# A New Approach to Time Dependent AR Modeling of Signals and Its Application to Analysis of the Fourth Heart Sound

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***Abstract*—In this paper, we present a new method to estimate spectrum transition between short-length signals of the succeeding frames in low SNR cases. If the transition pattern is complex and/or there are large differences in the transition patterns among the individual sets of multiframe signals, it is difficult to estimate the transition pattern stably by the previously proposed time-varying AR modeling because the results are considerably dependent on the choice of the basic functions to be used. We propose a new approach of modeling to estimate the spectrum transition of the multiframe signals by using a linear algorithm without any basic functions. Instead of basic functions we use the spectrum transition constraint and the SVD-based technique is applied on the proposed method to obtain more accurate estimates. By applying this method to the analysis of multiframe signals of the fourth heart sounds obtained during the stress test, significant differences of the transition patterns are clearly detected in the spectra between patients with myocardial infarction and normal persons. The significant characteristics of these transition patterns may be applied to acoustic diagnosis of heart diseases.**

## I. Introduction

Much work has been done on the parametric spectrum estimation of noisy signals using linear models of autoregressive (AR) transfer function. A strong restriction of these methods lies in the necessary assumption that the signals may be considered to be stationary over the observation interval. Time-varying parametric approaches of modeling have been proposed to overcome this limitation and to take the effects of nonstationary signals into account explicitly. To estimate the parameters using a linear algorithm, the unknown time-varying parameters are approximated by linearly weighted combinations of a small number of known functions. The choice of the basic functions is an important part of such modeling process. A convenient way is to replace the time-dependent coefficients with their second-order expansion [1], or an arbitrary order expansion [2], [6]. Legendre [3], [7], Fourier [4], prolate spheroidal [5], and B-spline [11] are usually chosen for the basic functions. Since the number of unknown parameters is large, efficient equivalent representations for the modeling have been also proposed such as lattice filters [2], [5], [8].

However, if the spectrum transition pattern is complex and/or there are large differences in the transition patterns among the individual sets of multiframe signals, it is difficult to estimate the transition pattern stably by choosing a set of basic functions *a priori*.

In this paper we propose a new approach of modeling to estimate the spectrum transition of the multiframe signals by using a linear algorithm without any basic function, as described in Section II. We also propose a singular-value-decomposition (SVD)-based method to obtain more accurate multiframe AR coefficients according to the above-mentioned linear optimization in low signal-to-noise (SNR) cases. In Section III, simulation results show that estimation error obtained with this new method is about 10 dB less than that obtained with the ordinary SVD-based method even in the cases where the SNR is low (about 10 dB) and the signal length is very short (less than 30 points).

In order to diagnose left ventricular dysfunction based on the acoustic characteristics of the ventricle of the heart muscle, it is necessary to detect the spectrum transition between the succeeding frames during the stress test. We apply the method to analyzing the multiframe signals of the fourth heart sounds and then diagnosing myocardial infarction. The experimental results are described in Section IV. Significant differences of the transition patterns are obtained in the spectra between the normal persons and the patients. These characteristics of the transition patterns may be applied to the acoustic diagnosis of heart diseases.

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## II. Principle

Let $\{x(n;j), n = 0, 1, \cdots, N-1\}$ denote a $j$th frame ($j = 0, 1, \cdots, F-1$) autoregressive signal of order

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$M$, represented by the backward recursion such that

$$x(n;j) = -\sum_{i=1}^{M} a_i(j)\cdot x(n+i;j) + e(n;j) \quad (1)$$

where $F$ is the number of frames, $e(n;j)$ is a stationary white noise process of the $j$th frame with zero mean and variance $\sigma_e^2$, and $\{a_i(j),\ i = 0, 1, \cdots, M\}$ are linear predictive coefficients of the $j$th frame signal ($a_0(j) = 1$). We assume that the signal $x(n;j)$ is stationary over the $j$th frame. The AR coefficients $\{a_i(j)\}$ are slowly time varying between the succeeding frames. When the length of each frame signal is short and the SNR is low, it is difficult to obtain stably the spectrum estimate of each frame by applying an ordinary spectrum estimate approach, such as the covariance method [14], to each frame signal independently by minimizing the cost function $J_0$ such as

$$J_0 = \sum_{n=0}^{N-1-M} |e(n;j)|^2 = \sum_{n=0}^{N-1-M} \left| \sum_{j=0}^{M} a_i(j)\cdot x(n+i;j)\right|^2. \quad (2)$$

