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Non-contact measurement of particle velocity distribution of ultrasonic waves along an elastic bar using laser Doppler velocimetry

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A new method is described for non-contact measurement of particle velocity distribution along an elastic bar excited by a PZT transducer. By analyzing two signals measured by a laser Doppler velocimetry (LDV) from two incident angles, the particle velocity distribution for each of the two components, normal and parallel to the bar axis, is separately obtained. The principle of the proposed measurement method is confirmed by the experiments using a Langevin type PZT transducer having a resonant frequency of 17 kHz. The sensitivity of this method is enough high to measure the particle velocity less than 12 μm/s at the resonant frequency of 17 kHz.

Keywords: Laser Doppler, Velocity distribution, Retroreflective tape

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1. INTRODUCTION

The measurement of particle velocity distributions of the ultrasonic vibration provides useful information for the analysis of characteristics of a vibrating elastic body in various applications. Many methods have been proposed to measure the particle velocity or the displacement such as:

1) a photonic sensor method
2) a laser Doppler velocimetry (LDV)[1,2]
3) a speckle interference method[3,4]
4) a laser heterodyne system[5,6].

The methods of 1) and 2) have been used to measure the particle velocity component normal to the vibrating surface (out-of-plane). However, the particle velocity component parallel to the vibrating surface (in-plane) cannot be measured by these conventional methods.

On the other hand, the methods of 3) and 4) can be employed to measure the two velocity components, in-plane and out-of-plane, although the optical systems of these methods should be changed depending on the two particle velocity components. Moreover, the noise arisen in the optical system and electrical circuits for the modulation and the demodulation is not negligible. A measurement method based on the bispectral analysis was proposed to suppress the noise components.[7] However, it is not employed practically because the system is complicated and, moreover, the two velocity components cannot be measured simultaneously.

This paper proposes a new method for obtaining the two particle velocity components simultaneously which are normal and parallel to the vibrating surface using retroreflective tape.[8,9] These two components can be mathematically separated from the two velocity signals measured from two arbitrary incident directions with an LDV. By averaging the transfer function from the driving signal to output signal of the LDV, the signal-to-noise ratio has been improved.

2. MEASUREMENT SYSTEM

The measurement system using the LDV (Ono

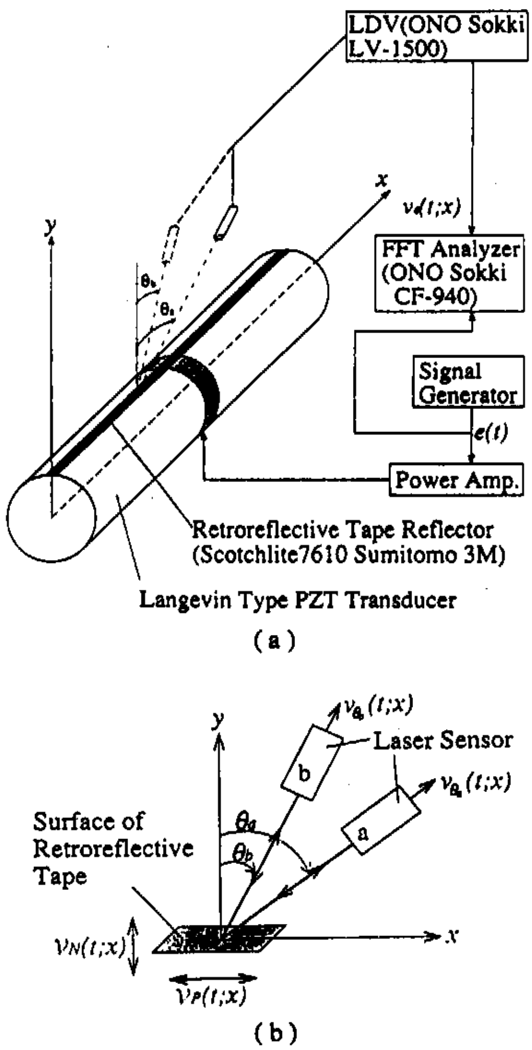

Fig. 1 A system for measuring non-contact particle velocity distribution. (a) Block diagram of the measurement system. (b) Components of the particle velocity and the direction of laser beam.

