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Preliminary investigation on shape estimation of concrete pile by vibration analysis

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A new method is proposed to estimate the shape of a pile by vibration analysis. This is the first method proposed for a pile-shape estimation, which can be used to diagnose the integrity of a pile with an enlarged, subterranean base. It is demonstrated, and verified by experimentation, that pile-shape can be estimated by using an acoustic tube model. Pile shape estimation by this method was found to be quite accurate for an experimental pile laid horizontally on ties, at ground level. This method could be used for estimating the shape of an underground pile, when the soil effect is removed from the measured vibration signal.

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1. INTRODUCTION

Foundation piles are widely used in the construction of modern buildings. Recently, a new construction method is often used, where a hole with the same dimensions as a pile is sunk and the diameter of the lower part of the hole is expanded. After the hole is filled with concrete milk, a pre-molded concrete girder is inserted, constructing a pile with an enlarged base. However, whether or not the pile really has an enlarged base cannot be examined easily, since it is buried under the ground. A traditional method to examine the integrity of a pile is the so-called loading test, where the underground pile is loaded with a heavy weight to evaluate the bearing capacity. Since this method requires both much labor and time to examine all the piles at a construction site, there has been a need for a new, more reliable method.

Several methods have been proposed[1-4] to inspect pile defects, usually involving a test-blow to the head of a pile using a small hammer. The reflected signal is picked-up at the head and fed through a low-pass filter, and the resultant waveform is inspected. However, such methods require the presence of a specialized inspector, and the accuracy of the inspection depends on his subjective judgment. Furthermore, the shape of the pile cannot be estimated by these methods.

A method is proposed here to estimate the shape of a concrete pile using an acoustic tube model, and as such is the first method proposed for pile-shape estimation. The assumptions required in the method are discussed in relation to the experiments performed, and it is shown that pile-shape can indeed be estimated using the proposed method.

2. CONDITIONS FOR APPLICATION OF THE ACOUSTIC TUBE MODEL

The vibration of a pile is a complex waveform, influenced both by the pile structure and the point receiving the hammer test-blow. The pile-head and its measurement are shown in Fig. 1. A cylindrical concrete pile is considered, bearing two

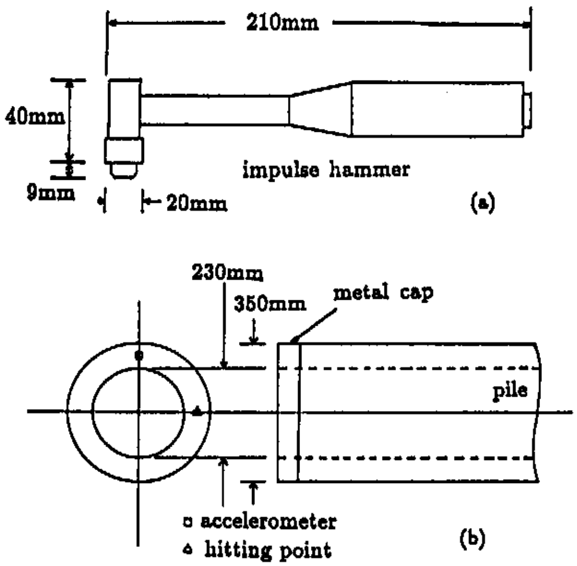

Fig. 1 The impulse hammer (a) and the pile-head (b) used in the experiments.

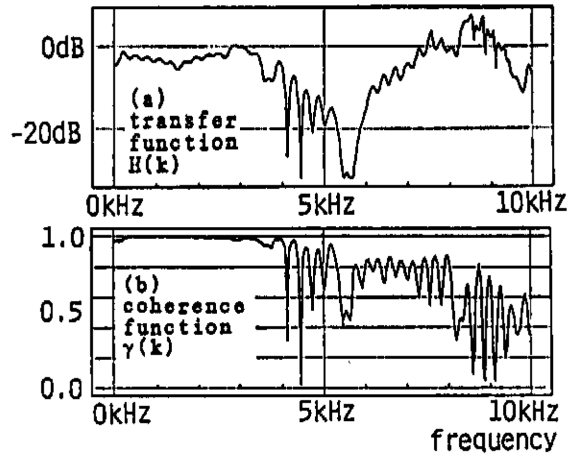

Fig. 2 Transfer function (a) and coherence function (b) of the concrete pile.

