectangular drawing

G has an inner rectangular drawing with sketched outer face ⟺ G_d has a perfect matching



an inner rectangular drawing of *G*



a decision graph G_d of *G*

Running time

$n_d = O(n)$
$m_d = O(n)$

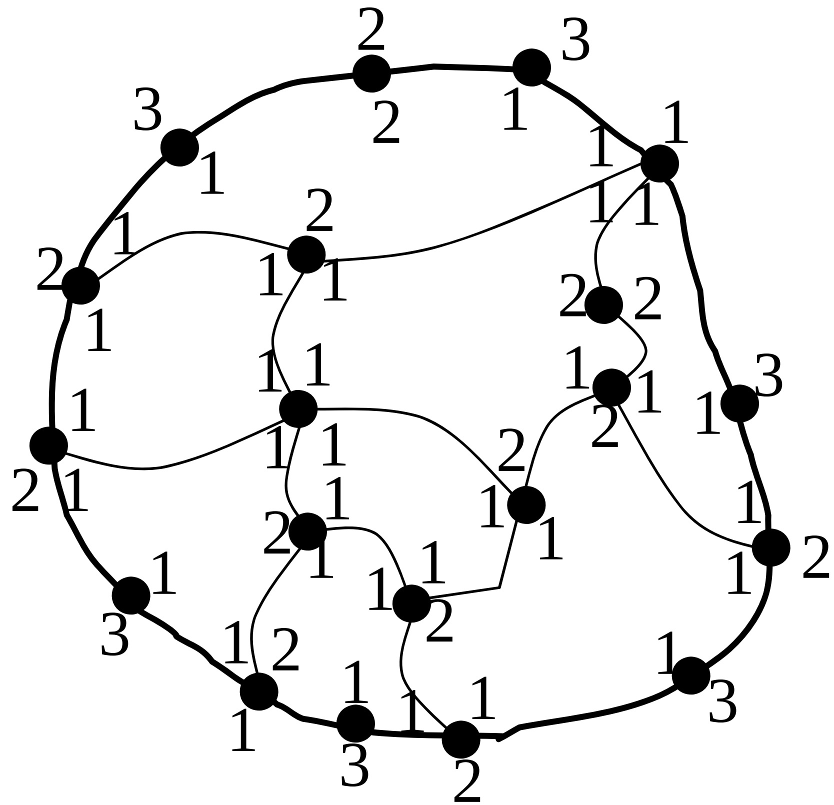
A perfect matching of G_d can be found in time $O(\sqrt{n_d}\, m_d)$

[HK73,MV80]

or in time $O(\sqrt{n_d}\, m_d / \log n_d)$

[FM91,Hoc04,HC04]

A perfect matching of G_d can be found in time $O(n^{1.5}/\log n)$



a regular labeling of G

Running time

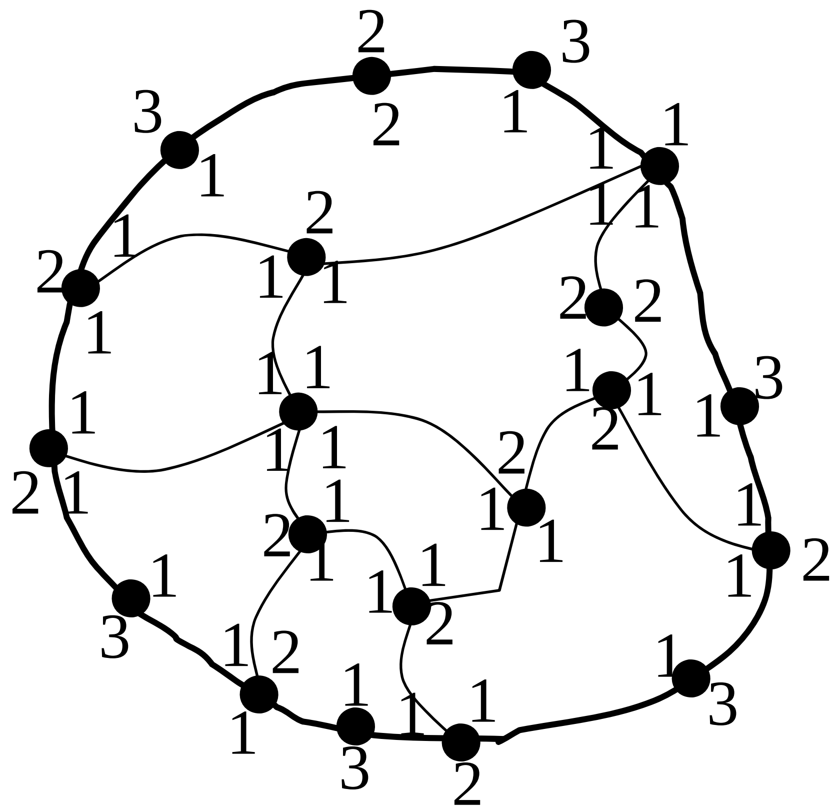
$n_d = O(n)$
$m_d = O(n)$

A perfect matching of G_d can be found in time $O(\sqrt{n_d m_d})$

[HK73,MV80]

or in time $O(\sqrt{n_d m_d}/\log n_d)$

[FM91,Hoc04,HC04]



a regular labeling of *G*

Running time

$n_d = O(n)$
$m_d = O(n)$

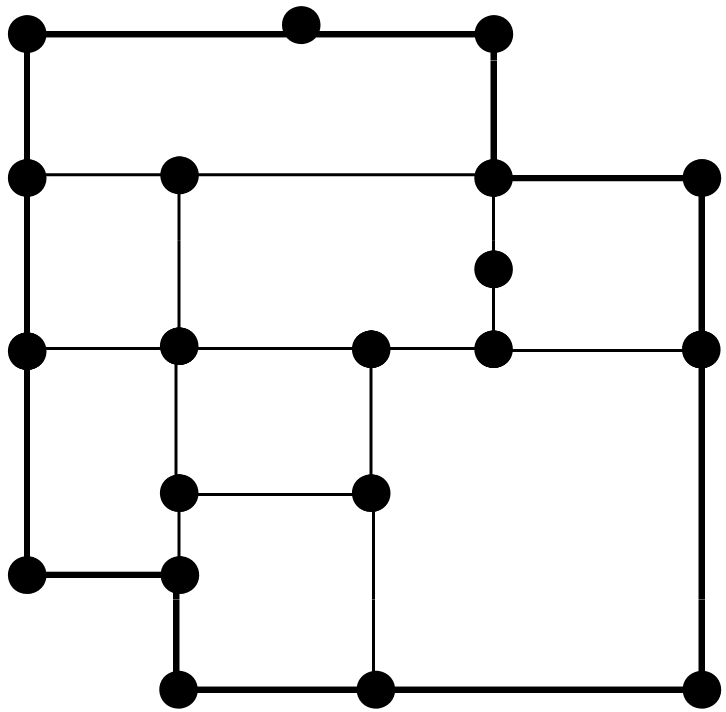
A perfect matching of G_d can be found in time $O(\sqrt{n_d m_d})$

[HK73,MV80]

or in time $O(\sqrt{n_d m_d} / \log n_d)$

[FM91,Hoc04,HC04]

An inner rectangular drawing of G can be found in time $O(n^{1.5}/\log n)$



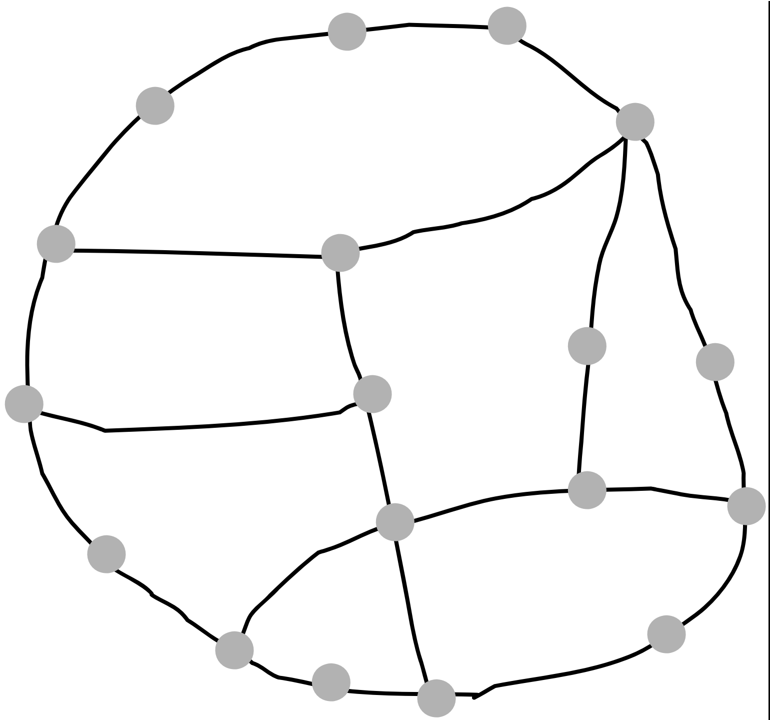
an inner rectangular drawing of G

1:A necessary and sufficient condition for the existence of an inner rectangular drawing of *G*.

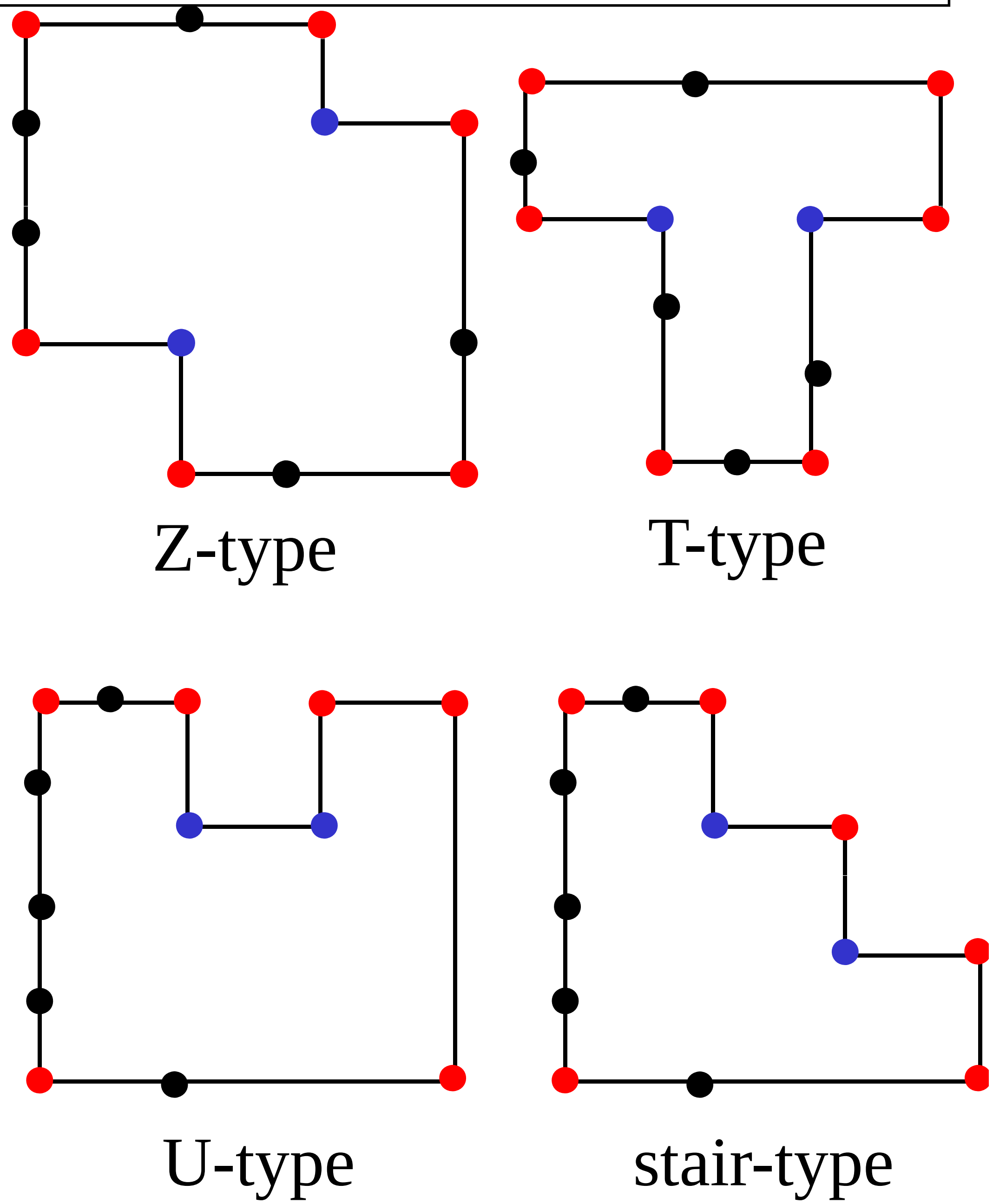
2: $O(n^{1.5}/\log n)$ time algorithm to find an inner rectangular drawing of *G* if a sketch of the outer face is given.

3: a polynomial time algorithm to find an inner rectangular drawing of *G* in a general case, where a sketch is not always given.

Case 1: the numbers of convex and concave outer vertices are given.



