lers to five significant figures are shown.

Table 2: Absolute and percentage errors in assigned closed-loop eigenvalues

λ_i	Minimising J		Minimising L	
	absolute	percentage	absolute	percentage
−0.1778	0.00088	0.495	0.0030	1.648
−0.5628	0.00190	0.337	0.0077	1.377
$-0.5113 \pm j0.1013$	0.00070	0.135	0.0027	0.510
$-1.6297 \pm j0.5605$	0.00100	0.129	0.0030	0.333
$-1.2715 \pm j0.3923$	0.000015	0.001	0.00065	0.049

Controller K_J is seen to be much more robust to rounding errors than controller K_L.

Conclusions: To obtain a state feedback controller that assigns desired distinct closed-loop eigenvalues to the closed-loop system and is robust to rounding errors in the elements of the controller, the product of the condition number of the closed-loop system and the controller gain should be minimised. An algorithm for minimising this product has been demonstrated.

© IEE 1995
22 March 1995
Electronics Letters Online No: 19950713

M.M. Zaytoun (*Department of Electrical Engineering, Faculty of Engineering, Alexandria University, Alexandria, Egypt*)

T.J. Owens (*Department of Electrical Engineering and Electronics, Brunel University, Uxbridge, Middlesex UB8 3PH, United Kingdom*)

References

1 OWENS, T.J., and MARSH, J.F.: 'Some computational issues in optimal control by nonlinear programming', *Ann. Oper. Res.*, 1993, **43**, pp. 249–257

2 ZAYTOUN, M.M.: 'Computational optimization for robust control'. PhD Thesis, Faculty of Engineering, Alexandria University, Egypt, 1994

3 BYERS, R., and NASH, S.G.: 'Approaches to robust pole assignment', *Int. J. Control*, 1989, **49**, pp. 97–117

Accurate determination of transit time of ultrasound in thin layers

H. Kanai, T. Kimura and N. Chubachi

Indexing terms: Acoustic wave velocity measurement, Piezoelectric transducers

The authors propose a new method to remove the transient response of the piezoelectric transducer in the pulse-echo method using the inverse Fourier transform acoustic input impedance and reflection coefficient so that the time resolution in the measurement of transit time in a thin object may be increased.

Introduction: The pulse-echo method is essential in the fields of ultrasonic testing of materials and medical ultrasonics. Since a piezoelectric transducer has a long transient response, it is necessary to remove this response if the transit time on the ultrasound in the thin layer is to be compared with the transient response. For this problem, numerous studies on inverse filtering have been reported [1] in which the transient response of the piezoelectric transducer is described. In these studies, a transfer system from voltage V_3 at the electrical port to force F_2 at the acoustical port is employed (Fig. 1). However, both the force and particle velocity are not effectively employed and the acoustic impedance cannot be considered.

A new method to accurately determine the transit time of the ultrasound in the thin layer of an object is proposed in which the acoustic input impedance from the voltage at the electrical port of

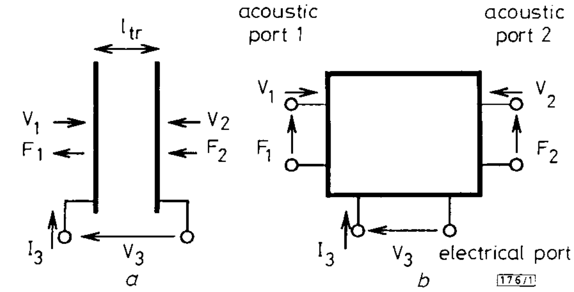

Fig. 1 *Transducer*

a Physical parameters of transducer
b Transducer regarded as three-port black box

the piezoelectric transducer is estimated. The reflection characteristics determined from the estimated acoustic impedance have an impulsive train in the time domain, the interval of which is double that of the transit time in the thin layer. Since the transient response of the transducer is completely removed in the resultant impulsive train, the accuracy of the determination of the transit time in the thin layer is improved by the proposed method even if the transit time in the layer is compared with the length of the transient response of the transducer. The method is applicable to both the pulse-echo method and the frequency-scanning method, which uses continuous sound.