To achieve a meaningful spectrum transient pattern of multiframe signals, we must design an alternative cost function $J_1$ is such a way that the resulting solution is sufficiently smooth under the assumption that the AR parameters over the succeeding frames do not very rapidly. In addition to this assumption, we assume that the difference $\{\Delta a_i(j) \overset{\text{def}}{=} a_i(j) - a_i(j-1)\}$ between $i$th AR parameters of the succeeding frames are a random drawing from a stationary normal distribution having zero mean and variance $\sigma_i^2$. Thus, the framed AR parameters $\{a_i(j),\ i = 1, 2, \cdots, M,\ j = 0, 1, \cdots, F-1\}$ are assumed to be from a process with independent increments. This model is, therefore, specified by a set of constant parameters $\{a_i(0),\ i = 1, \cdots, M\}$, $\{\Delta a_i(j),\ i = 1, \cdots, M,\ j = 0, 1, \cdots, F-1\}$, and $\{\sigma_i^2,\ i = 1, \cdots M\}$. To determine the spectrum transient pattern without any basic functions, as is done in existing time-varying spectrum estimation algorithms, we specify the cost function $J_1$ by the sum of the normalized total residual power $J_0$ and the total transition power, which is defined as the sum of the average power difference between the AR coefficients of the succeeding frames, such as

$$J_1 = \frac{E_{n,j}[|e(n;j)|^2]}{E_{n,j}[|x(n;j)|^2]} + \sum_{i=1}^{M} \gamma_i \cdot E_j[|\Delta a_i(j)|^2] \quad (3)$$

where

$$\Delta a_i(j) = a_i(j) - a_i(j-1), \quad (j = 1, 2, \cdots, F-1).$$

And where $\gamma_i$ is a constant, which decides the weight of the constraint on the spectrum transient, and $E_x[\cdot]$ denotes the averaging operating over various values of $x$. The resultant spectrum transition pattern obtained by minimizing the cost function $J_1$ depends on the value of $\gamma$, which is difficult to determine in the general case.

Thus, based on the concept of the constrained least squares filter [12], we modify the specification of the cost function $J_1$ of (3) as follows: The constrained least squares problem could be formulated as minimizing the normalized residual power subject to the condition that the transition power of the $i$th set of AR parameters equals the constant $\sigma_i^2$. Using the method of Lagrange multipliers

$$J = \frac{E_{n,j}[|e(n;j)|^2]}{E_{n,j}[|x(n;j)|^2]} - \sum_{i=1}^{M} \lambda_i \{E_j[|\Delta a_i(j)|^2] - \sigma_i^2$$

where $\lambda_i$ is a Lagrange multiplier. Let $E_{n,j}[|e(n;j)|^2]$, $E_{n,j}[|x(n;j)|^2]$, and $E_j[|\Delta a_i(j)|^2]$ be defined as follows:

$$E_{n,j}[|e(n;j)|^2] = \frac{1}{F(N-M)} \sum_{j=0}^{F-1} \sum_{n=0}^{N-M-1} |e(n;j)|^2$$

$$E_{n,j}[|x(n;j)|^2] = \frac{1}{F(N-M)} \sum_{j=0}^{F-1} \sum_{n=0}^{N-M-1} |x(n;j)|^2$$

and

$$E_j[|\Delta a_i(j)|^2] = \frac{1}{F-1} \sum_{j=1}^{F-1} |\Delta a_i(j)|^2.$$

The cost function $J$ can then be described as follows:

$$J = \frac{1}{P_0 F(N-M)} \sum_{j=0}^{F-1} \sum_{n=0}^{N-1-M} \left| \sum_{i=0}^{M} a_i(j)\cdot x(n+i;j)\right|^2 - \sum_{i=1}^{M} \lambda_i \left\{ \frac{1}{F-1} \sum_{j=1}^{F-1} |\Delta a_i(j)|^2 - \sigma_i^2 \right\} \quad (4)$$

where

$$P_0 = E_{n,j}[|x(n;j)|^2].$$

Using a $(N-M)$-dimensional vector $\boldsymbol{x}_j = [x(0;j), x(1;j), \cdots, x(N-1-M;j)]^T$, a $M$-dimensional vector $\boldsymbol{a}_j = [a_1(j), \cdots, a_M(j)]^T$, a $M$-dimensional vector $\boldsymbol{\Delta a}_j = [\Delta a_1(j), \cdots, \Delta a_M(j)]^T$, where the superscript $T$ denotes the matrix transpose, and $(N-M)$-by-$M$ matrix $X_j$:

$$X_j = \begin{pmatrix} x(1;j) & x(2;j) & \cdots & x(M;j) \\ x(2;j) & x(3;j) & \cdots & x(M+1;j) \\ \vdots & \vdots & & \vdots \\ x(N-M;j) & x(N-M+1;j) & \cdots & x(N-1;j) \end{pmatrix} \quad (5)$$