metal caps: one on the pile-head and the other at the pile-toe. The impulse hammer used for the test-blow is only 175 g in weight. To apply the acoustic tube model to the estimation of pile-shape, the following assumptions are necessary.[5] (1) The energy losses of the wave propagating through the pile can be neglected. (2) The wave propagating through the pile can be regarded as a plane wave, and the effects of other vibration modes can be ignored.

The energy losses from the wave propagating through the pile are estimated by hitting the head of a pile with an impulse hammer and measuring the vibration at the pile-head. The vibrations recorded include the exciting pulse and the first, second and further reflection waves. The signal from the exciting pulse up to the point just before the first reflected peak is defined as the input. The signal from the first reflected peak until just before the second reflected peak is defined as the output. The transfer function is computed using these input and output values (Fig. 2(a)): the internal losses of the propagation wave are shown to be less than 6 dB for the components with frequencies less than 3.5 kHz.

From the coherence function of Fig. 2(b), it can also be seen that the frequencies above 3.5 kHz are not useful for pile-shape estimation. The vibration components with frequencies greater than 3.5 kHz correspond to waves traveling around the axis of the cylindrical pile, because the lowest frequency of this vibration mode is equal to 3.8 kHz, as calculated using the outer diameter of the pile and the propagation speed. A sound speed in the pile of 4,200 m/s was estimated by experiment.[6] This is quite close to the value calculated from the Young's Modulus and the density of the pile. The Young's Modulus of the pile is estimated from the combined Young's Moduli of the iron and of the concrete, and similarly for calculation of the density of the pile. The vibrations in concrete and in reinforcing ribs do not appear to travel separately.

From these experiments it is reasonable to assume that the waves traveling in the pile consist mainly of plane waves with frequencies of less than 3.5 kHz. So, in these experiments, the other vibration components, greater than 3.5 kHz, were removed by a low-pass filter, and the pile-shape was estimated using the acoustic tube model. The assumption that the vibrations traveling in a pile are plane waves is discussed later in relation to the experimental results.

3. THEORETICAL CONSIDERATIONS

The acoustic tube model is adapted to construction piles, for the estimation of the cross-sectional area of each section, by analyzing the reaction of the pile-head to a hammer pulse. That is, the pile is assumed to be made of sequential sections of elastic column of the same length, as shown in Fig. 3. To apply this model to the present problem, the following assumptions are made:

(1) The length of each column section is equal to

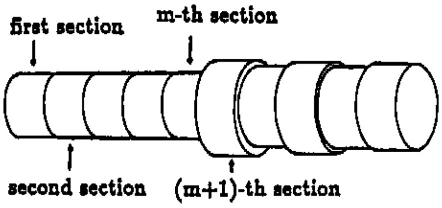

Fig. 3 Pile model.

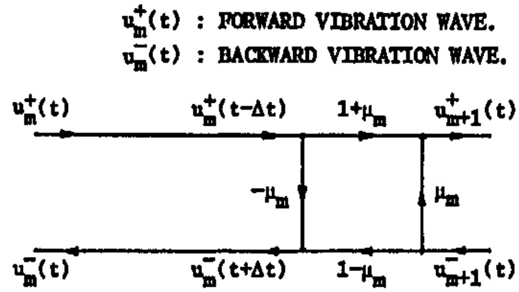

Fig. 4 Mathematical model of the pile.

the distance which the elastic wave in the pile propagates in $T/2$ s, where T is the sampling period. (2) The transverse dimension of each section is shorter than the wavelength, so that the wave propagating through each section can be regarded as a plane wave. (3) Energy losses in the wave due to heat conduction within the pile are negligible. (4) The cross-sectional area varies only at the border between adjacent sections.