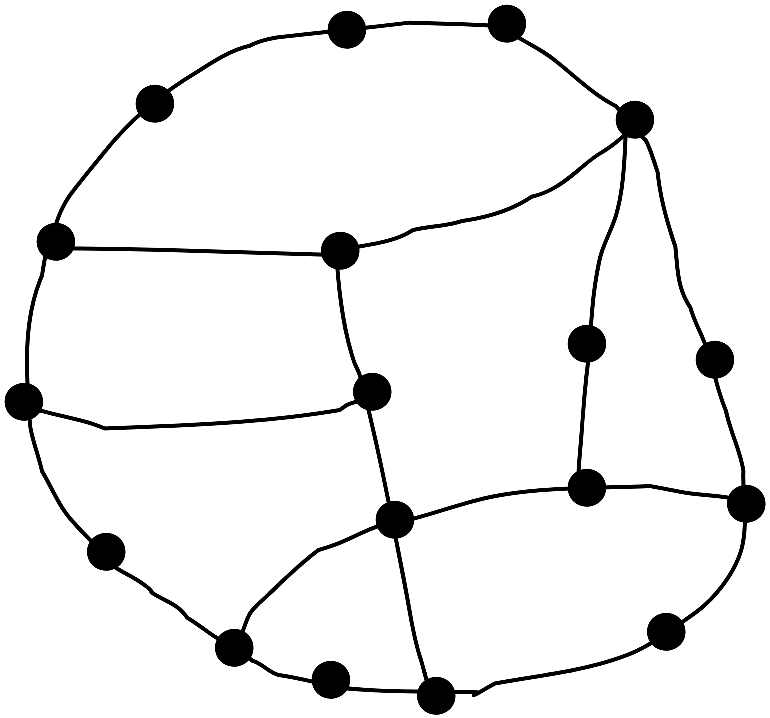
$n_{cv} = 6$,
$n_{cc} = 2$



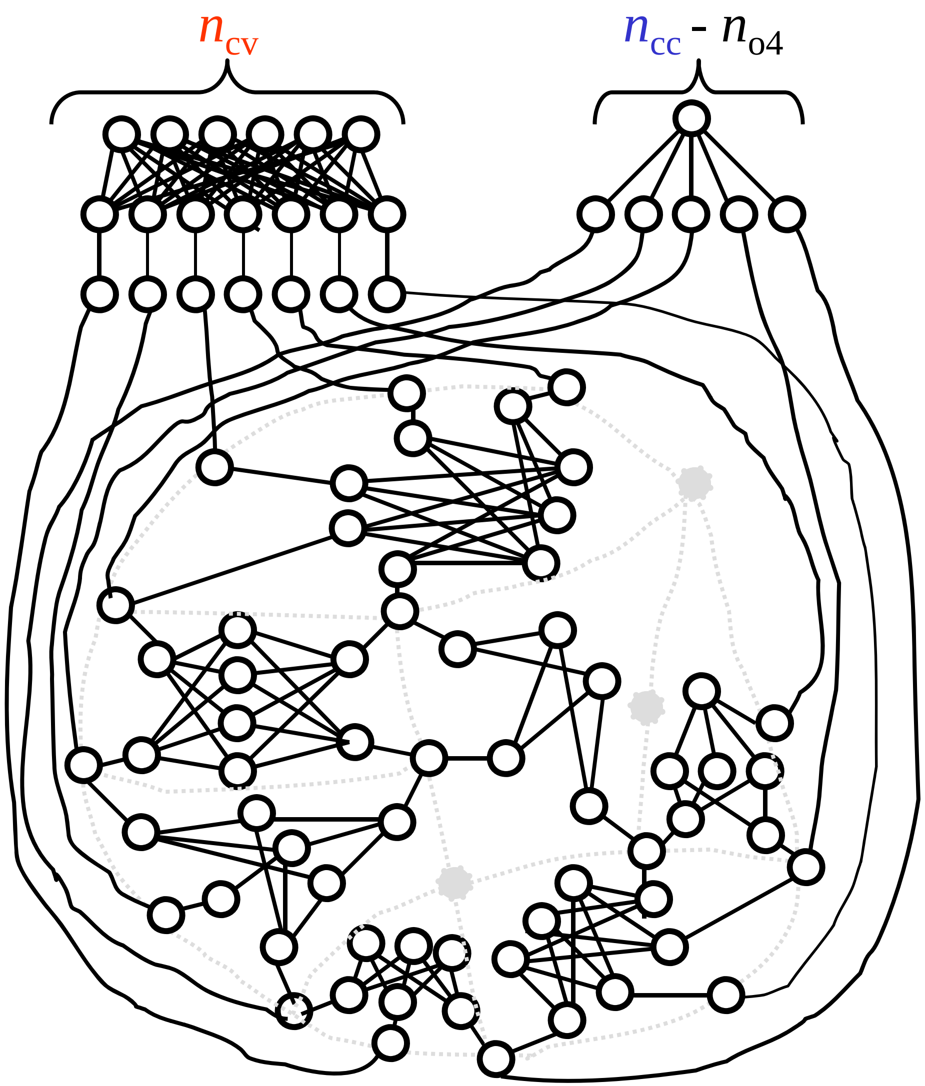
Case 1: the numbers of convex and concave outer vertices are given.

$n_{\mathrm{cv}} = 6$,

$n_{\mathrm{cc}} = 2$



a plane graph G

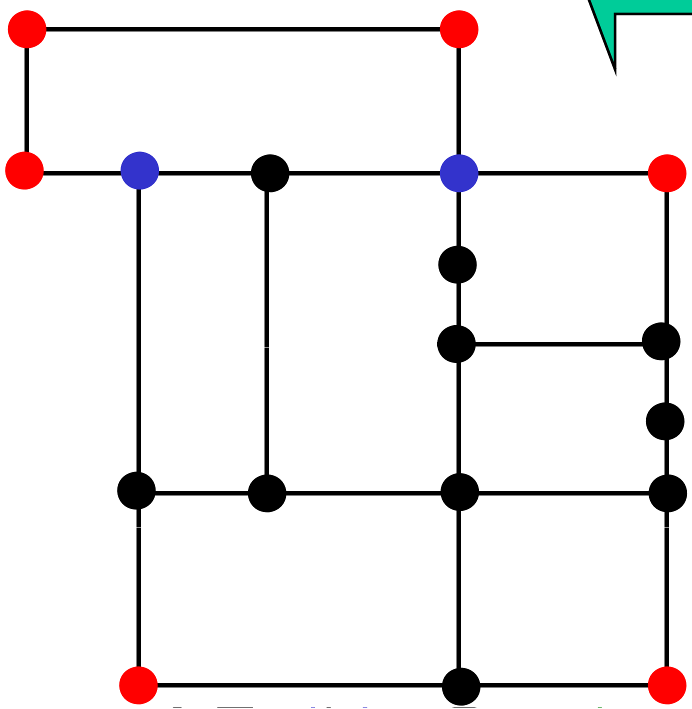


a decision graph G_{d} of G

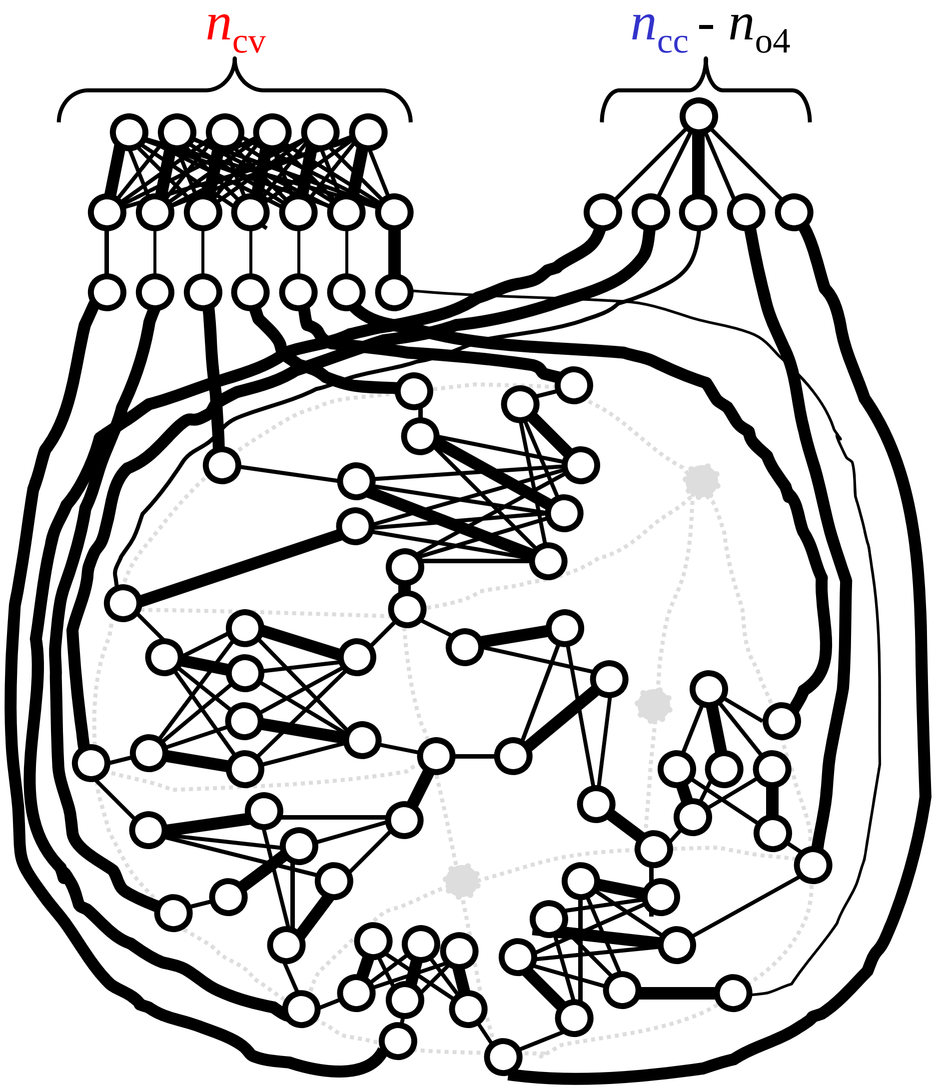
Case 1: the numbers of convex and concave outer vertices are given.

$n_{cv} = 6,$

$n_{cc} = 2$



an inner rectangular drawing of *G*



a decision graph G_d of *G*

Case 1: the numbers of convex and concave outer vertices are given.

Running time

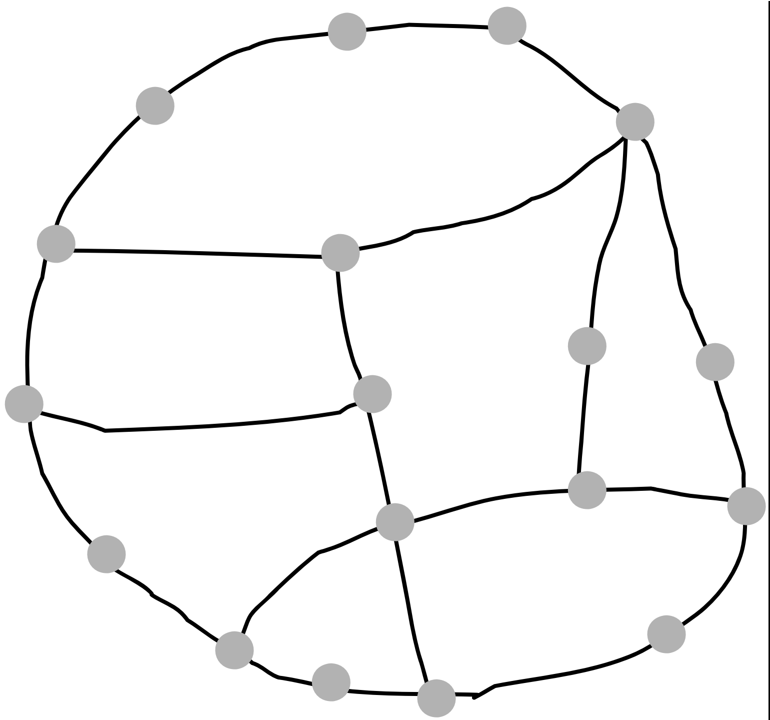
$n_d = O(n)$

$m_d = O(N)$, $N = n + n_{\mathrm{cv}} n_o$ (n_o: the number of outer vertices).

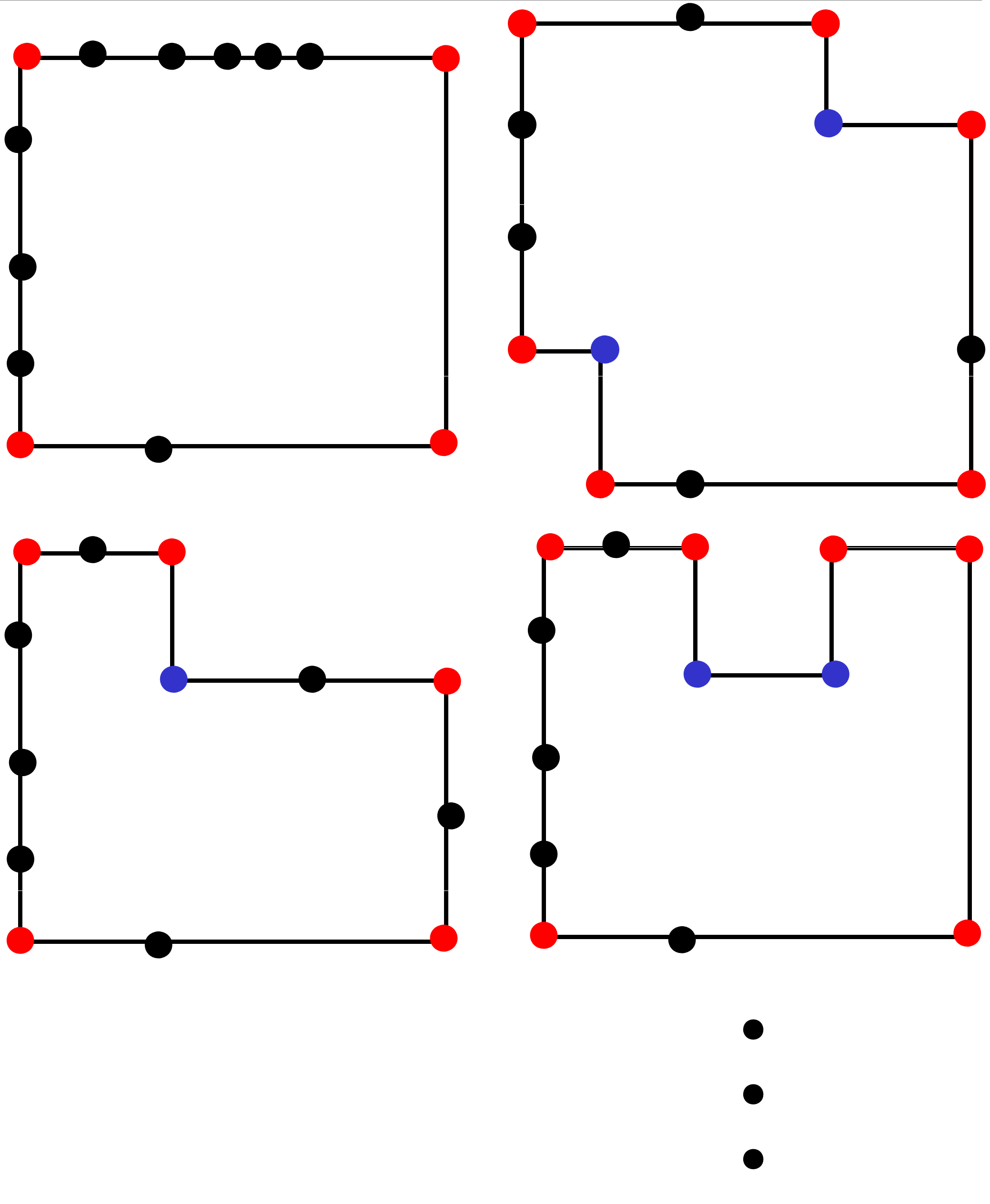
An inner rectangular drawing of G can be found in time

$O(\sqrt{n} N / \log n)$.

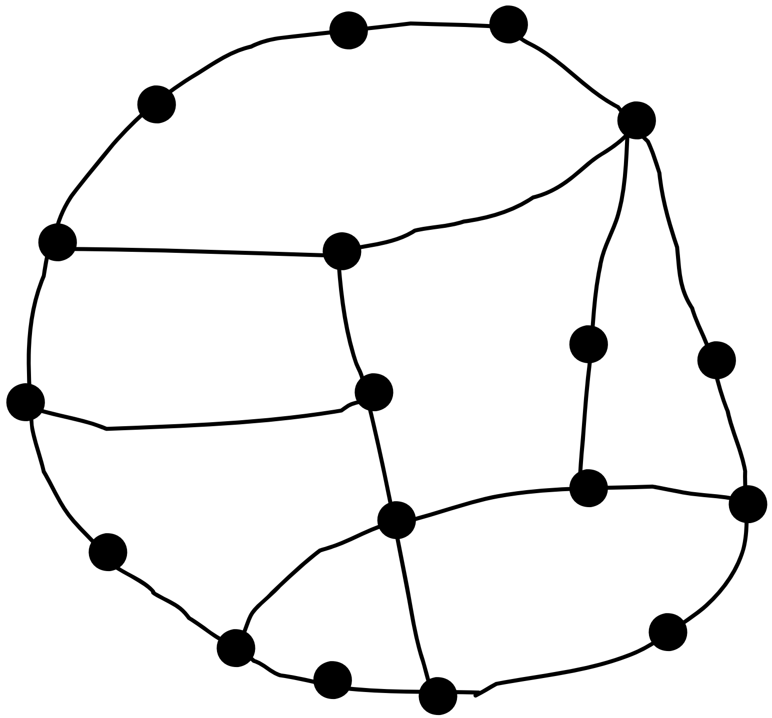
Case 2: neither the outer sketch nor the numbers of corners are given.