the cost function $J'$, which is obtained by multiplying (4) by $P_0F(N - M)$, is given by

$$J' = P_0F(N - M)J$$
$$= \sum_{j=0}^{F-1} \|x_j + X_j a_j\|^2 - \sum_{i=1}^{M} \lambda_i' \left\{ \sum_{j=1}^{F-1} \|\Delta a_i(j)\|^2 - \sigma_i'^2 \right\} \quad (6)$$

where

$$\lambda_i' = \lambda_i \cdot \frac{P_0F(N - M)}{F - 1}$$

and

$$\sigma_i'^2 = \sigma_i^2(F - 1).$$

Using the relation

$$a_j = a_0 + \sum_{k=1}^{j} \Delta a_k, \qquad (j = 1, 2, \cdots, F - 1)$$

and taking the derivatives of $J'$ with respect $a_0$, $\Delta a_j$, and $\{\lambda_i'\}$, the following simultaneous equations are obtained:

$$\frac{1}{2} \cdot \frac{\partial J'}{\partial a_0} = \mathbf{0} = \sum_{m=0}^{F-1} (X_m^T x_m + X_m^T X_m a_0) + \sum_{m=1}^{F-1} X_m^T X_m \left( \sum_{k=1}^{m} \Delta a_k \right) \quad (7a)$$

$$\frac{1}{2} \cdot \frac{\partial J'}{\partial \Delta a_j} = \mathbf{0} = \sum_{m=j}^{F-1} \left\{ X_m^T x_m + X_m^T X_m \cdot \left( a_0 + \sum_{k=1}^{m} \Delta a_k \right) \right\} - \begin{pmatrix} \lambda_1' \Delta a_1(j) \\ \lambda_2' \Delta a_2(j) \\ \vdots \\ \lambda_M' \Delta a_M(j) \end{pmatrix}$$
$$(j = 1, 2, \cdots, F - 1) \quad (7b)$$

$$\frac{1}{2} \cdot \frac{\partial J'}{\partial \lambda_i'} = 0 = \sum_{j=1}^{F-1} |\Delta a_i(j)|^2 - \sigma_i'^2$$
$$(i = 1, 2, \cdots, M). \quad (8)$$

Let $G$ be a $F(N - M)$-by-$FM$ matrix

$$G = \begin{pmatrix} X_0 & \mathbf{0}_{N-M,M} & \cdots & \cdots & \mathbf{0}_{N-M,M} \\ X_1 & X_1 & \mathbf{0}_{N-M,M} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \mathbf{0}_{N-M,M} \\ X_{F-1} & X_{F-1} & \cdots & \cdots & X_{F-1} \end{pmatrix}$$

and let $\Lambda$ be a $FM$-by-$FM$ matrix

$$\Lambda = \begin{pmatrix} \mathbf{0}_{M,M} & \cdots & \cdots & \cdots & \mathbf{0}_{M,M} \\ \vdots & I_{M,M}\vec{\lambda}' & \mathbf{0}_{M,M} & \cdots & \vdots \\ \vdots & \cdots & \ddots & \ddots & \mathbf{0}_{M,M} \\ \mathbf{0}_{M,M} & \cdots & \cdots & \cdots & I_{M,M}\vec{\lambda}' \end{pmatrix}$$

where $\mathbf{0}_{N_1N_2}$ denote a $N_1$-by-$N_2$ zero matrix and a $N_1$-by-$N_2$ unit matrix, respectively, and $\vec{\lambda}'$ is a $M$-dimensional vector $(\lambda_1', \lambda_2', \cdots, \lambda_M')^T$. By using these matrices, (7a) and (7b) are arranged as follows:

$$(G^TG - \Lambda)A = -G^TX \quad (9)$$

where $A = (a_0^T, \Delta a_1^T, \cdots, \Delta a_{F-1}^T)^T$ and $X = (x_0^T, x_1^T, \cdots, x_{F-1}^T)^T$. It is concluded that using the transition constraint of (4) the solutions of the AR coefficients of $F$ frames are simultaneously obtained from