Sokki, LV-1500) is shown in Fig. 1(a). The retroreflective tape reflector (Sumitomo 3M, Scotchlite 7610, less than 0.1 mm thick) which reflects an incident ray back along the incident ray direction, is attached along the line on the cylindrical surface of the Langevin type PZT transducer shown in the figure. Thus, the LDV detects the particle velocity signals $v_\theta(t;x)$ at a point x on the surface of the retroreflective tape, where θ denotes the angle determined by the incident ray and the line perpendicular to the bar surface as shown in Fig. 1(b). The detected signal $v_\theta(t;x)$ contains the parallel component $v_P(t;x)$ in the x-direction and the normal component $v_N(t;x)$ in the y-direction. Therefore, $v_\theta(t;x)$ can be represented by the weighted sum of these two components as follows:

$$v_\theta(t;x)=v_N(t;x)\cos\theta+v_P(t;x)\sin\theta\,. \qquad (1)$$

In the experiments of this paper, a Langevin type PZT transducer in Fig. 1(a) is stationarily excited at a resonant frequency f_0. The transfer function $H_\theta(f_0;x)$ from the input signal $e(t)$ of the power amplifier to the measured velocity signal $v_\theta(t;x)$ is obtained by a standard FFT analyzer. Thus, $H_\theta(f_0;x)$ is independent of time t. From the phase characteristics $\angle H_\theta(f_0;x)$ of the resultant $H_\theta(f_0;x)$, let us define the spectrum components $w_\theta(f_0;x)$ at the resonant frequency f_0 as follows:

$$w_\theta(f_0;x)=P_\theta(f_0;x)\exp\{j\angle H_\theta(f_0;x)\}\,, \qquad (2)$$

where $P_\theta(f_0;x)$ is the power spectrum of the measured signal $v_\theta(t;x)$ and is employed in order to evaluate the absolute amplitude of the velocity on the sample surface. Figure 2 shows the relation among the phase components in this measurement system. The phase component $\angle H_\theta(f_0;x)$ in Eq. (2) is given by the sum of the phase delay Ω_0, which is defined from the signal generator to the elastic body of the Langevin type PZT transducer, and the phase component $\Omega_\theta(f_0;x)$ of the standing wave between the PZT transducer and the point x on the beam as follows:

$$\angle H_\theta(f_0;x)=\Omega_0+\Omega_\theta(f_0;x) \qquad (3)$$

The first term Ω_0 in the right hand side is a constant value which depends on the frequency f_0 but which does not depend on θ and x. Since the second term $\Omega_\theta(f_0;x)$ corresponds to the phase of the

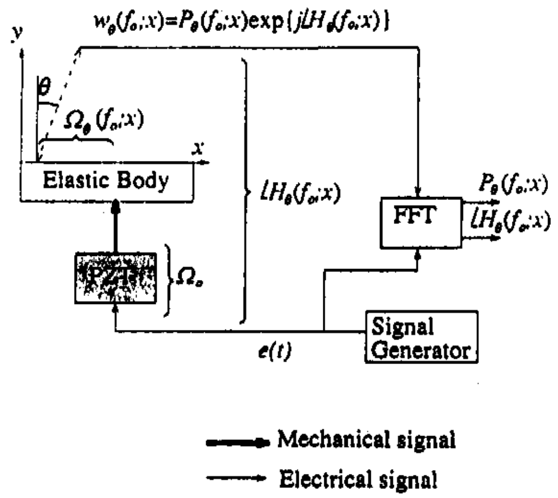

Fig. 2 The relation among the phase components in the measurement system.