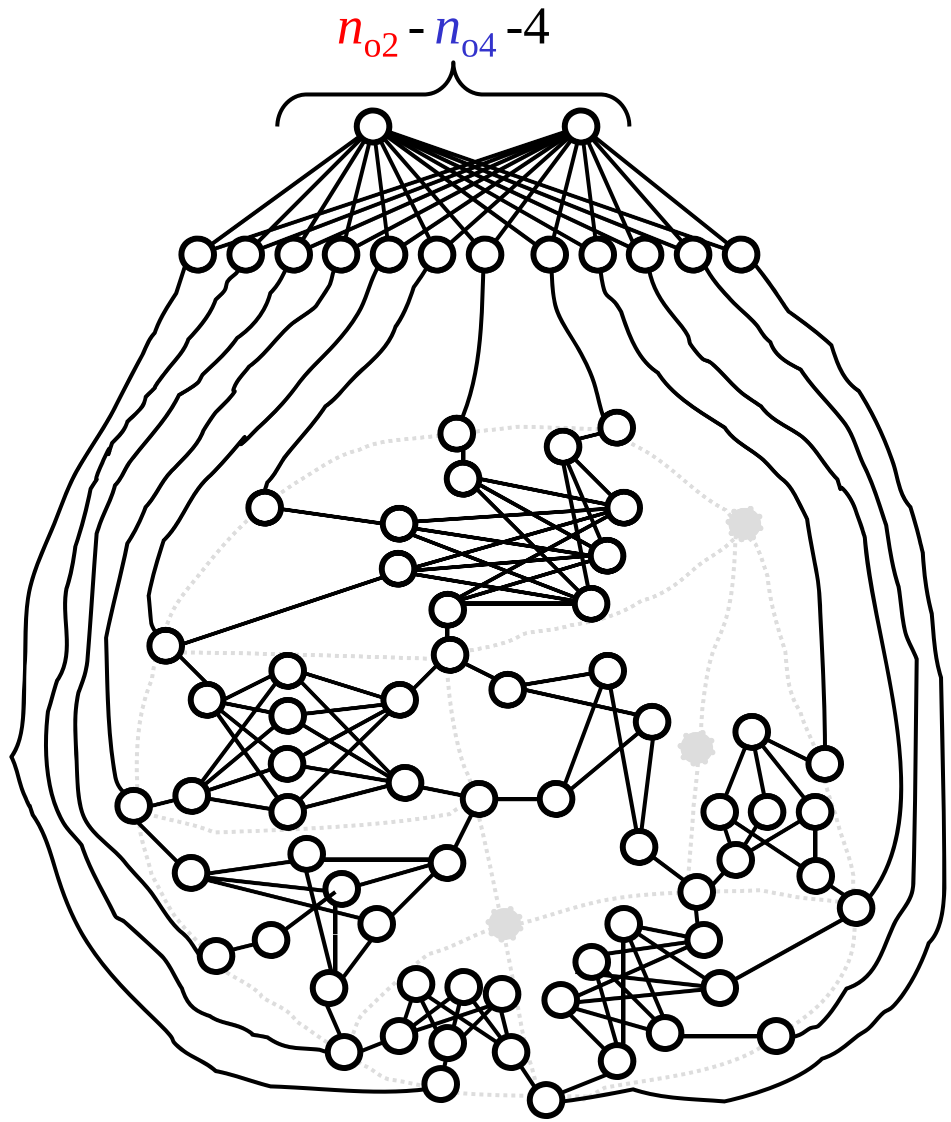
n_{cv} , n_{cc} are arbitrary



Case 2: neither the outer sketch nor the numbers of corners are given.

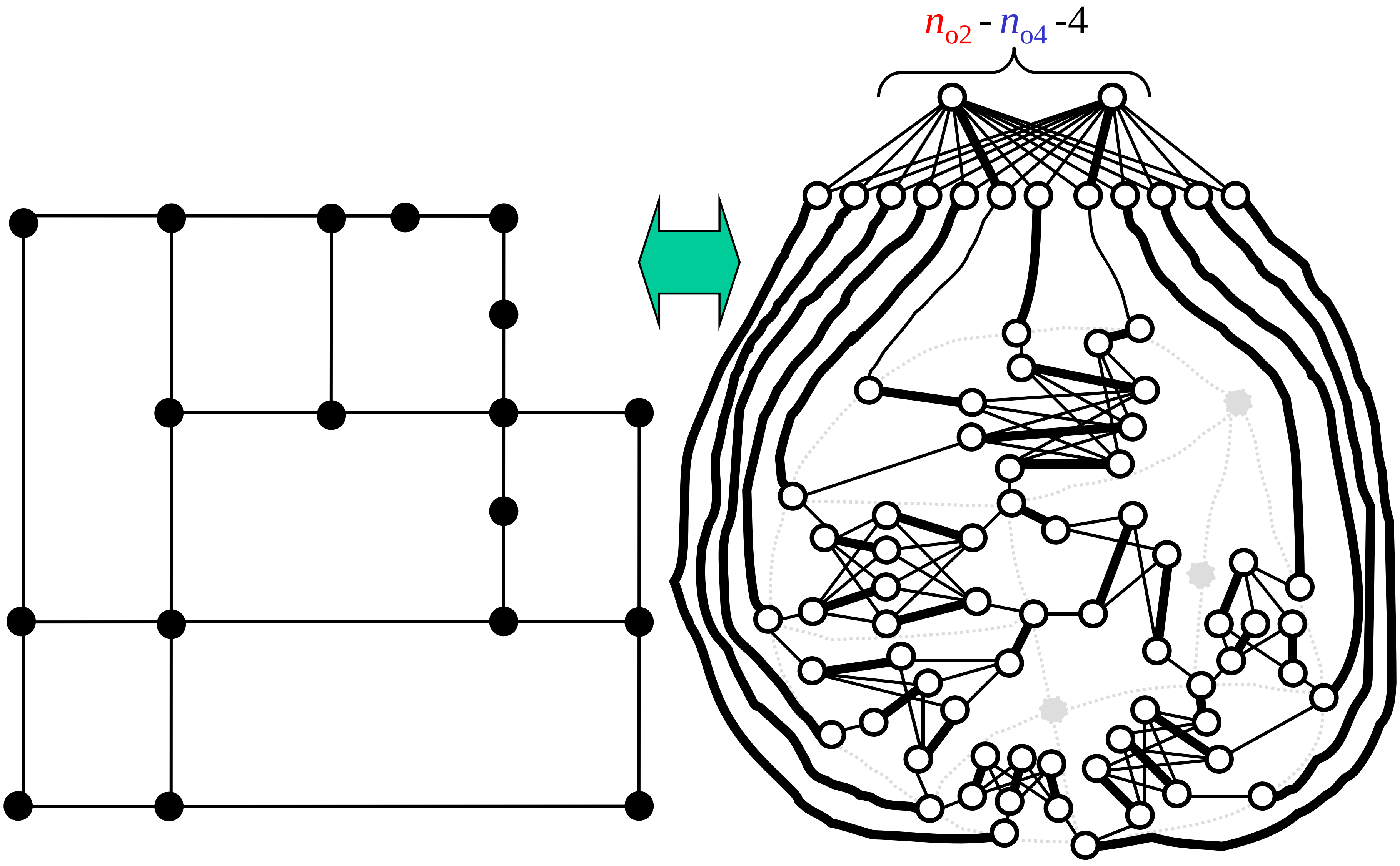


a plane graph G



a decision graph G_d of G

Case 2: neither the outer sketch nor the numbers of corners are given.



an inner rectangular drawing of G

a decision graph G_d of G

Case 2: neither the outer sketch nor the numbers of corners are given.

Running time

$n_d = O(n)$

$m_d = O(N')$, $N' = n + (n_{o2} - n_{o4} - 4)n_o$

(n_o: the number of outer vertices,

n_{o2} and n_{o4}: the numbers of outer vertices of degrees 2 and 4)

An inner rectangular drawing of *G* can be found in time

$O(\sqrt{n}N' / \log n)$.

Conclusion

(1) G has an inner rectangular drawing ⟺ G_d has a perfect matching

(2) An inner rectangular drawing can be found in time

- $O(n^{1.5} / \log n)$ if the outer face is sketched.
- $O(\sqrt{n}N / \log n)$ if (n_{cv}, n_{cc}) is prescribed.

  $N = n + n_{cv} n_o$ n_o: the number of outer vertices

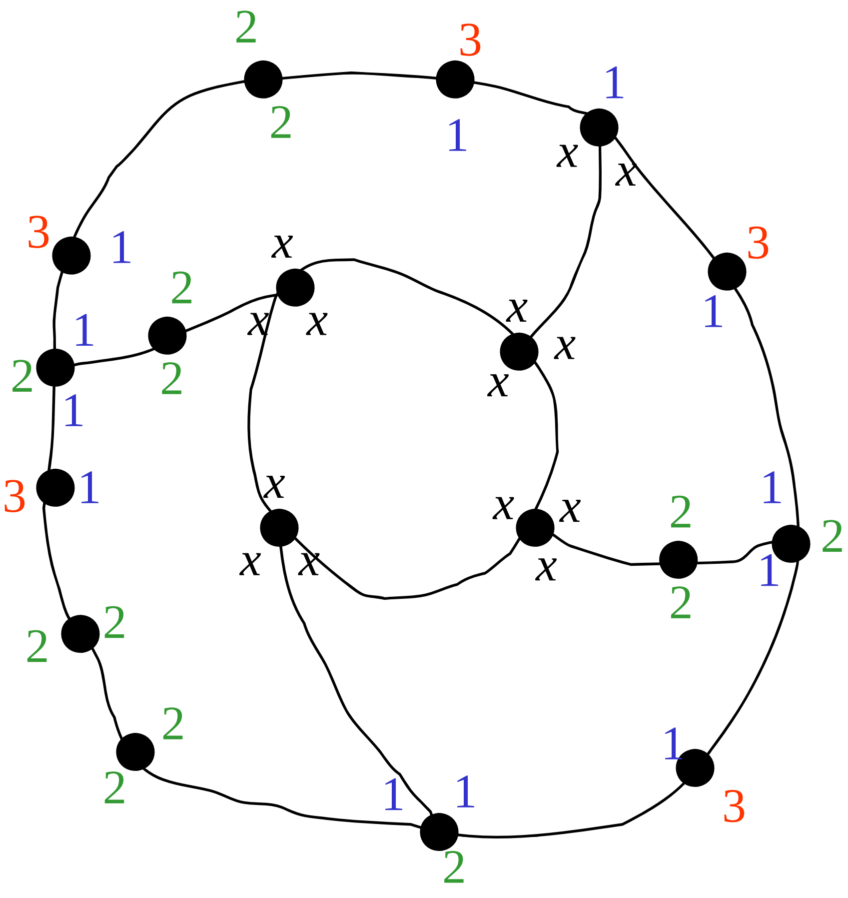
- $O(\sqrt{n}N' / \log n)$ for a general case.

  $N' = n + (n_{o2} - n_{o4} - 4) n_o$

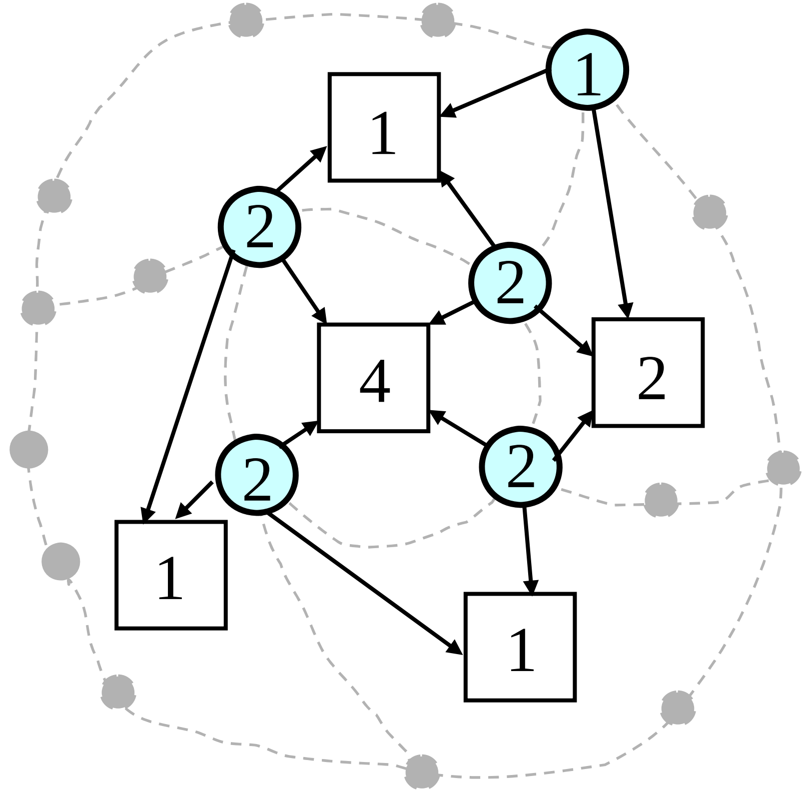
  n_{o2} and n_{o4}: the numbers of outer vertices of degrees 2 and 4

(3) Linear algorithm ?