$$A = -(G^TG - \Lambda)^+ \cdot G^TX \quad (10)$$

where $^+$ denotes the generalized inverse operation. Thus, the spectrum transient pattern is determined from the estimates of the AR coefficients of each frame. After determining the value of the variance $\sigma_i^2$ for each order of AR parameters, the $M$ Lagrange multipliers $\{\lambda_i'\}$ of (9) must be adjusted so that the constraint of the second term of (6) or (8) is satisfied. In this paper, we assume that the variance of every coefficient of the polynomial $A_j(z) = \sum_{i=0}^{M} a_i(j)z^{M-i}$ has the same value, that is, $\sigma_1'^2 = \sigma_2'^2 = \cdots = \sigma_M'^2 \overset{\text{def}}{=} \rho'^2/M$, and then $\lambda_1' = \lambda_2' = \cdots = \lambda_M' \overset{\text{def}}{=} \lambda'$. Using these assumptions, (6) can be written

$$J' = \sum_{j=0}^{F-1} \|x_j + X_j a_j\|^2 - \lambda' \left\{ \sum_{j=1}^{F-1} \|\Delta a_j\|^2 - \rho^2 \right\}. \quad (6')$$

The simultaneous equations obtained by taking the derivatives of (6') with respect to $a_0$, $\Delta a_j$, and $\lambda'$ are arranged as follows:

$$\zeta \overset{\text{def}}{=} \frac{1}{2} \cdot \frac{\partial J'}{\partial \lambda'} = \sum_{j=1}^{F-1} \|\Delta a_j\|^2 - \rho'^2 = 0 \quad (8')$$

$$(G^TG - \lambda' I_{FM}')A = -G^TX \quad (9')$$

where $I_{FM}'$ is a $FM$-by-$FM$ matrix:

$$I_{FM}' = \begin{pmatrix} \mathbf{0}_{M,M} & \cdots & \cdots & \cdots & \mathbf{0}_{M,M} \\ \vdots & I_{M,M} & \mathbf{0}_{M,M} & \cdots & \vdots \\ \vdots & \cdots & \ddots & \ddots & \mathbf{0}_{M,M} \\ \mathbf{0}_{M,M} & \cdots & \cdots & \cdots & I_{M,M} \end{pmatrix}$$

The AR coefficients of $F$ frames are simultaneously obtained from

$$A = -(G^TG - \lambda' I_{FM}')^+ \cdot G^TX. \quad (10')$$

This can be done in an iterative fashion such as the bisection method. We describe how to choose the value of $\rho^2$ in Section IV.

When the SNR is low, in order to improve the accuracy of the estimates, we apply the SVD-based approach [13] to the above modeling as follows: Let $L$ be a number such that $M \le L \le N - M$ (typically $L$ is several times larger than $M$). Redefine the matrix $X_j$ by the $(N - L)$-by-$L$ matrix and redefine the vectors $\boldsymbol{x}_j$, $\boldsymbol{a}_j$, $\Delta \boldsymbol{a}_j$, and $\vec{\lambda}'$ by $\boldsymbol{x}_j = [x(0;j), \cdots, x(N-1-L;j)]^T$, $\boldsymbol{a}_j = [a_1(j), \cdots, a_L(j)]^T$, $\Delta \boldsymbol{a}_j = [\Delta a_1(j), \cdots, \Delta a_L(j)]^T$, and $\vec{\lambda}' = [\lambda_1', \lambda_2', \cdots, \lambda_L']^T$, respectively. Next, the SVD of the resultant $FL$-by-$FL$ matrix $(G^T G - \Lambda)$ of (9) is computed. By truncating the nonsignificant singular values other than the $FM$ largest singular values of the matrix $(G^T G - \Lambda)$, we obtain the $FL$-dimensional vector $\boldsymbol{A}$ from (10). The AR coefficients $\{a_i(j)\}$ of each frame signal are obtained from the vector $\boldsymbol{A}$. Then, the roots of the polynomial

$$A_j(z) = z^L + a_1(j) \cdot z^{L-1} + \cdots + a_L(j)$$

are calculated. Based on the SVD-based method, when there is no measurement noise, this polynomial has exactly $M$ roots outside the unit circle on the $z$ plane, which are the reciprocals of the poles. The extra roots of the polynomial $A_j(z)$ generalized by noise are guaranteed to be inside the unit circle. Based on this observation, the poles of each frame signal are estimated by the reciprocals of the roots of the polynomial $A_j(z)$ outside the unit circle.