If the cross-sectional area of a section is different from that of the next section, the waves are reflected at the border between the sections. Letting S_m and S_{m+1} be the cross-sectional areas of the m-th and $(m+1)$-th sections respectively, the reflection coefficient, μ_m, at the border between the sections may be expressed as

$$\mu_m = \frac{S_{m+1}-S_m}{S_{m+1}+S_m} \tag{1}$$

When the pile model is described by the acoustic tube model, the elastic waves in the adjacent m-th and $(m+1)$-th sections are described by the mathematical model as shown in Fig. 4, where $u_m^+(t)$ and $u_m^-(t)$ denote the forward- and backward-propagating wave in the m-th section respectively. From Fig. 4, the waves are expressed as linear combinations of a forward- and a backward-traveling wave:

$$\begin{cases} u_{m+1}^+(t) = (1+\mu_m)u_m^+(t-\Delta t) + \mu_m u_{m+1}^-(t) \\ u_m^-(t+\Delta t) = -\mu_m u_m^+(t-\Delta t) + (1-\mu_m)u_{m+1}^-(t) \end{cases} \tag{2}$$

where Δt is half the sampling period, T, and also the one-way traveling time for a pulse propagating through a section. By applying the z-transform to sampled sequences of the waves of Eq. (2), the following is obtained (cf. Robinson[7]):

$$\begin{bmatrix} U_{m+1}^+(z) \\ U_{m+1}^-(z) \end{bmatrix} = \frac{z^{1/2}}{1-\mu_m} \begin{bmatrix} z^{-1}, & \mu_m \\ \mu_m z^{-1}, & 1 \end{bmatrix} \begin{bmatrix} U_m^+(z) \\ U_m^-(z) \end{bmatrix} \tag{3}$$

where $U_m^+(z)$ and $U_m^-(z)$ are the z-transforms of the sampled sequences of $u_m^+(t)$ and $u_m^-(t)$ respectively, and the delay Δt is expressed by $z^{-1/2}$ in the z-domain.

The equations describing the forward and backward waves at each section are derived after Robinson.[7] From the definition of the reflection coefficients, the following equation is obtained:

$$\mu_m = -\frac{\sum_{j=0}^{m-1} a_{m-1,j}\,x(m-j)}{\sum_{j=0}^{m-1} a_{m-1,j}\,x(j)}, \quad (\text{for } m=1, 2, \ldots,) \tag{4}$$

where $x(n)$ is the signal calculated from the observed vibration $y(n)$, as defined below in Eq. (6), and the coefficients $a_{i,j}$ are computed recursively from the equation:

$$\sum_{i=0}^{m} a_{m,i} z^{-i} = \sum_{i=0}^{m-1} a_{m-1,i} z^{-i} + \mu_m \sum_{i=0}^{m-1} a_{m-1,i} z^{m-i}\,. \tag{5}$$

The velocity, $y(n)$, is obtained by integrating the vibration picked-up by the accelerometer at the pile-head, which is the sum of $u_1^+(n)$ and $u_1^-(n)$, that is:

$$y(u) = u_1^+(n) + u_1^-(n) = 2u_1^-(n) + \delta(n)$$

where $\delta(n)$ is an exciting pulse corresponding to the hammer pulse. The relation between the velocity signal, $y(n)$, and $x(n)$ of Eq. (4) is described as follows:

$$x(n) = \begin{cases} y(n), & \text{for } n=0 \\ y(n)/2, & \text{for } n>0 \end{cases} \tag{6}$$

If in Eq. (4), $m=1$ and $a_{0,0}=1$, the reflection coefficient, μ_1, is obtained as follows:

$$\mu_1 = x(1)/x(0) \tag{7}$$

Then, the coefficients $\{a_{1,j}\}$ $(j=0, 1)$ are calculated by using Eq. (5). In the same way, for the m-th section $(m=2, 3, \ldots)$, the reflection coefficient, μ_m, is calculated recursively by using Eqs. (4) and (5). Thus, the cross-sectional area of the m-th section, S_m, can be obtained by using Eq. (1), because the cross-sectional area of the first section, S_0 is equal to that of the pile-head. Therefore, the pile-shape can be estimated even if the pile has an enlarged base.