Network Flow

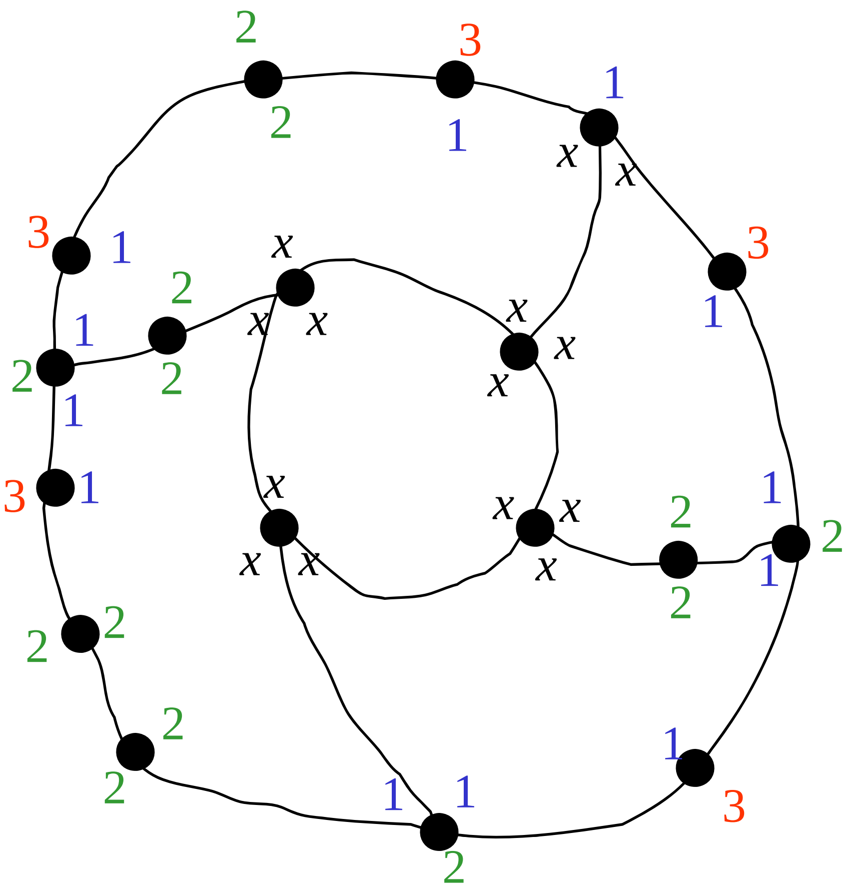


G

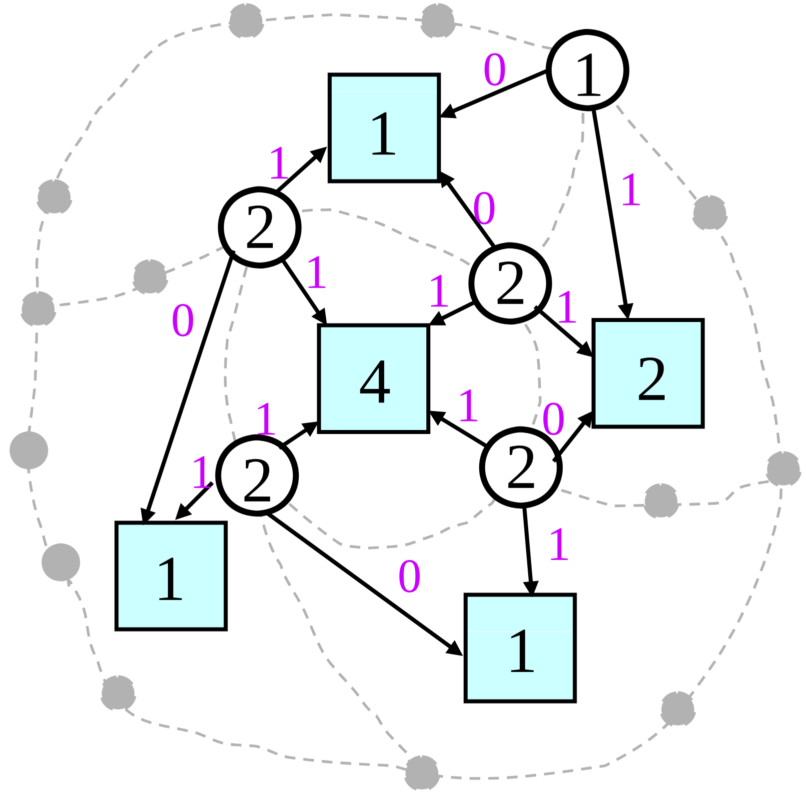


Network *N*

Network Flow

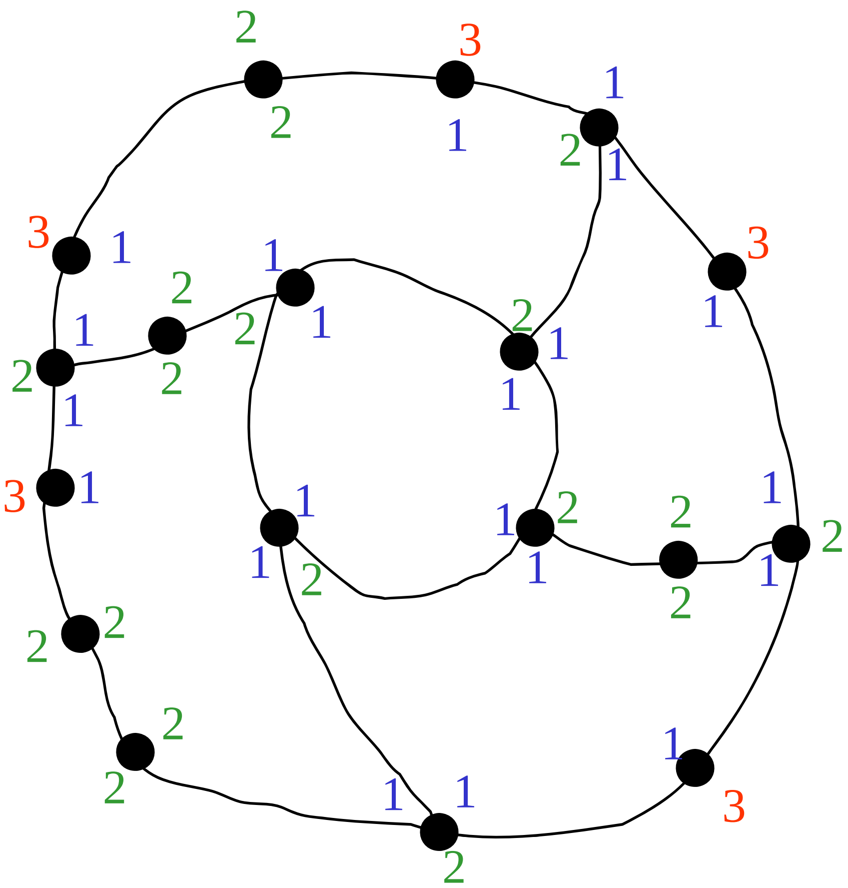


G

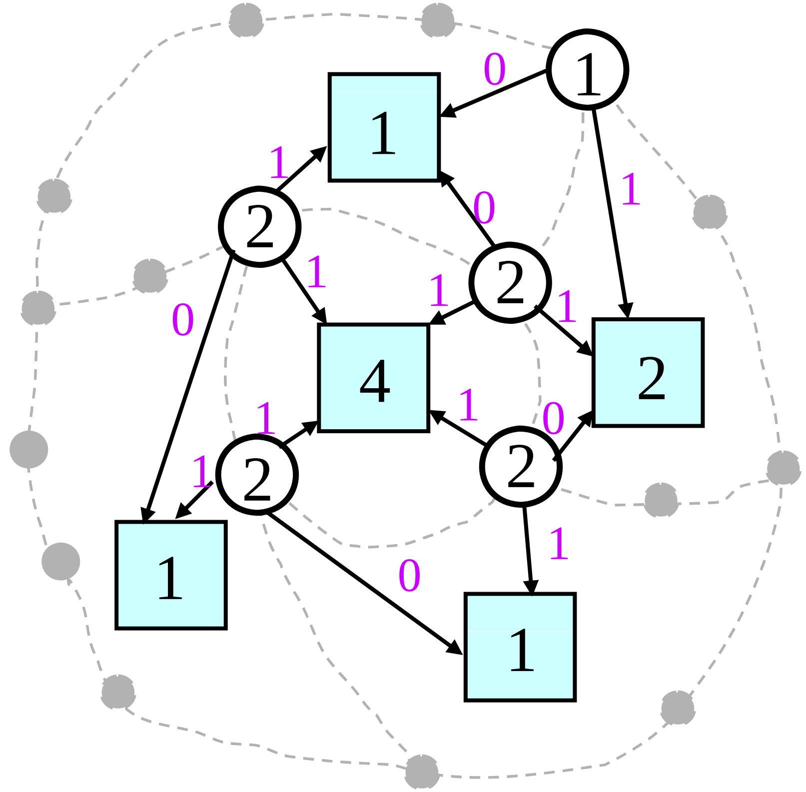


Network *N*

Network Flow

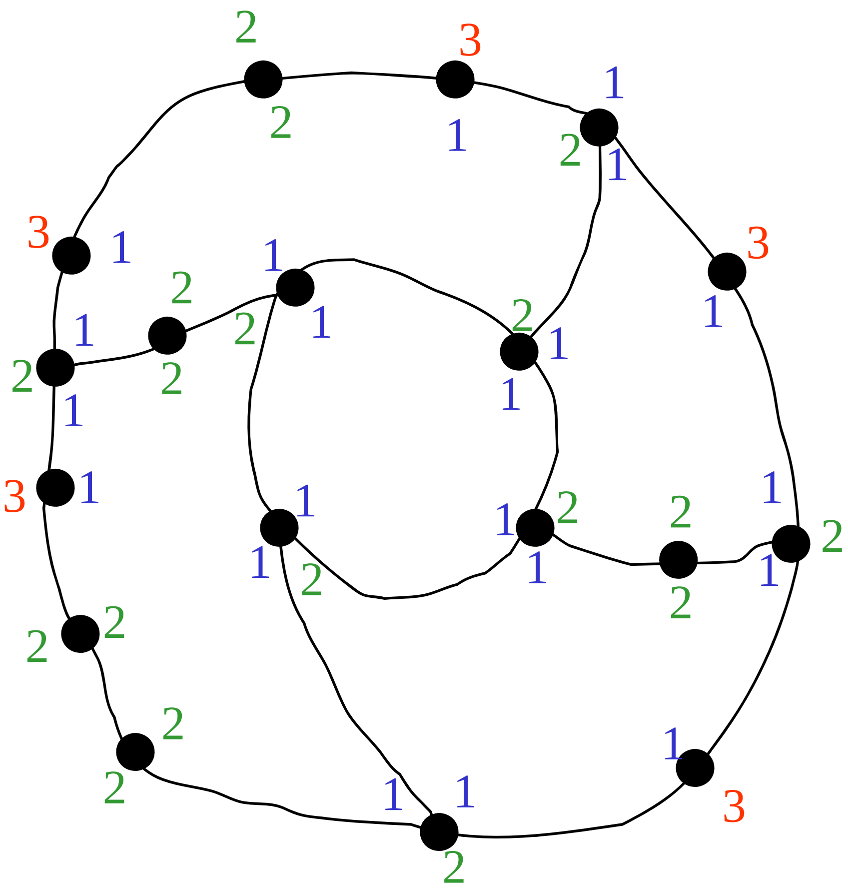


G

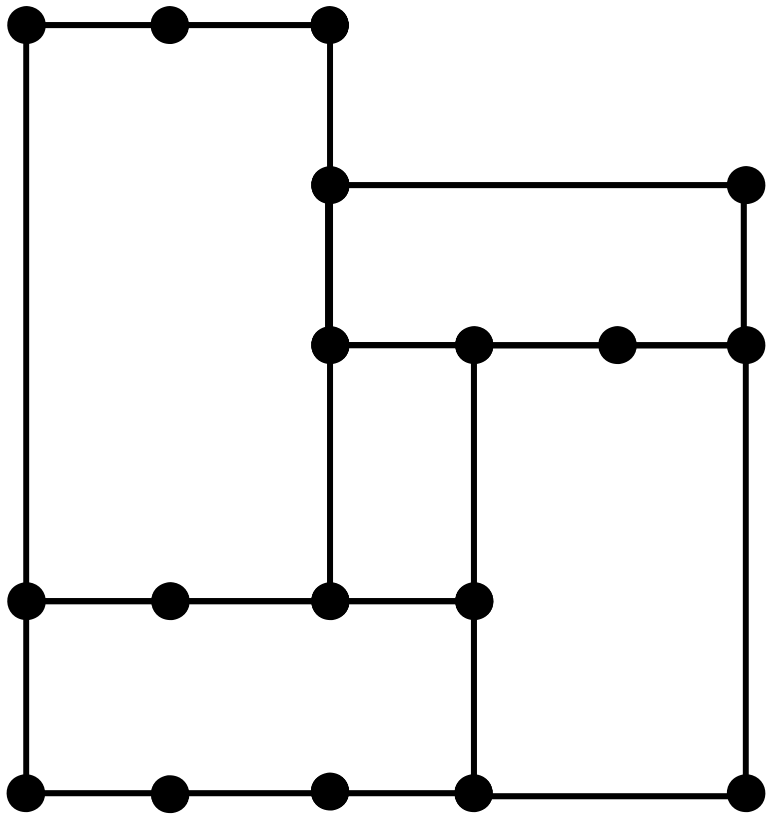


Network *N*

Network Flow

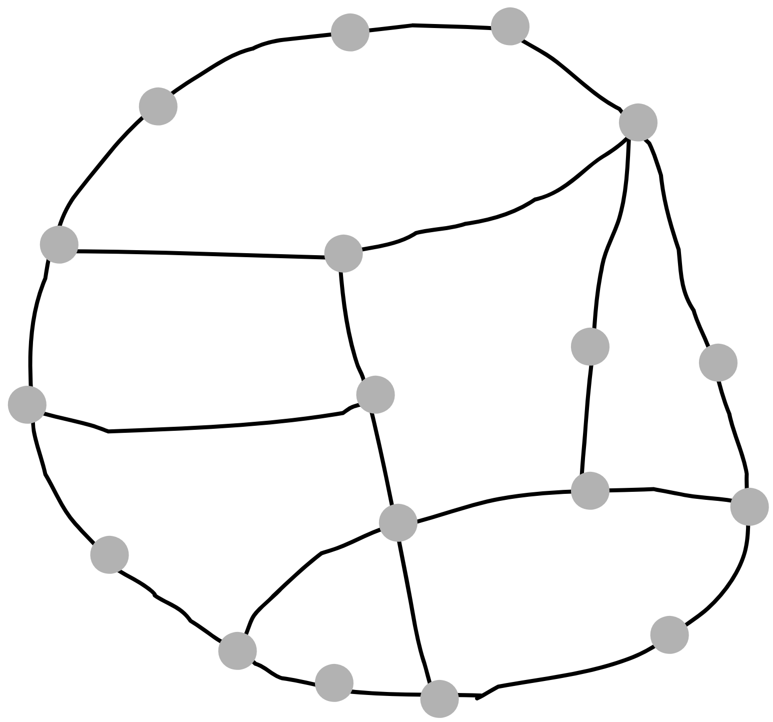


G



An inner rectangular drawing of *G*

Case 1: the numbers of convex and concave outer vertices are given.