## III. Simulation Experiments

In order to illustrate the advantage of the proposed method to estimate spectrum transient pattern of multi-frame signals, we choose the example of the fourth order all-pole model, of which poles are $0.96\exp(\pm j0.18\pi)$ and $0.96\exp(\pm j0.35\pi)$. The AR coefficients $\{a_i(0)\}$ of the first frame are decided from the 4 poles. The AR coefficients $\{a_i(j)\}$ of the succeeding $j$th frame are decided as the sum of $\{a_i(j-1)\}$ of the previous frame and random process $\{\Delta a_i(j)\}$ with zero mean and the variance $\sigma_\Delta^2$. The signal $x(n;j)$ of each frame is generated based on (1) by using the resultant AR coefficients $\{a_i(j)\}$ and white noise process $e(n;j)$. Each frame signal $x(n;j)$ is contaminated by measured white noise which is uncorrelated with driving white noise $e(n;j)$. The estimation error $\eta$ of the estimated AR parameters $\{\hat{a}_i(j)\}$ of $\{a_i(j)\}$ are evaluated by the following normalized mean-square error as

$$\eta = E\left[\frac{\sum_{i,j} |\hat{a}_i(j) - k \cdot a_i(j)|^2}{\sum_{i,j} |a_i(j)|^2}\right] \tag{11}$$

where $k$ is the scaling constant chosen to minimize $\{(\Sigma_{i,j} |\hat{a}_i(j) - k \cdot a_i(j)|^2)/(\Sigma_{i,j} |a_i(j)|^2)\}$ by using the true value of the AR parameters. The number $F$ of the frames is equal to 13 and the length $N$ of each frame signal is equal to 28. The SVD-based method is used and the order $L$ is equal to 14. The average estimation error $\eta$ is calculated with regard to the 5 independent trials.

Fig. 1(a) shows the estimation error $\eta$ for the various values of $\rho^2$ of (4), when the variance $\sigma_\Delta^2$ of $\{a_i(j)\}$ is equal to $-30$ dB. Even when the SNR is low (20 and 10 dB) and the signal length is very short, the proposed method's estimation error is about 10 dB less than that obtained by applying the ordinary SVD-based method to each frame signal independently without any spectrum transition constraint as shown in the leftmost part ($\lambda = 0$) of the horizontal axis.

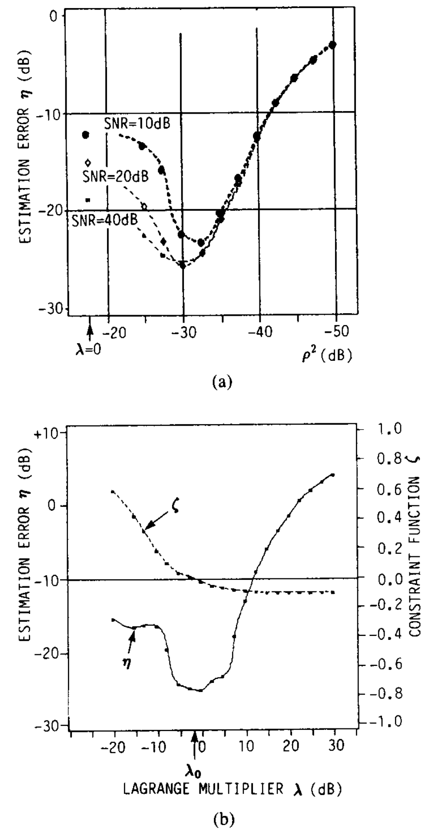


Fig. 1. (a) The estimation error $\eta$ of the AR parameters as a function of the $\rho^2$ of (4) for various values of SNR ($\sigma_\Delta^2 = -30$ dB.) The leftmost part ($\lambda = 0$) of the horizontal axis shows the results obtained by applying the ordinary SVD-based method to each frame signal independently without any spectrum transition constraint. (b) The relation between the estimation error $\eta$ of the AR parameters and the term $\zeta = \Sigma_j \|\boldsymbol{a}_j\|^2 - \rho'^2$ of the constraint in (8′) as a function of the $\lambda = \lambda' \times (F-1)/\{P_0 F(N-M)\}$ of (9′). ($\sigma_\Delta^2 = -30$ dB, SNR $= 10$ dB.) The $\lambda_0$ shows the value for which the constraint function $\zeta$ equals zero. Around the $\lambda_0$, the square of the estimation error $\eta$ takes minimum values.

Fig. 1(b) shows the relation between the estimation error $\eta$ of the AR parameters and the squared value of $\zeta$ (8′) as a function of $\lambda = \lambda' \times (F-1)/\{P_0 F(N-M)\}$ of (9′). In (10′), the Lagrange multiplier $\lambda$ must be adjusted so that the constraint is satisfied. In Fig. 1(b), the Lagrange multiplier $\lambda_0$ is taken as the value for which (8′) equals zero. At $\lambda_0$, the square of the estimation error $\eta$ takes minimum values. Thus, by finding a zero of the function $\zeta = \Sigma_{j=1}^{F-1} \|\boldsymbol{a}_j\|^2 - \rho'^2$ which changes sign based on the bisection method, the cost function $J'$ of (4) is optimized under a well-behaved estimation situation.