4. EXPERIMENTS AND RESULTS

4.1 Piles and Measurements

Figure 5 shows two piles used in the experiments. One is a uniformly cylindrical concrete pile: a normal pile, as used for construction foundations. The other has an enlarged base to sustain a larger bearing force, as described earlier. The piles were laid horizontally on ties. The vibration reaction at the pile-head to a pulse administered with a small hammer was picked up by an accelerometer attached to the head. Figure 6 shows the signals measured from the piles. Each signal was A to D converted at a sampling rate, f_s, of 25.6 kHz with 12-bit accuracy.

4.2 Shape-Estimation Algorithm

As mentioned earlier, some pre-processing is required to apply the proposed method to the signal obtained because it contains unwanted components. Firstly, a low-pass filter is used to suppress the components at frequencies higher than 3 kHz, because these components do not satisfy the required conditions. Each of the resultant vibration signals, which are acceleration functions, are integrated to obtain velocities, as shown in Fig. 7. Next, the velocity signal is de-sampled, at one fourth the original sampling rate, which removes the components with frequencies greater than the de-sampling frequency, $f_s/4$. The pulse sequence obtained is shown in Fig. 8. From this pulse sequence, the reflection coefficients, the corresponding cross-

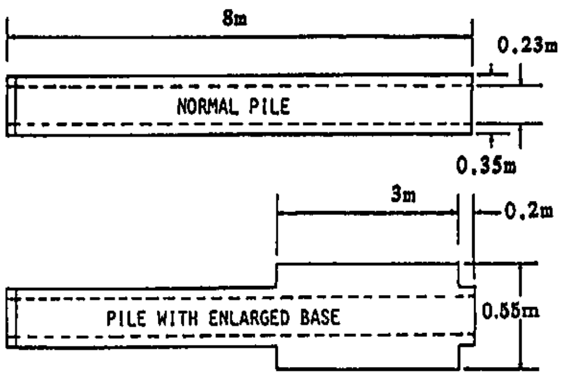

Fig. 5 The piles used in the experiments.

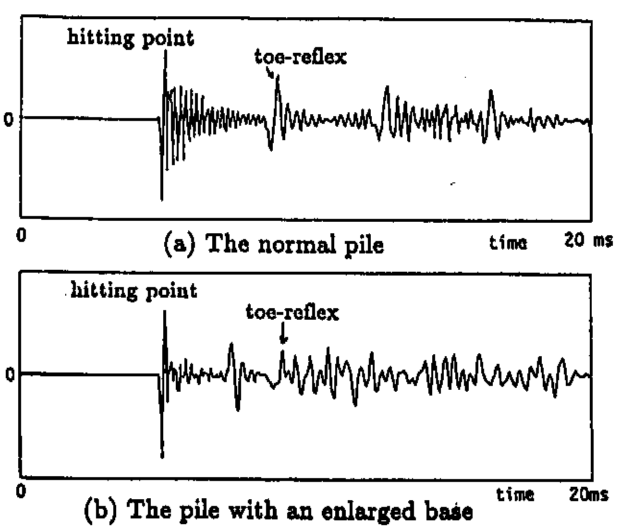

Fig. 6 Observed waveforms at the pile-head.