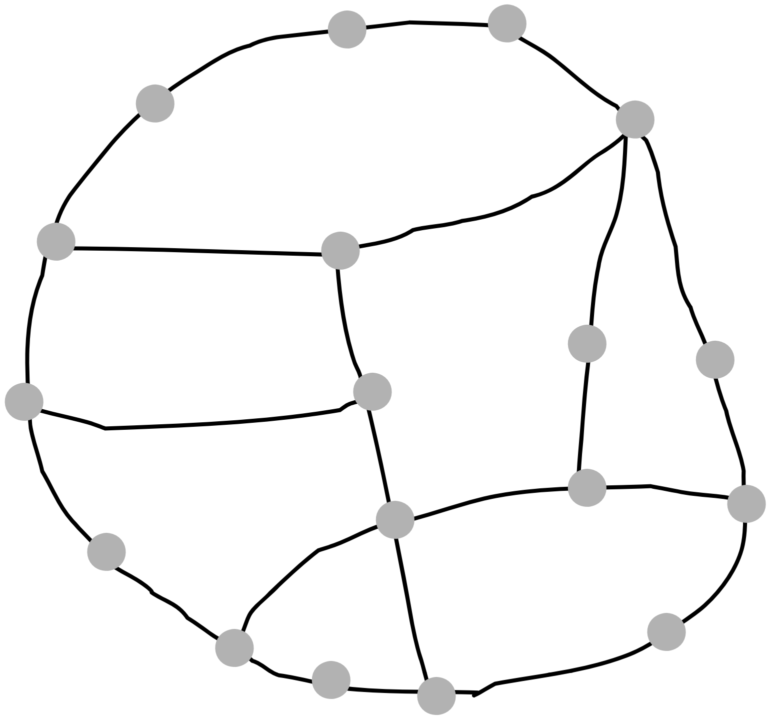


$n_{cv} = 6$, $n_{cc} = 2$

Case 2: general case

Inner rectangular drawing with prescribed numbers n_{cv} and n_{cc}

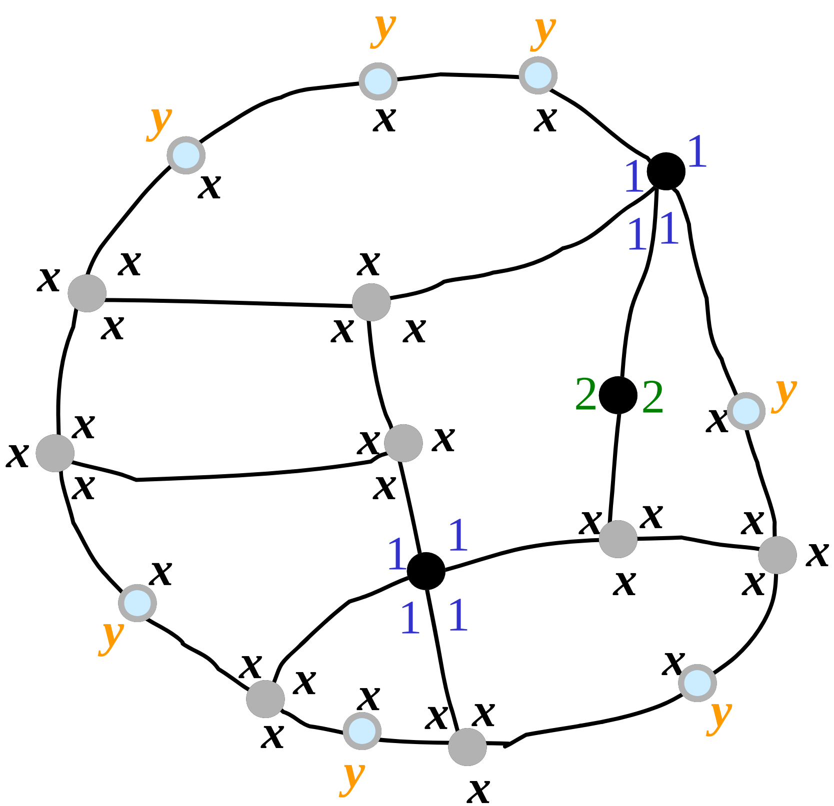
$n_{cv} = 6$, $n_{cc} = 2$



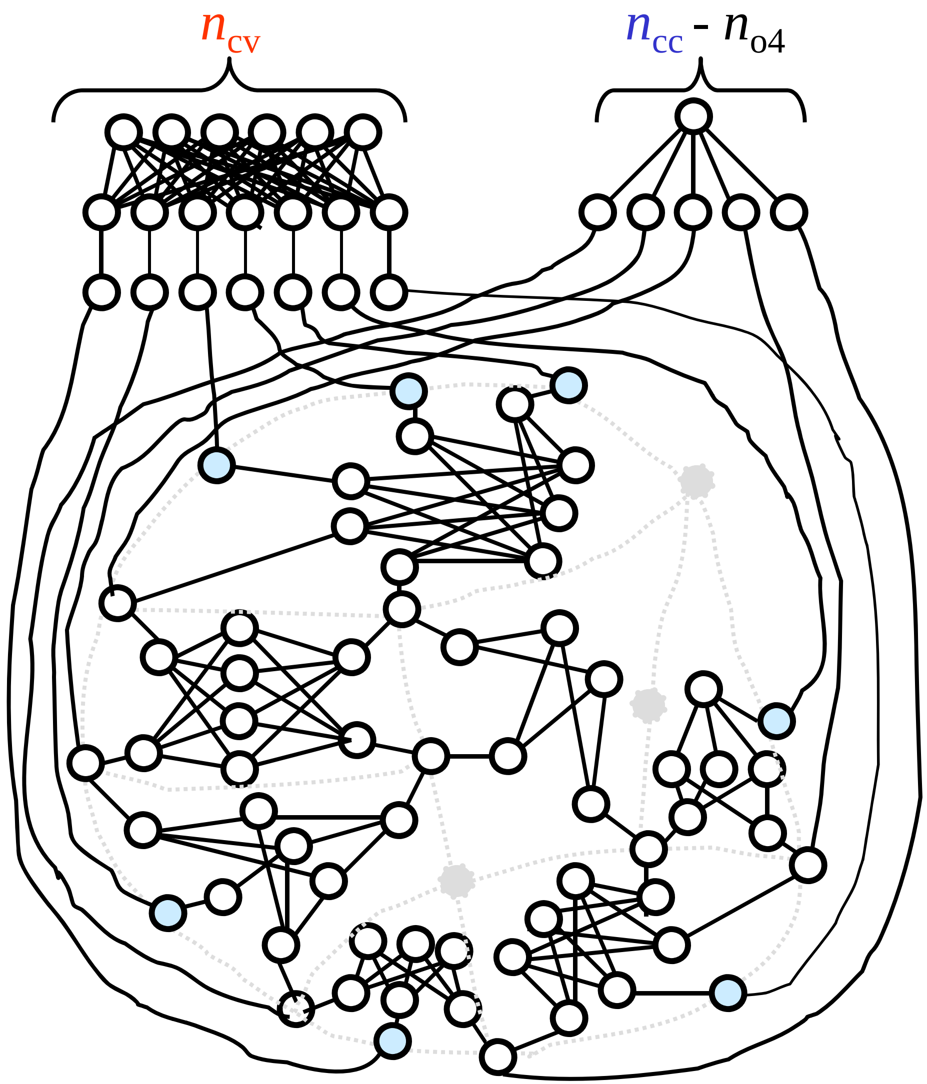
a plane graph G

Inner rectangular drawing with prescribed numbers n_{cv} and n_{cc}

$n_{cv} = 6$, $n_{cc} = 2$



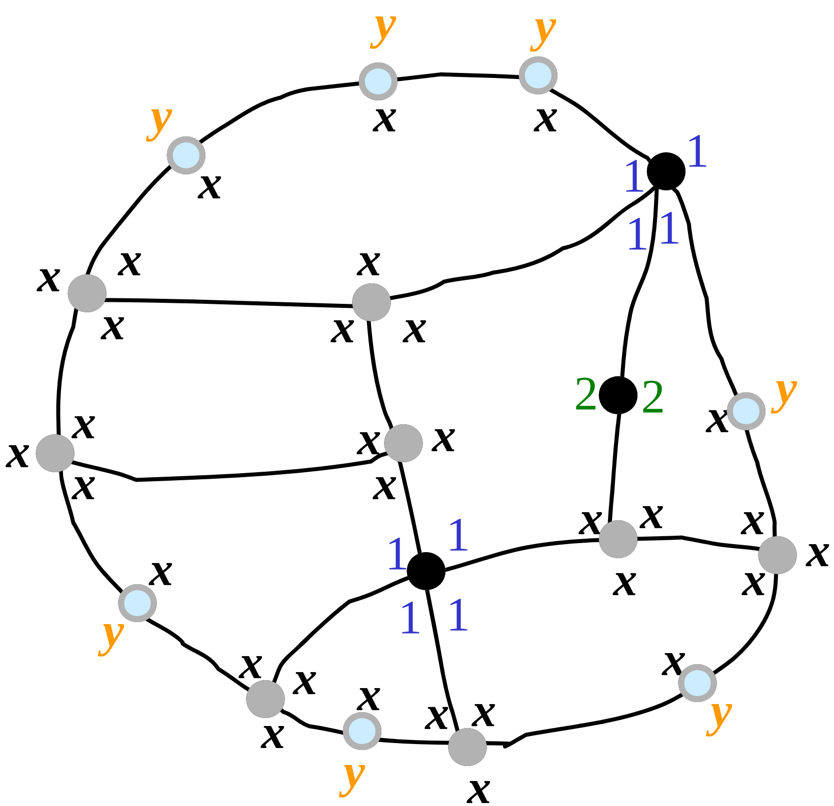
a plane graph G



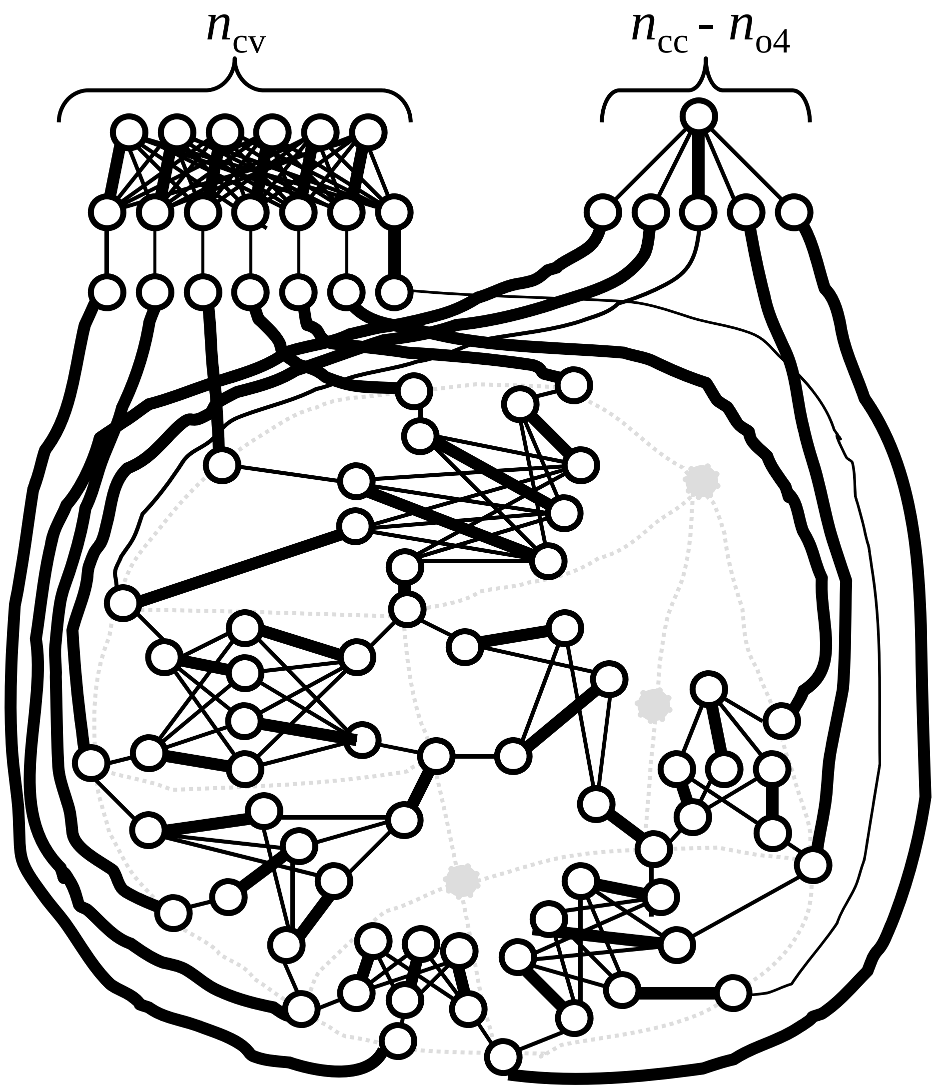
a decision graph G_d* of G

Inner rectangular drawing with prescribed numbers n_{cv} and n_{cc}

$n_{cv} = 6$, $n_{cc} = 2$



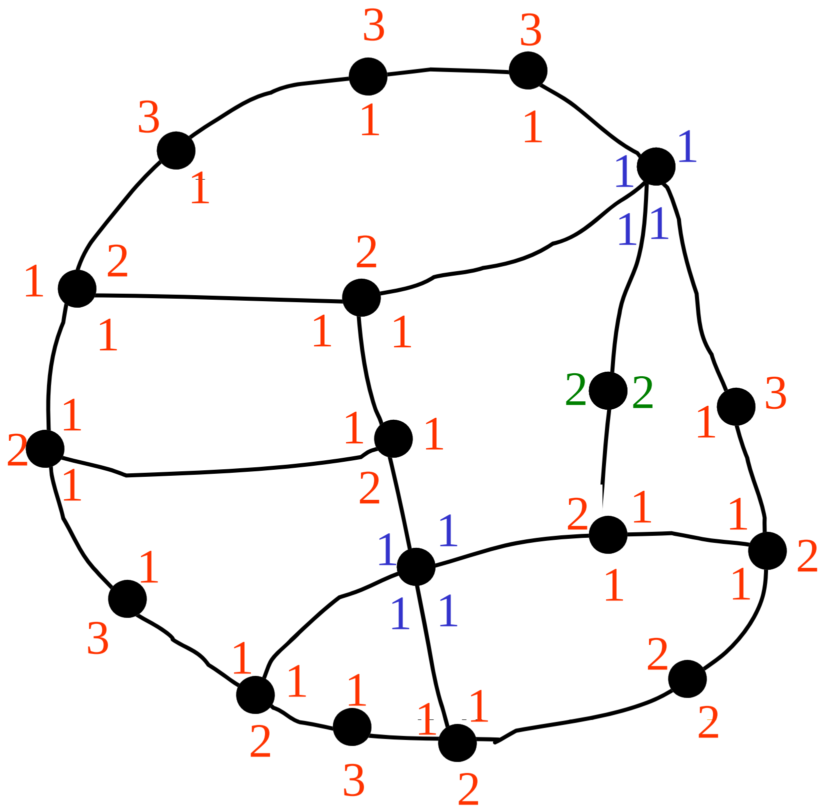
a plane graph G



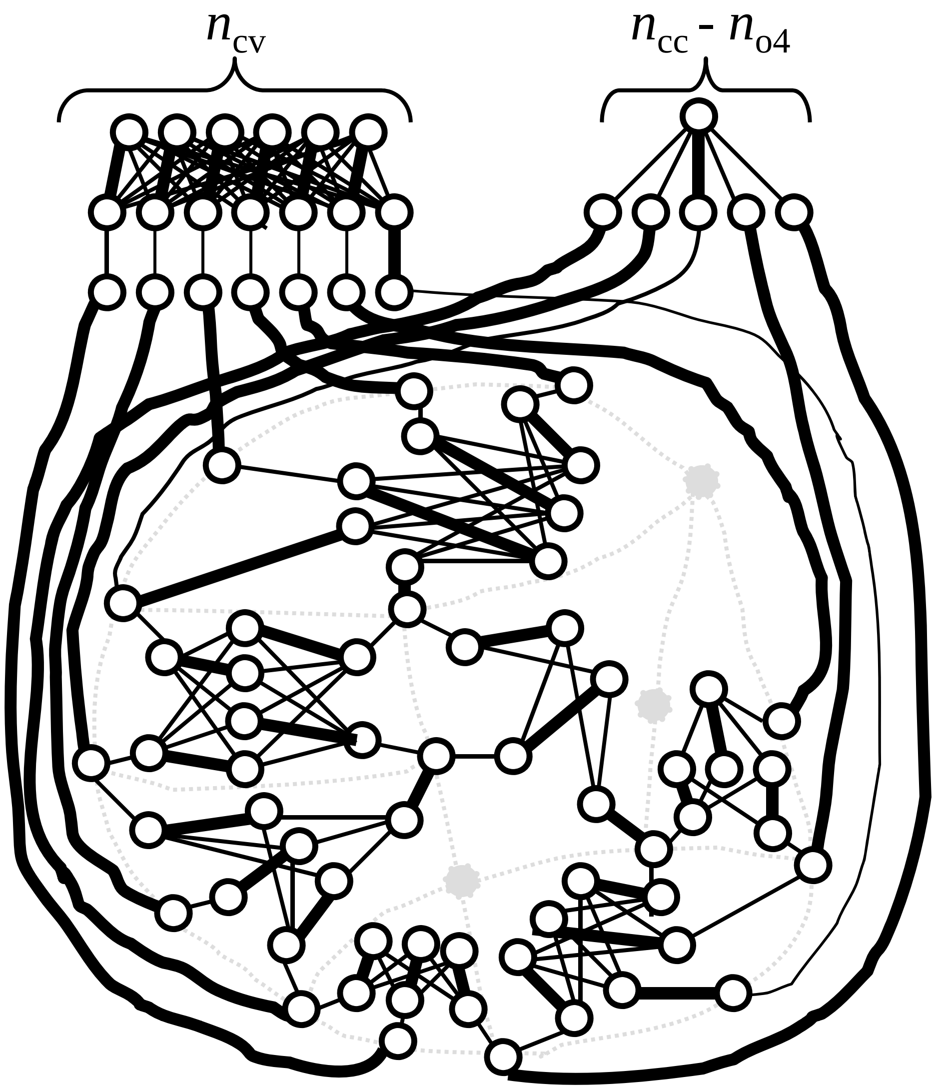
a decision graph G_d* of G