Figs. 2(a)–(c) show the singular values of the matrix $G^T G$, which is obtained when any spectrum transition constraint such as the second term in (3) or (4) is not used. By comparing Figs. 2(b) and (c), it is found that the high order singular values are affected significantly by additive

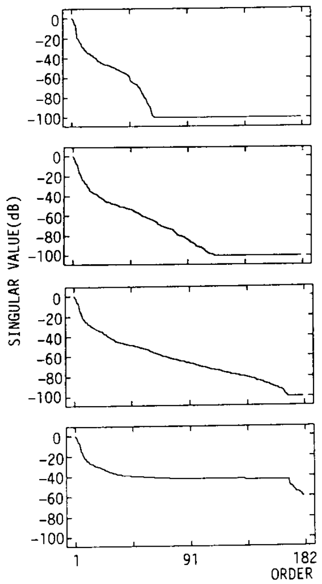


Fig. 2. The singular values of the matrix ($G^TG - \lambda I'_{FL}$) of (10') for various cases. (a) Impulse driven, SNR = $\infty$, $\lambda = 0$. (b) White noise driven, SNR = $\infty$, $\lambda = 0$. (c) White noise driven, SNR = 10 dB, $\lambda = 0$. (d) White noise driven, SNR = 10 dB, $\rho^2 = -30$ dB.

noise. As shown in Fig. 2(d), however, these singular values affected by additive noise are masked by introducing the spectrum transition constraint and then the resultant estimates show the spectrum transition more accurately as show in Fig. 1.

## IV. Experimental Results

We applied this method to the analysis of multiframe signals of the fourth heart sound for the diagnosis of myocardial infarction. As shown in Fig. 3, each frame signal was cut off from one beat signal, which was detected at one-minute intervals during a stress test which consists of an exercise period of 5 minutes and a succeeding rest period of 10 minutes. Each frame signal is about 140 ms in length and is A/D converted at a sampling period of 5 ms. In order to diagnose left ventricular dysfunction based on the acoustic characteristics of the ventricle of the heart muscle, it is necessary to clearly detect the spectrum transition between the succeeding frames during the stress test.

Since SNR is low and the duration time of each fourth heart sound is very short, there are large fluctuations and many phantom peaks appear in spectra estimated by independently applying the fast Fourier transform (FFT) and the maximum entropy method (MEM) ($M = 6$) to each frame signals ($N = 28$, $F = 13$) detected from a normal person and a patient with myocardial infarction as shown in Fig. 4.

On the other hand, by applying the proposed method ($M = 6$) to the same multiframe signals as analyzed in Fig. 4, the resultant spectrum transition patterns are shown in Fig. 5, respectively, for various values of $\rho^2$ of (4). In

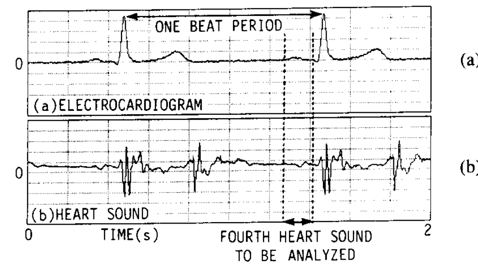


Fig. 3. Each frame signal of the fourth heart sound was sampled from one beat signal, which is detected at 1-min intervals. (a) An electrocardiogram. (b) A heart sound.

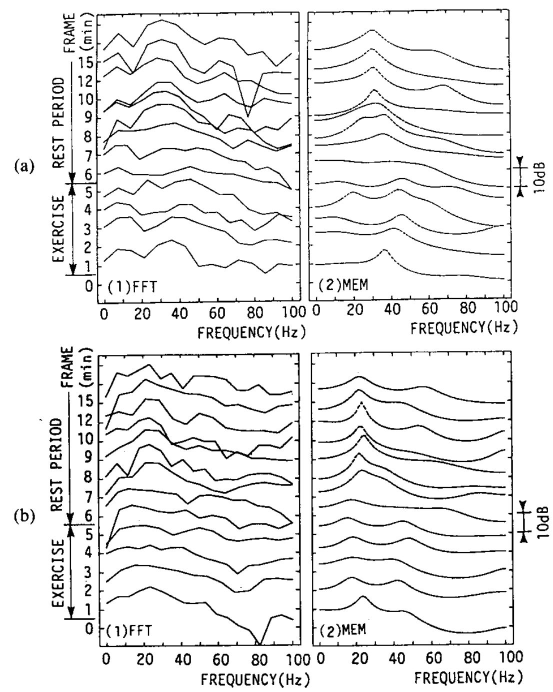


Fig. 4. The spectra of multiframe signals of the fourth heart sound obtained by using the FFT (1) and the MEM (2). (a) Normal, (b) myocardial infarction.

these figures, as the value of $\rho$ is decreased, the transition restriction becomes stronger and then the smoother spectrum transition pattern is obtained. When the value of $\rho^2$ is equal to $-40$ dB, however, the spectrum transition of the spectra is not estimated any more and the spectrum common to all frame signals is obtained instead.