standing wave, $\Omega_e(f_0; x)$ is equal to either 0 or π, that is, the value $\exp\{j\Omega_e(f_0; x)\}$ is equal to either $+1$ or -1. Thus, by deleting the phase characteristics $\exp(j\Omega_0)$ from $\angle H_e(f_0; x)$ in Eq. (2), the measured complex component $w_e(f_0; x)$ in Eq. (2) is transformed into the real component $v_e(f_0; x)$ as follows:

$$\begin{aligned} v_e(f_0; x) &= w_e(f_0; x)\exp(-j\Omega_0) \\ &= P_e(f_0; x)\exp\{j\angle H_e(f_0; x) - j\Omega_0\} \\ &= P_e(f_0; x)\exp\{j\Omega_e(f_0; x)\} \end{aligned} \quad (4)$$

On the other hand, in order to separately obtain the parallel component $v_P(t; x)$ and the normal component $v_N(t; x)$ from the output signal of LDV, the velocity spectrum components $v_e(f_0; x)$ are measured for the two incident angles θ_a and θ_b as shown in Fig. 1(b). From Eq. (1), these two components are described by

$$\begin{bmatrix} v_{ea}(f_0; x) \\ v_{eb}(f_0; x) \end{bmatrix} = \begin{bmatrix} \cos\theta_a & \sin\theta_a \\ \cos\theta_b & \sin\theta_b \end{bmatrix} \begin{bmatrix} v_N(f_0; x) \\ v_P(f_0; x) \end{bmatrix}. \quad (5)$$

For the case of $\theta_a \neq \theta_b$, the parallel component and the normal component of the velocity spectrum components $v_e(f_0; x)$ are given by

$$\begin{bmatrix} v_N(f_0; x) \\ v_P(f_0; x) \end{bmatrix} = \frac{1}{\sin(\theta_b - \theta_a)} \begin{bmatrix} \sin\theta_b & -\sin\theta_a \\ -\cos\theta_b & \cos\theta_a \end{bmatrix} \cdot \begin{bmatrix} v_{ea}(f_0; x) \\ v_{eb}(f_0; x) \end{bmatrix}. \quad (6)$$

To obtained the velocity distribution for the parallel and normal components, the velocities $v_{ea}(f_0; x)$ and $v_{eb}(f_0; x)$ are measured at each point x along the line of the attached reflector using Eq. (4). The two components $v_P(f_0; x)$ and $v_N(f_0; x)$ are then determined by Eq. (6).

The Langevin type PZT transducer employed as an elastic bar is constructed in the following experiments as shown in Fig. 3. The two arms are made of aluminum and the PZT ceramic layers are clamped by a steel bolt. The resonant frequency f_0 of the transducer is 17 kHz.

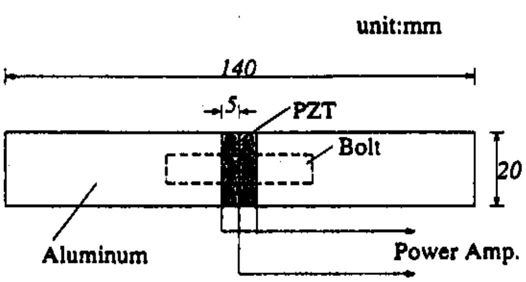

Fig. 3 Construction of the Langevin type PZT transducer.

3. EXPERIMENTAL RESULTS

Figures 4(a) and 4(b) show the distribution of $w_{ea}(f_0; x)$ and $w_{eb}(f_0; x)$ in Eq. (2) measured at various points x for the incident angle $\theta_a = 55°$ and $\theta_b = 0°$, respectively. After averaging 16 power spectra and cross spectra by the FFT analyzer, these values of $w_{ea}(f_0; x)$ and $w_{eb}(f_0; x)$ are obtained. From the results in Fig. 4, the phase delay Ω_0 is determined as the constant value of 47° for the fre-