Inner rectangular drawing with prescribed numbers n_{cv} and n_{cc}

$n_{cv} = 6$, $n_{cc} = 2$



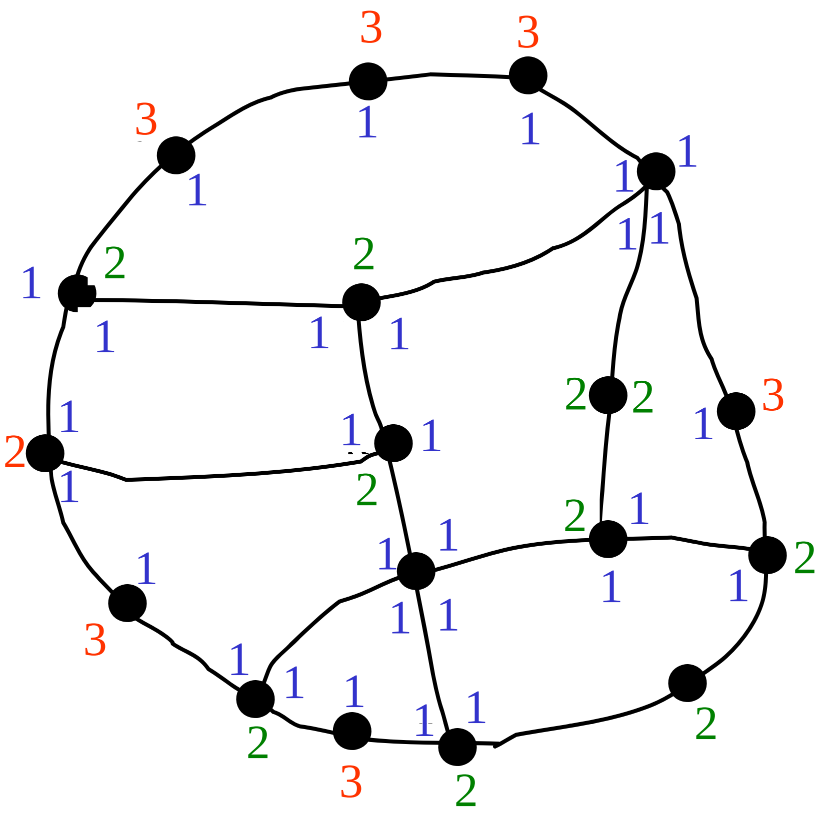
a plane graph G



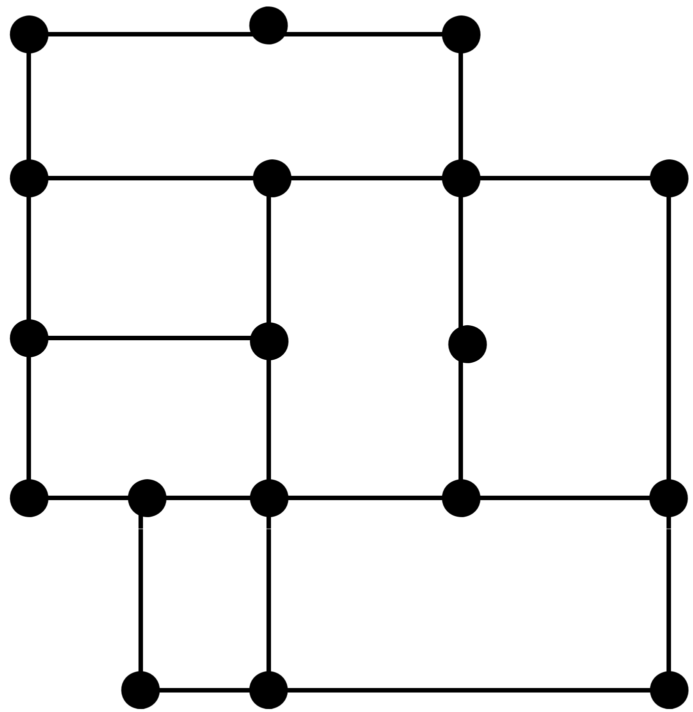
a decision graph G_d* of G

Inner rectangular drawing with prescribed numbers n_{cv} and n_{cc}

$n_{cv} = 6, \ n_{cc} = 2$



a plane graph G



an inner rectangular drawing of G

Running time

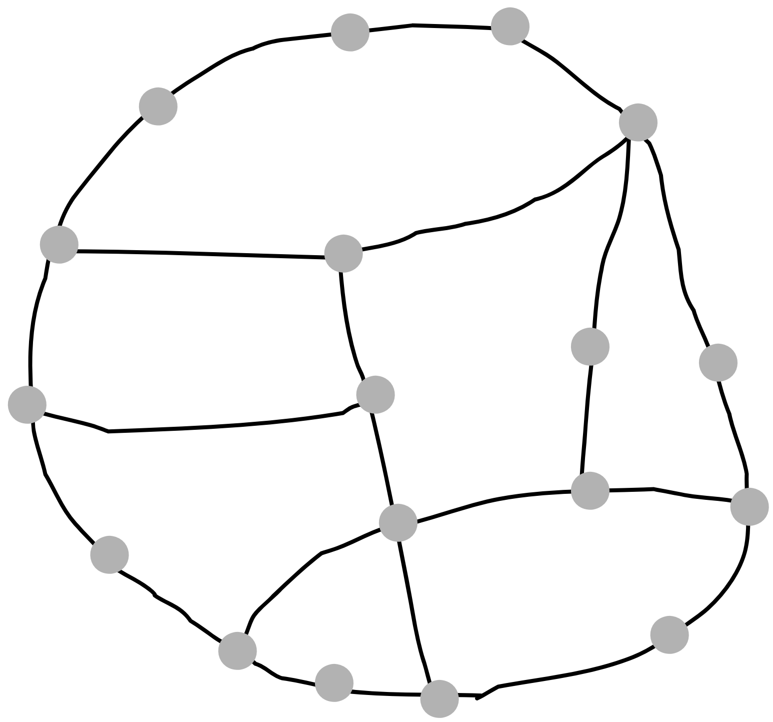
G_d* has an $O(n)$ number of vertices and $O(N)$ (N=n + $n_{\mathrm{cv}}n_o$

n_o: the number of outer vertices) number of edges.

An inner rectangular drawing D of G can be found in time

$O(\sqrt{n}N / \log n)$.

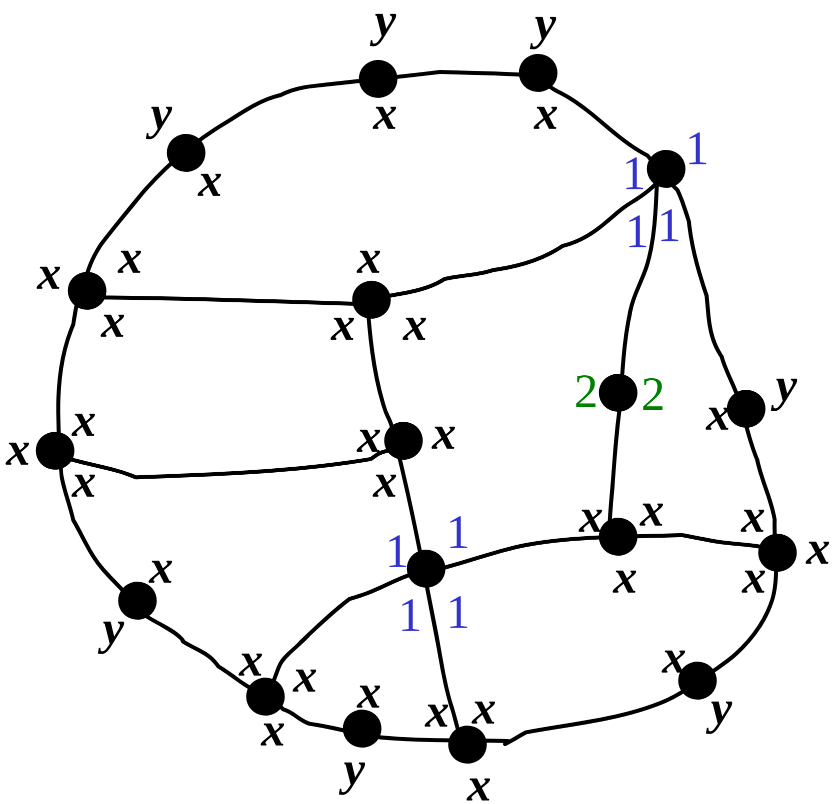
Case 1: the numbers of convex and concave outer vertices are given.