Therefore, by using an appropriate value of $\rho^2$, which seems to be about $-20$ dB, the transition of the spectrum is clearly detected as shown in Figs. 5(a-2) and (b-2). In these experiments we found that the optimum value of $\rho^2$ is chosen systematically as follows: For each frame, three different signals were cut off from succeeding three beat periods. We assume that all of the three signals have the same set of the AR parameters. By applying the proposed method independently to each of the three sets of the multiframe signals, the resultant estimates of the AR param-

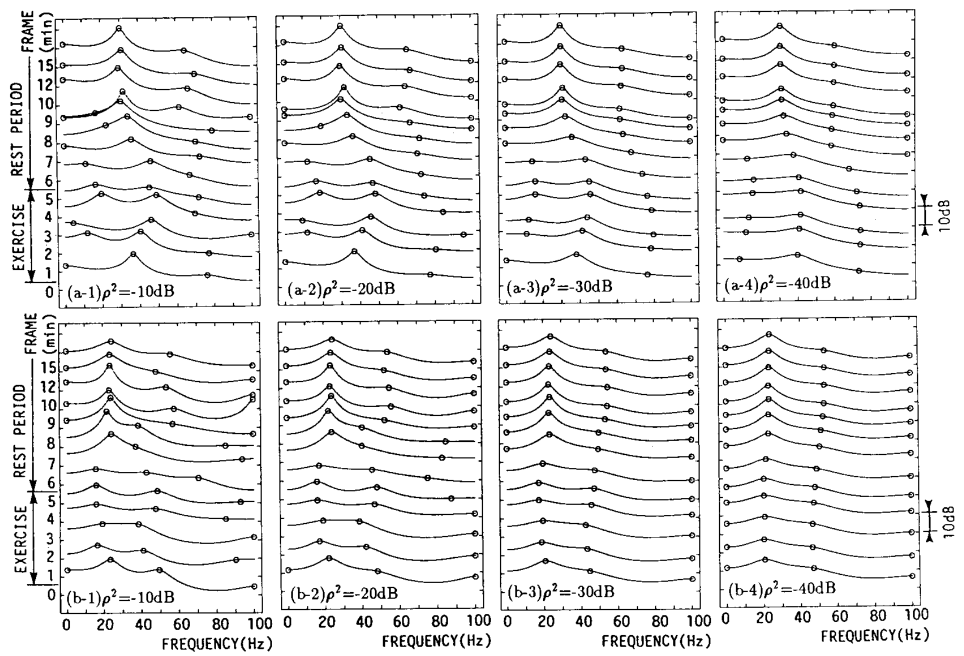

Fig. 5. The spectrum transition of multiframe signals of the fourth heart sound obtained by the proposed method for various values of $\rho^2$ of (4). Each pole position is indicated by the mark "o." (a) Normal, (b) myocardial infarction. Each signal is the same as that used in Fig. 4. As the value of $\rho^2$ is decreased, the transition restriction becomes stronger. (When a pole is on the real axis on the $z$ plane, the positions of the four poles are indicated in the spectrum.)

eters must agree with each other. However, even for small values of $\rho^2$, that is, under the strong transition restriction, there are differences between them, as shown in Fig. 6. The variance $\sigma_E^2$ of the AR parameters, which is calculated from the estimates of the three sets of the multiframe signals, indicates the power of the measurement error. If the variance of $\{\Delta a_i(j)\}$, which is equal to $\rho^2$, is constrained as to be less than the error power $\sigma_E^2$, the resultant estimates are meaningless. Thus, we choose the value of $\rho^2$ as the noise power $\sigma_E^2$, which is obtained for sufficiently large value of $\rho^2$. In Fig. 6 the value is about $-18$ dB.

By comparing the transition pattern in Fig. 5(b-2) with that in Fig. 5(a-2), there are significant differences between the normal person and the patient with myocardial infarction. For the exercise period of first 5 minutes, the frequency of main vibration of the normal person shifts to higher frequency, while for the patient with myocardial infarction the frequency shifts to lower frequency. By applying the proposed method to a group of 20 people, 10 of whom suffered from myocardial infarction, these significant characteristics of the transition patterns were observed for nine normal persons and eight patients with myocardial infarction.