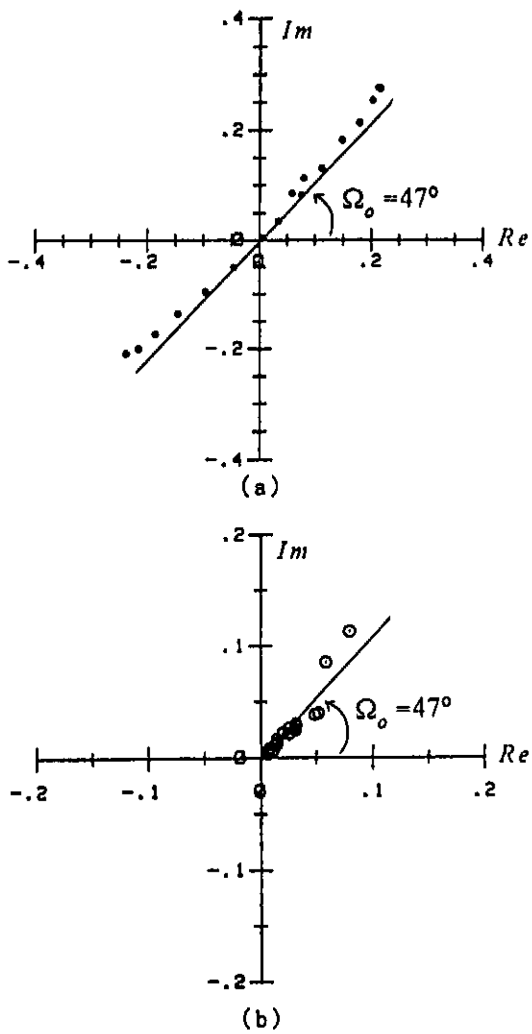

Fig. 4 Experimental results to determine the phase delay Ω_0 from the power amplifier to the PZT transducer. (a) Distribution of the complex spectrum components $w_{ea}(f_0; x)$ measured from the incident angle of $\theta_a = 55°$. (b) Distribution of the complex spectrum components $w_{eb}(f_0; x)$ measured from the incident angle of $\theta_b = 0°$.