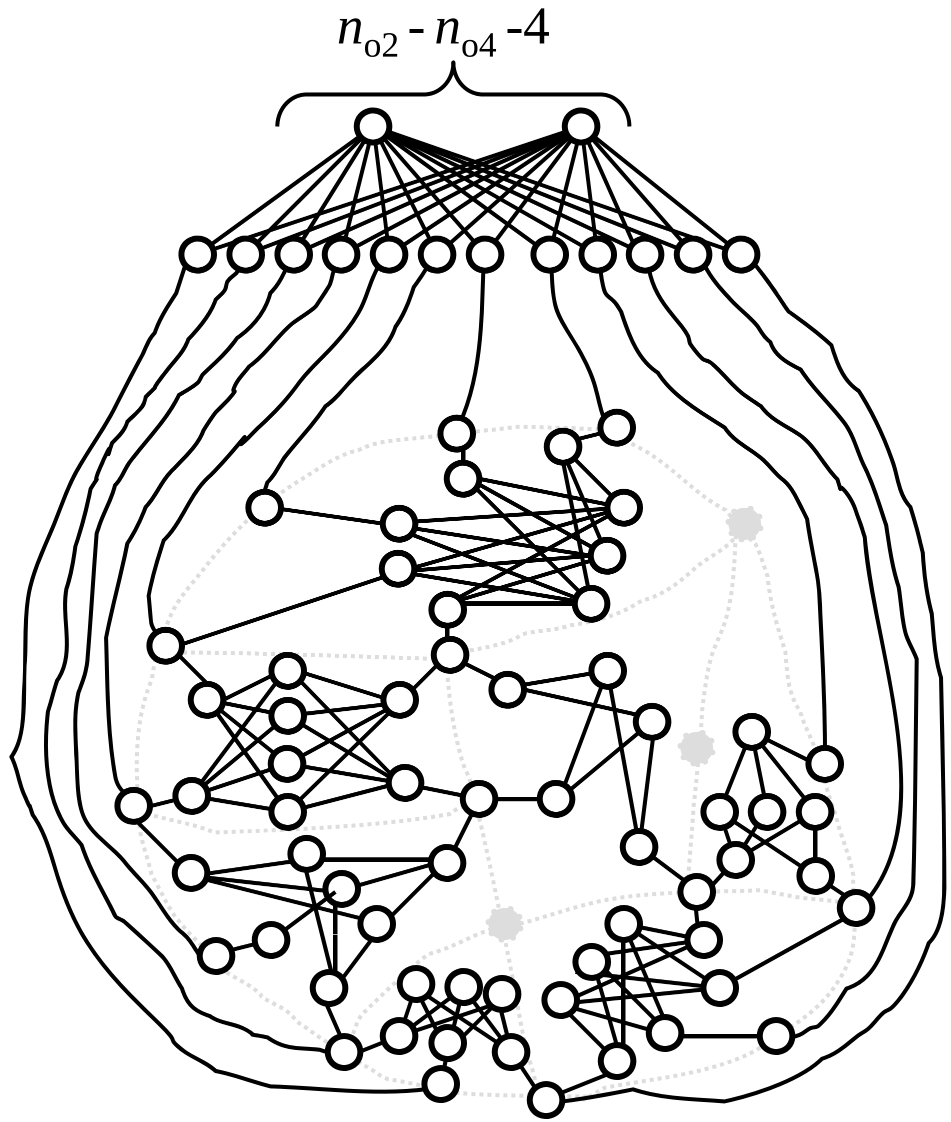
$n_{cv} = 6$, $n_{cc} = 2$

Case 2: in general case

Inner rectangular drawing

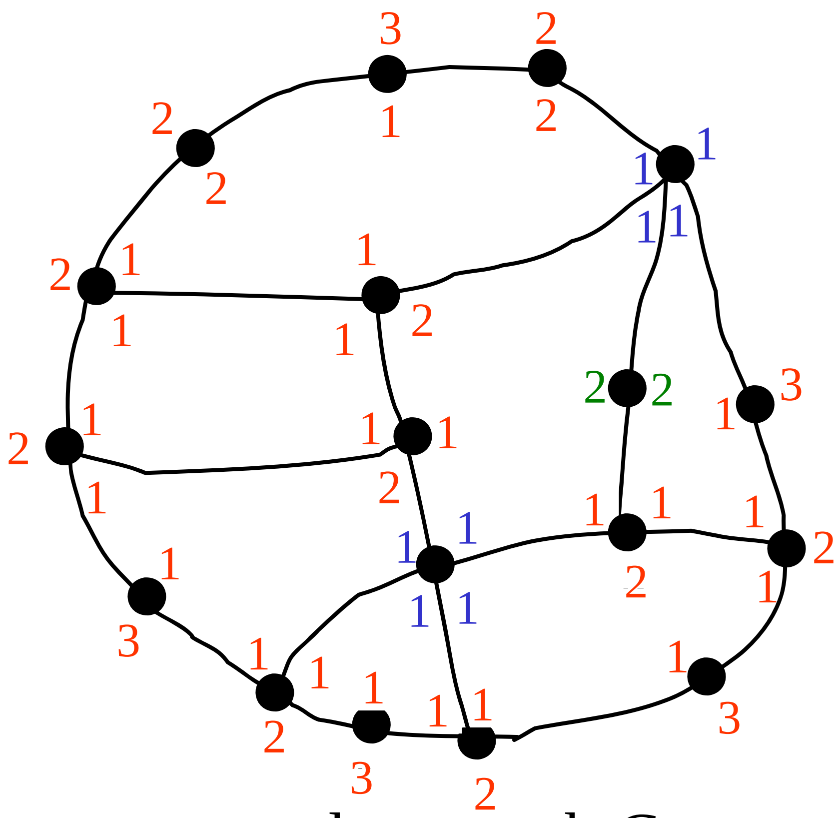


a plane graph G

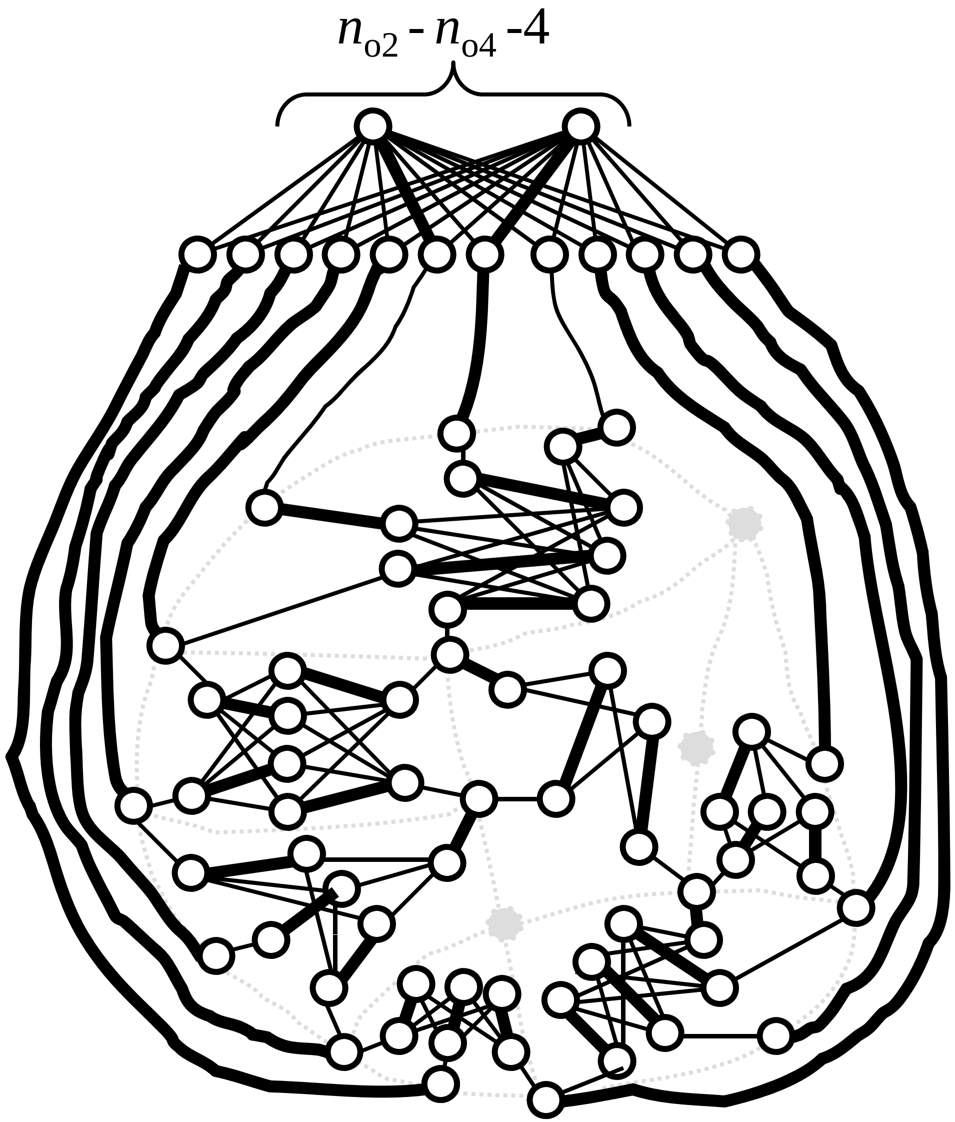


a decision graph $G_d^\star$ of G

Inner rectangular drawing

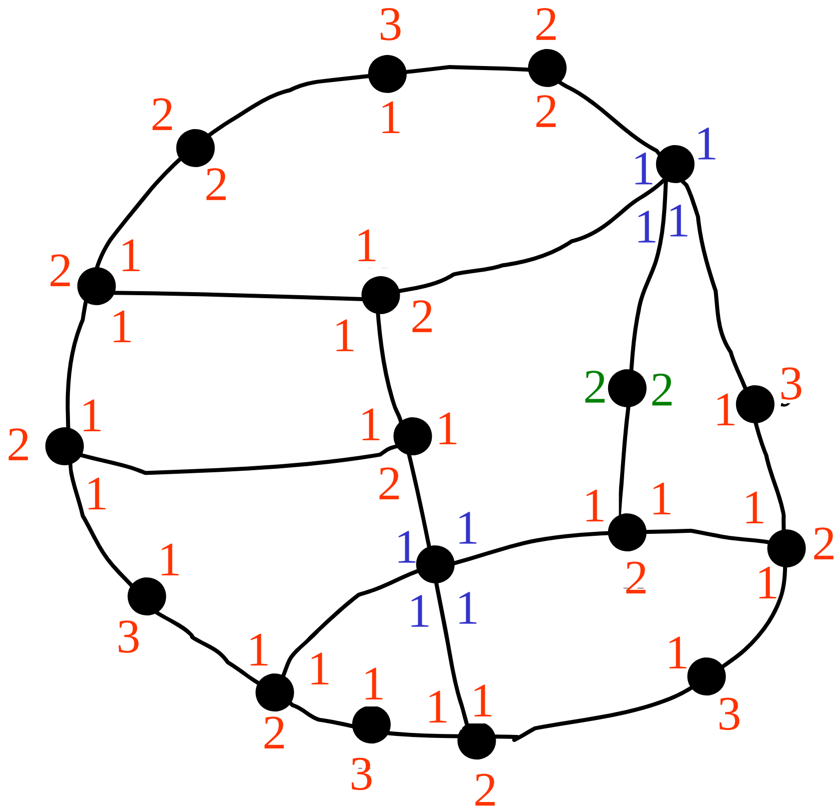


a plane graph G

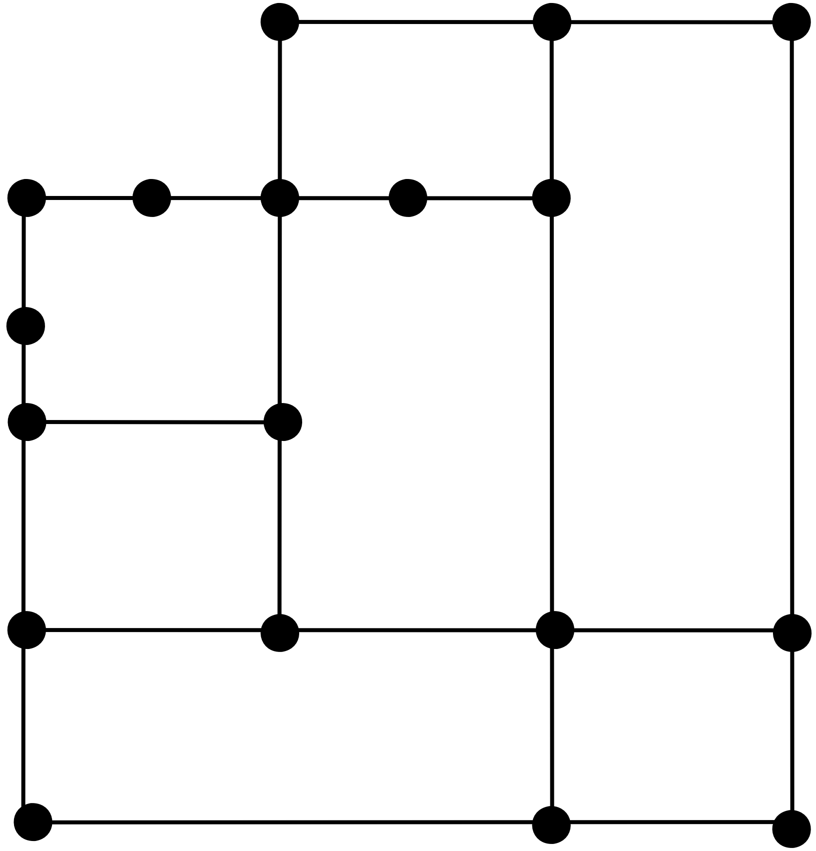


a decision graph $G_d^{\bigstar}$ of G

Inner rectangular drawing



a plane graph *G*



an inner rectangular drawing of *G*

Running time

$G_d^\bigstar$ has an $O(n)$ number of vertices and $O(N')$

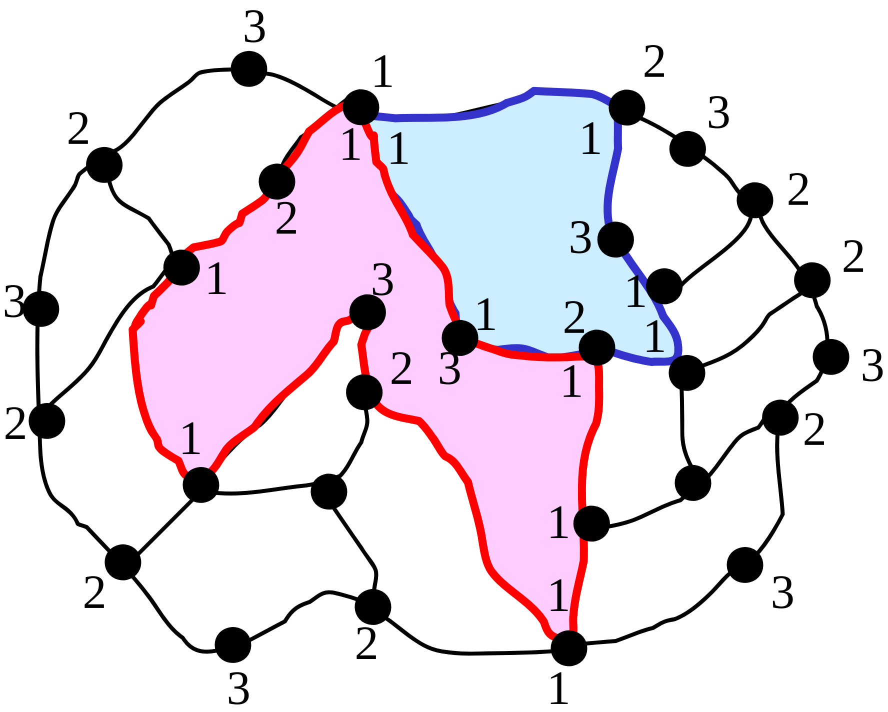
($N'=n + (n_{o2} - n_{o4} - 4)n_o$ n_o: the number of outer vertices

n_{o2} and n_{o4}: the numbers of outer vertices of degrees 2 and 4)
number of edges.

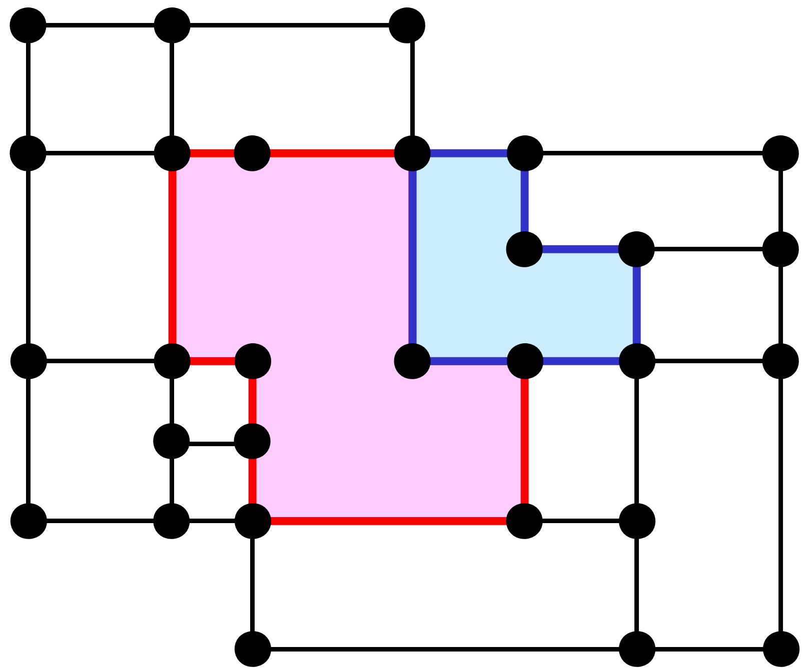
An inner rectangular drawing D of G can be found in time

$O(\sqrt{n}N' / \log n)$.