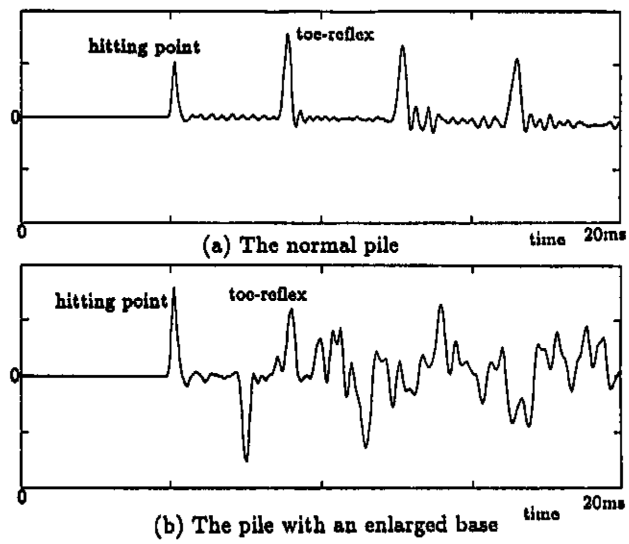

Fig. 7 Velocity waveforms.

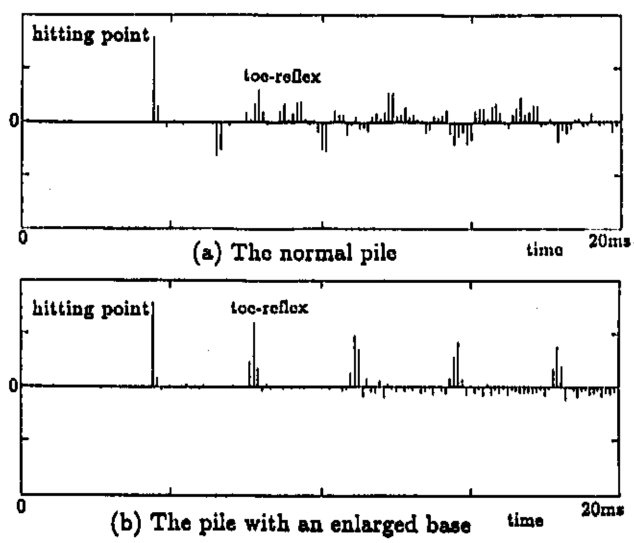

Fig. 8 De-sampled velocity signals.

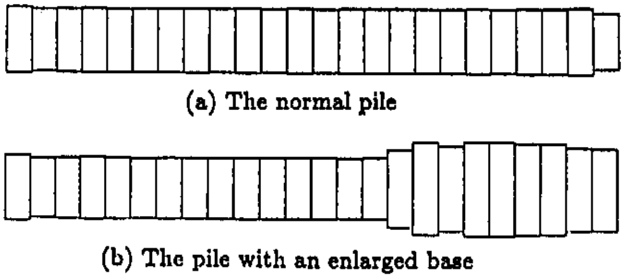

Fig. 9 The pile-shapes estimated from the velocity signals.

sectional areas, and diameter of each section are estimated by the method proposed above (Chap. 3). Due to the rate of de-sampling, $f_s/4$, a pile with a length of 8 m is divided into 24 sections. The estimated pile-shapes are shown in Fig. 9.

4.3 A Simple Improvement

The proposed method estimates the shape of a pile under the assumption that the driving signal, $\delta(n)$, is an impulse, and that the ideal reflected signal, $y(n)$, should be a sequence of pulses. However, the observed signal is not a sequence of pulses since the real signal generated by a hammer is not an ideal impulse, and also the characteristics of the low-pass filter are not ideal. Consequently, the diameter of sections near the borders of cross-sectional area variations is not estimated accurately, as can be seen in Fig. 9. Therefore, a further step of pre-processing has been added: all the peaks in the velocity waveform as shown in Fig. 7 are changed to pulses with the same amplitude and at the same location as shown in Fig. 10. This approximation works well if the cross-sectional areas do not increase or decrease over more than one section. Moreover, this method saves much computation time. Figure 11 shows the estimated shapes of the piles using this approximate method.