During the exercise period of 5 minutes, the amount of blood flow increases in the ventricle and the coronary artery and the acoustic characteristics of the ventricle wall become hard. For this reason the frequency of the main vibration of normal persons shift to higher frequency. In the cases of patients with myocardial infarction, however,

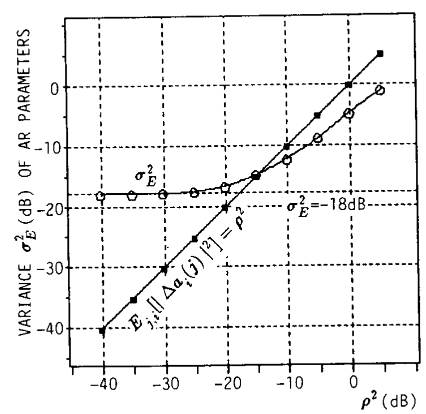

Fig. 6. The variance $\sigma_E^2$ of the AR parameters, estimated from three sets of multiframe signals of a person, as a function of $\rho^2$.

there are at least two components with different physical and acoustic nature in the ventricle wall, one corresponds to a normal myocardium and the other corresponds to the diseased heart muscle. During the exercise period such differences appeared remarkably and then the vibrations with lower frequency were also obtained.

Similar phenomena were detected in the literature [15] using an isolated canine left ventricle with regional ischemia. The instantaneous impedance frequency curve [16] changed from a single-peak configuration to a double-peak configuration after a coronary flow cessation and returned to the single-peak configuration with the reperfusion of coronary flow. The method proposed in this paper provides a new approach to detect such myocardial physical heterogeneity of a ventricle by analyzing the human heart sound obtained during the stress test.

## V. Concluding Remarks

We present a new method to estimate spectrum transition between short-length multiframe signals in low SNR cases by using a linear algorithm without any basic function. If the spectrum transition between the succeeding frames during the stress test is detected accurately, the left ventricular dysfunction can be diagnosed based on the acoustic characteristics of the ventricle of heart muscle. By applying the proposed method to the diagnosis of myocardial infarction, we found that there are significant differences of the transition patterns in the spectra between the normal persons and the patients. The characteristics of these transition patterns may be applied to acoustical diagnosis of heart diseases.

Six issues remain for future research as follows:

1) The model for describing the framed AR process introduced in the beginning of Section II is by no means the best one in general cases. Further work is needed to develop an alternative model which describes the relation between the AR parameters of the succeeding frames.

2) In Section II, the innovation process $e(n; j)$ of the AR transfer system is stationary normal with the zero mean and the same variance values over the frames. However, this assumption is not always justified. If the frequency characteristics of the innovation process is known *a priori*, the parameters of the framed AR model are determined by the so-called weighted (generalized) least mean square technique for minimizing the cost function $J$ of (4) or $J'$ of (6). However, it is also necessary to develop an alternative method to estimate the framed AR parameters when the characteristics of the innovation process is not known.

3) In (6′), (9), and (10′), which were used in the simulation experiments in Section III and the experiments in Section IV, we assumed that the variance $\sigma_i^2$ for different value of the order $i$ has the same value. However, this assumption does not hold generally. Thus, it is necessary to develop the appropriate method to determine the variances $\{\sigma_i^2\}$ of the AR parameters from the observed data based on the relation between the coefficients of the polynomial $A_j(z)$ and the positions of the poles.

4) In the experiment of Section IV, the length of the fourth heart sound of each frame is very short and then only almost one period signal is contained in the analyzed period of each frame signal. Moreover, the signal-to-noise ratio is very low. For these circumstances, it is very important to develop or select an appropriate technique for deciding whether the derivation from the assumed stationarity should be regarded as certainly caused or not.

5) It is also necessary to extend the proposed method so as to incorporate the multichannel case.

6) For computational reduction, it is necessary to exploit the special structure of the resulting linear system.

These important issues are currently under investigation.

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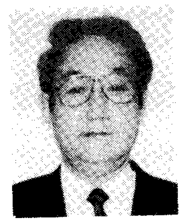

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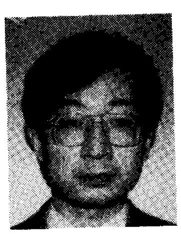

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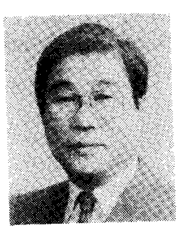

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