Related result

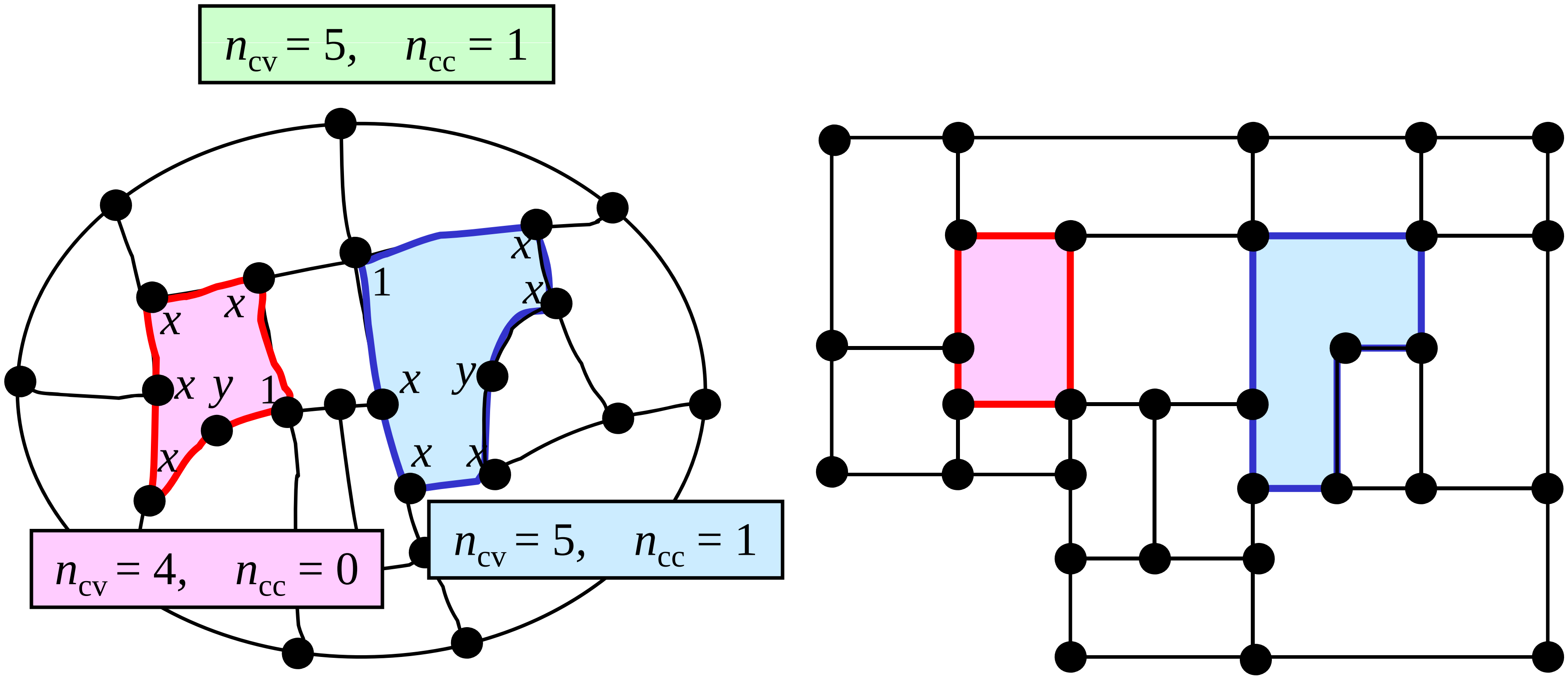


a plane graph *G*



If a sketch of several faces of *G* including the outer face is prescribed, then one can examine whether *G* has a drawing such that each of the other face is a rectangle.

Related result



a plane graph G

If faces $F_0, F_1, \ldots F_i$ of G are vertex-disjoint and the numbers of convex and concave vertices are prescribed, then one can examine whether G has a drawing such that each of $F_0, F_1, \ldots F_i$ has prescribed numbers of convex and concave vertices and each of the other faces is a rectangle.

Regular labeling

We call f a regular labeling of G if f satisfies the following three conditions (a)-(c)

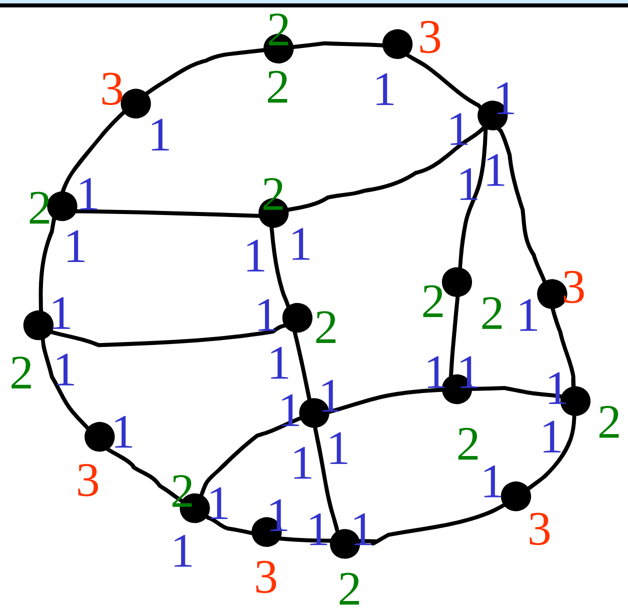
(a) the labels of any vertex in G total to 4;

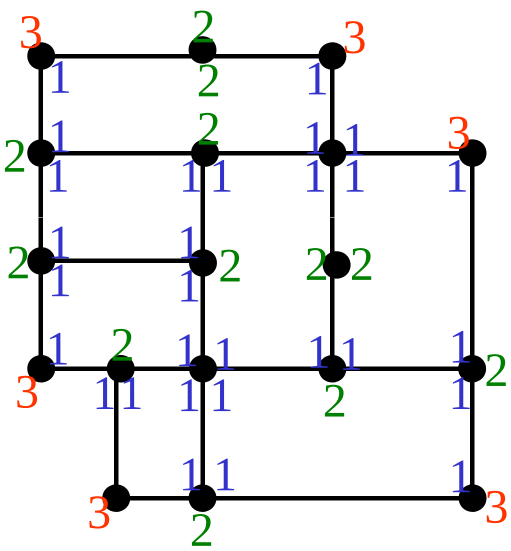
(b) the labels of any inner angles is 1 or 2, and any inner face has exactly four angles of label 1;

(c) n_{cv} - n_{cc} = 4.

n_{cv}: the number of outer angles having label 3

n_{cc} : the number of outer angles having label 1

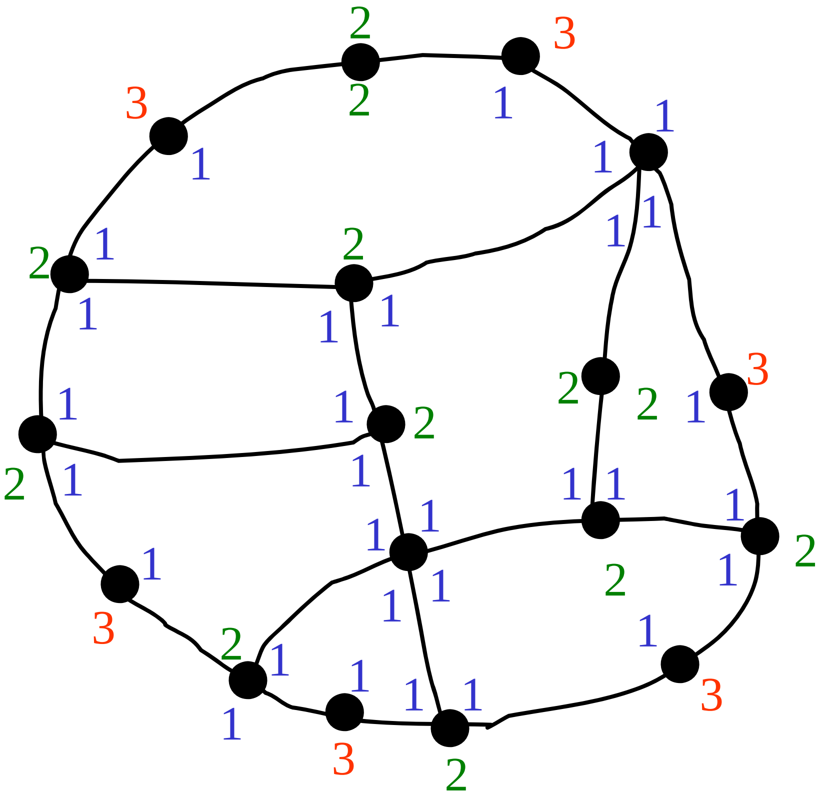




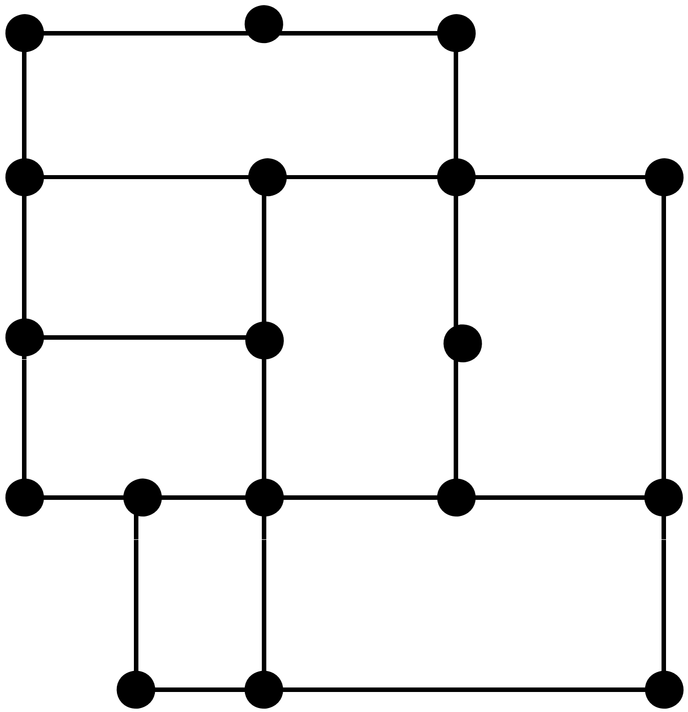
A necessary and sufficient condition for the existence of an inner rectangular drawing of *G*

A plane graph *G* has an inner rectangular drawing

if and only if *G* has a regular labeling



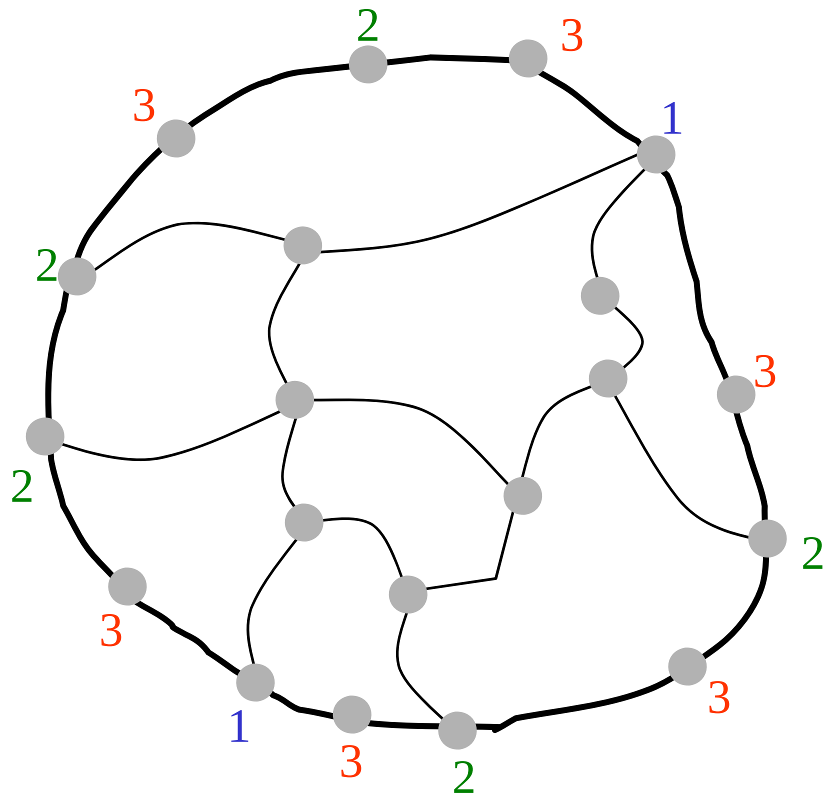
a plane graph *G*



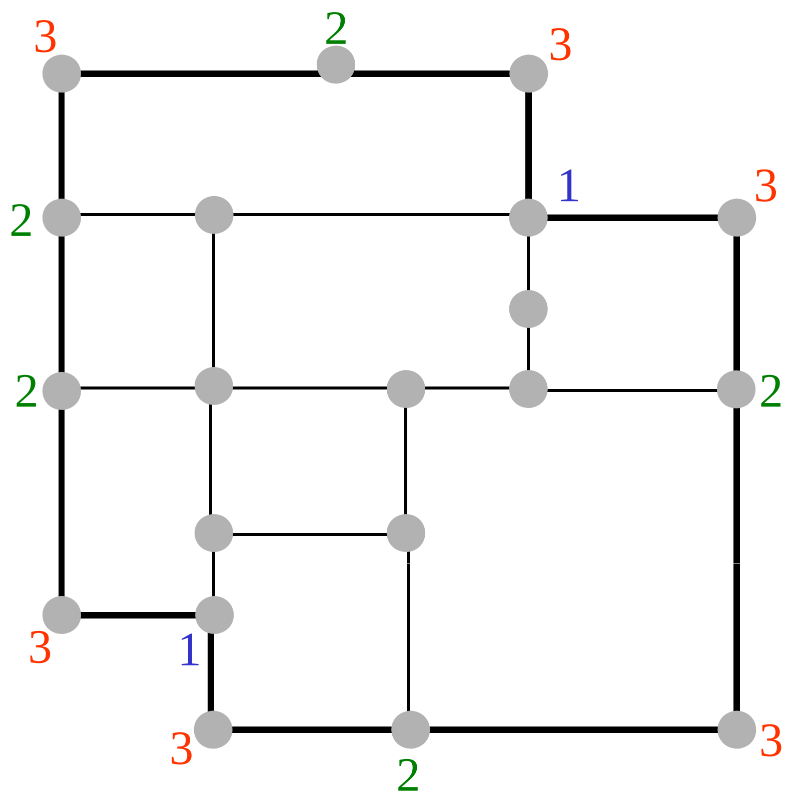
an inner rectangular drawing of *G*

Construct a decision graph G_d

Some of the inner angles of *G* can be immediately determined

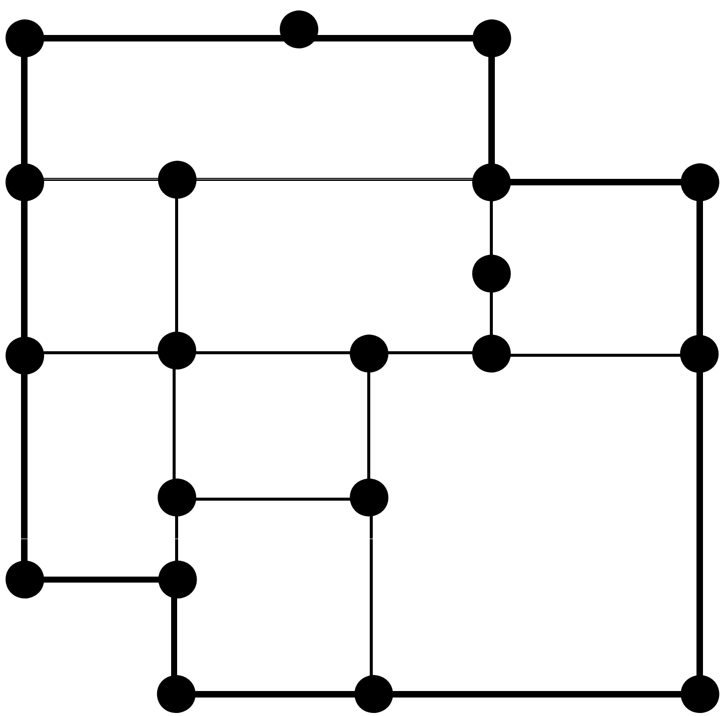


a plane graph *G*



Construct a decision graph G_d

Some of the inner angles of *G* can be immediately determined



an inner rectangular drawing of *G*