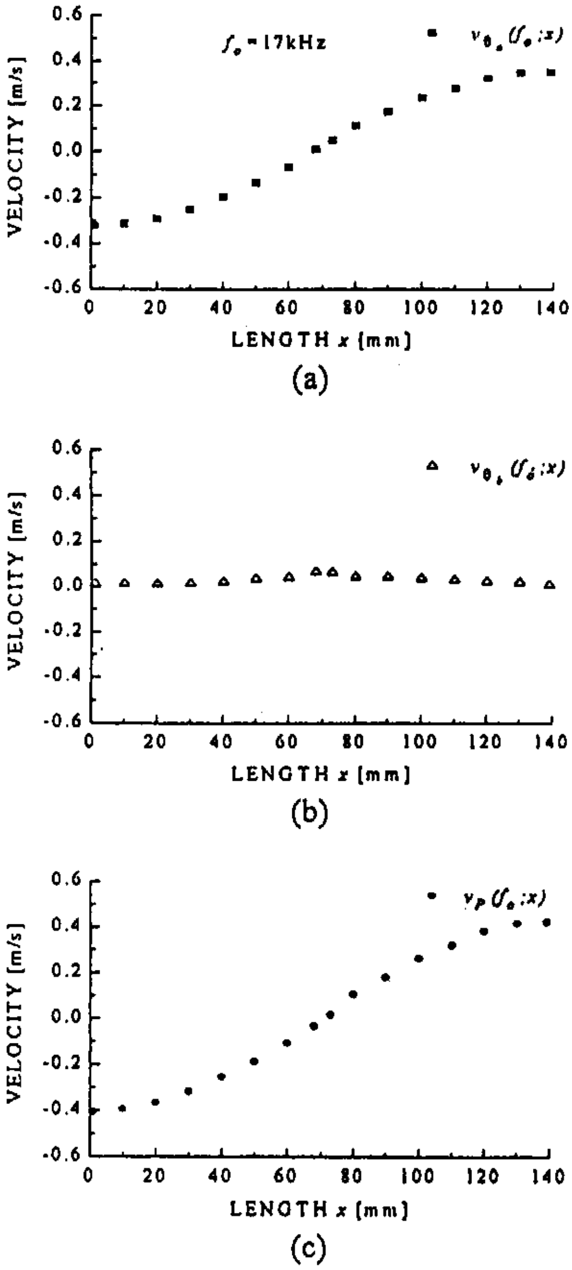

Fig. 5 Particle velocity distributions of the Langevin type PZT transducer measured from the incident angles of θ_a and θ_b, and the resultant parallel velocity distribution. (a) Particle velocity distribution $v_{\theta_a}(f_0; x)$ measured from the incident angle θ_a. (b) Particle velocity distribution $v_{\theta_b}(f_0; x)$ measured from the incident angle θ_b. (c) Parallel particle velocity distribution $v_P(f_0; x)$.

quency $f_0=17$ kHz. Using Eq. (4) and Ω_0, each velocity distribution of $v_{\theta_a}(f_0; x)$ and $v_{\theta_b}(f_0; x)$ is obtained as shown in Fig. 5(a) and 5(b), respectively.

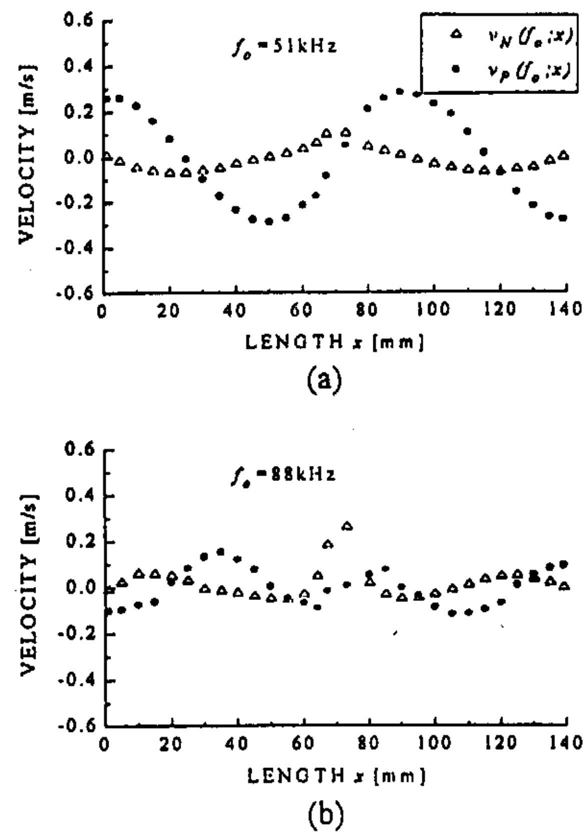

Fig. 6 Particle velocity distributions of the Langevin type PZT transducer for harmonic resonant frequencies.

In this case, $v_{\theta_b}(f_0; x)$ coincides with $v_N(f_0; x)$ in Eq. (5). By substituting $v_{\theta_a}(f_0; x)$ and $v_{\theta_b}(f_0; x)$ into Eq. (6), the parallel component of velocity $v_P(f_0; x)$ is successfully separated as shown in Fig. 5(c).

The x-direction components of particle velocities at both end surfaces of the Langevin type PZT transducer are additionally measured with normal incident beam of LDV, and found to be -0.421 m/s (at $x=0$ mm) and 0.435 m/s (at $x=140$ mm), respectively, which almost coincide with the values shown in Fig. 5(c).

Similarly, the velocity distributions for harmonic resonant frequencies in 51 kHz and 88 kHz are obtained as shown in Fig. 6. The results of Fig. 6 show that the velocity components of $v_N(f_0; x)$ are due to the radial mode of PZT, and these values are prominent at higher frequency.