4.4 Results and Discussion

As shown in Fig. 11(a), the estimated shape is almost straight for the normal pile. The estimated diameters of the column sections vary between 0.33 m and 0.37 m, and the averaged value is 0.343 m. For the pile with an enlarged base, Fig. 11(b), the estimation includes an easily-recognizable enlarged region. The averaged diameter from the pile-head to the 15-th section is 0.35 m and that from the 16-th section to the pile-toe is 0.46 m, as shown in

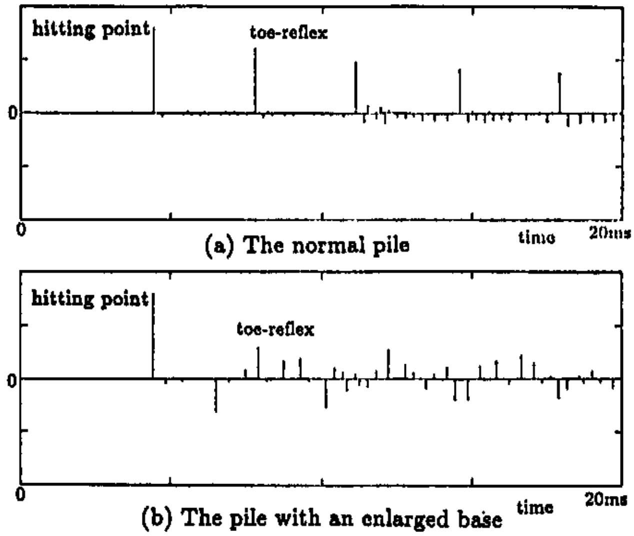

Fig. 10 Pulse sequences.

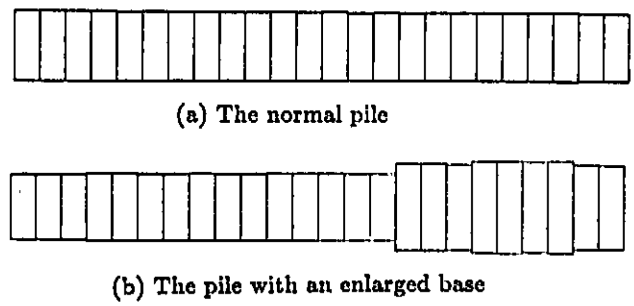

Fig. 11 Pile shapes estimated from the pulse sequences.

Table 1 Results of verification experiments.

	Normal pile diameter	Pile with an enlarged base	
		Diameter*	Length*
Pile size	0.35 m	0.35 m/0.55 m	4.80 m/3.20 m
Estimated size	0.343 m	0.349 m/0.460 m	5.00 m/3.00 m

* normal part/enlarged part.

Table 1. Thus, the shapes of both piles are estimated quite accurately.

The experimental results therefore show that an acoustic tube model can be applied successfully to the estimation of pile-shape. It is difficult to produce more precise estimations because of the following three reasons: —

(1) In the experiment, the frequency range is limited to 3.2 kHz. However, the speed of sound in the pile is 4,200 m/s. The half-wave length of the 3.2 kHz wave is about 0.66 m, and the accuracy of

estimation of resolution in length is about 0.66 m. Higher frequency components would therefore be necessary to achieve a higher resolution, but the acoustic tube model cannot be used for such a higher frequency range, as mentioned earlier (Chap. 2). The diameter of the pile used in the experiments is 0.35 m, which is the smallest size used in Japan. For piles with a larger size, the upper frequency limit should be lowered.

(2) Since the attenuation of the wave due to propagation through the pile is not completely negligible, as was assumed for calculations, the pile-shape estimated by the proposed method was affected slightly by this attenuation.

(3) The simple pre-processing described in Sect. 4.3 causes an error, because the position of the peak in a waveform does not always exactly correspond to the point where the cross-sectional area varies.

These important issues are currently under investigation.

5. CONCLUSION

This paper proposes a new method to estimate the pile-shape by analyzing the response of the pile-head to a hammer pulse. This is the first method proposed for pile-shape estimation. An acoustic tube model is applied for the estimation, and the conditions for this application are discussed. The experiments for shape estimation were carried out with piles laid horizontally above ground, and the results proved that the conditions for the application of an acoustic tube model are satisfied.

Estimation of the shape of underground piles is currently under investigation.

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