4. SENSITIVITY OF THE PROPOSED METHOD

Let us define the sensitivity of the proposed method by the minimum values of $v_P(f_0; x)$ obtained when the coherence function between the driving signal $e(t)$ and the output signal $v_\theta(t; x)$ of the LDV exceeds a pre-define constant γ_0^2 at the frequency f_0. It is well known that the random error $\epsilon_r[\angle H_\theta(f_0; x)]$ of the phase of the transfer function is approximately given in radian by[10]

$$\epsilon_r[\angle H_\theta(f_0; x)] \approx \frac{\sqrt{1-\gamma^2(f_0; x)}}{|\gamma(f_0; x)|\sqrt{2n_d}}, \qquad (7)$$

where n_d is the number of averaging operation and $\gamma^2(f_0; x)$ is the true value of the coherence function. Since the true value of the coherence function is unknown, let us assume that the true value is approximated by its estimate. When the constant γ_0^2 is 0.9 and the averaging number n_d is 16, the uncertainty $\epsilon_r[\angle H_\theta(f_0; x)]$ of the phase estimates in Eq. (7) is equal to 0.059 radian = 3.3°, which gives the uncertainty in the second term of the obtained value $w_\theta(f_0; x)$ in Eq. (2). For the first term $P_\theta(f_0; x)$ in Eq. (2), on the other hand, its normalized random error is given by $1/n_d^{1/2}$, which depends only on the number of averaging operation, and it can be decreased by increasing n_d. Thus, the coherence function is significant in the evaluation of the sensitivity of the proposed method as described above.

Figure 7 shows the results of $v_P(f_0; x)$ obtained when the constant γ_0^2 is equal to 0.9 and the incident angles θ_a are 30°, 40°, 50°, 60° and 70° for various resonant frequencies of f_0 = 17 kHz, 51 kHz and 88 kHz at x = 40 mm. From these results, it is found out that the measurable minimum amplitude is less than about 0.111 nm for the frequency range 17 kHz ~ 88 kHz.

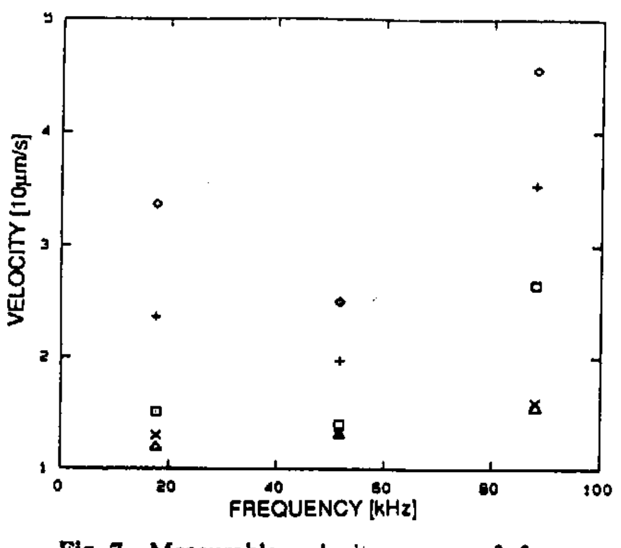

Fig. 7 Measurable velocity range of frequency for various values of the incident angle. ◇: θ_a = 30°, +: θ_a = 40°, □: θ_a = 50°, ×: θ_a = 60°, △: θ_a = 70°.

5. CONCLUSIONS

A new non-contact method for determining particle velocity distributions for a vibrating elastic body has been developed. The principle that two particle velocity components can be separated by analyzing two measured signals from two incident angles is confirmed experimentally with the Langevin type PZT transducer having a fundamental resonant frequency of 17 kHz. The parallel and the normal components of the particle velocities were determined by analyzing two measured signals obtained by the LDV from two incident angles. The resultant sensitivity of this method is enough high to measure the particle velocity less than 12 μm/s at the resonant frequency of 17 kHz.

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