Principle of optimum inverse filtering: We consider a uniform piezoelectric transducer with cross-sectional dimensions of many wavelengths as shown in Fig. 1*a*. As reported in [2], the characteristics are described by a three-port network as shown in Fig. 1*b*. Forces F_1 and F_2 at the acoustical ports and voltage V_3 across the transducer are described in matrix form using particle velocities v_1 and v_2 and current I_3 as follows:

$$\begin{bmatrix} F_1 \\ F_2 \\ V_3 \end{bmatrix} = -j \begin{bmatrix} Z_C \cot \beta_a l_{tr} & Z_C \operatorname{cosec} \beta_a l_{tr} & h/\omega \\ Z_C \operatorname{cosec} \beta_a l_{tr} & Z_C \cot \beta_a l_{tr} & h/\omega \\ h/\omega & h/\omega & 1/\omega C_0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ I_3 \end{bmatrix} \quad (1)$$

where $C_0 = \varepsilon^S A/l_{tr}$ and $Z_C = Z_0 A = A\sqrt{\rho_m c^D}$ are the clamped capacitance of the transducer and the acoustic impedance of an area A of the piezoelectric material having a thickness l_{tr}, a density ρ_m and a piezoelectric stress constant ε^S. The parameters c^D, $\beta_a = \omega\sqrt{\rho_m/c^D}$, and $h = e/\varepsilon^S$ are the piezoelectrically stiffened elastic constant, the propagation constant, and the transmitting constant, respectively.

We assume that acoustical port 1 is terminated by backing material with an acoustic impedance Z_b. By substituting the relation $F_1 = -Z_b v_1$ into eqn. 1, V_3 and I_3 at the electrical port are described by F_2 and $v'_2 \triangleq -v_2$ at acoustical port 1 by

$$\begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} A_{tr} & B_{tr} \\ C_{tr} & D_{tr} \end{bmatrix} \begin{bmatrix} F_2 \\ -v_2 \end{bmatrix} = K_{tr} \begin{bmatrix} F_2 \\ v'_2 \end{bmatrix} \quad (2)$$

where

$$\begin{aligned} A_{tr} &= \{-(Z_C/\omega C_0 \cot \beta_a l_{tr} + h^2/\omega^2 - jZ_b/\omega C_0\}/\Delta \\ B_{tr} &= \{\Delta - (h^2/\omega^2) D_{tr}\}/(jZ_C \cot \beta_a l_{tr} - Z_b) - jD_{tr}/\omega C_0 \\ C_{tr} &= (Z_b - jZ_C \cot \beta_a l_{tr})/\Delta \\ D_{tr} &= Z_C(Z_C - jZ_b \cot \beta_a l_{tr})/\Delta \\ \Delta &= -j(h/\omega)\{jZ_C \tan(\beta_a l_{tr}/2) + Z_b\} \end{aligned} \quad (3)$$

and K_{tr} is a 2-by-2 cascade matrix as shown in Fig. 2*b*.

Assuming that length l_w and acoustic impedances Z_w and $Z_{w'}$ in media 1 and 2 are known, we estimate the thickness l_{obj} of the object thin layer by removing the transit response of the transducer as follows: In the actual measurement system, a matching layer and acoustic medium 1 are between the transducer and the object layer. The total characteristics of these two media are given by the cascade of each distributed-parameter system. By describing these characteristics with cascade matrices K_m and K_w, respectively, eqn. 2 is rewritten by force F_{obj} and particle velocity v_{obj} at the surface of the object as follows:

$$\begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = K_{tr} K_m K_w \begin{bmatrix} F_{obj} \\ v_{obj} \end{bmatrix} \quad (4)$$

where K_m and K_w are defined by

$$K_m = \begin{bmatrix} \cosh\gamma_m l_m & Z_m \sinh\gamma_m l_m \\ 1/Z_m \sinh\gamma_m l_m & \cosh\gamma_m l_m \end{bmatrix} \quad (5)$$

$$K_w = \begin{bmatrix} \cosh\gamma_w l_w & Z_w \sinh\gamma_w l_w \\ 1/Z_w \sinh\gamma_w l_w & \cosh\gamma_w l_w \end{bmatrix} \quad (6)$$

and γ_m and γ_w are each propagation constants, and l_m and l_w are the thickness of the matching layer and length of medium 1, respectively. Thus, F_{obj} and v_{obj} are estimated from V_3 and I_3 at the electrical port as follows:

$$\begin{bmatrix} \hat{F}_{obj} \\ \hat{v}_{obj} \end{bmatrix} = (K_{tr} K_m K_w)^{-1} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \quad (7)$$

In many applications, it is not easy to measure the RF current I_3. Thus, we connect voltage source V_e, having an internal impedance Z_e with the electrical port as shown in Fig. 2. By substituting the relation $I_3 = (V_e - V_3)/Z_e$ at the port into eqn. 7, the input impedance $Z_{in-obj}(\omega)$ is estimated from V_3 and V_e as follows:

$$\hat{Z}_{in-obj}(\omega) \triangleq \frac{\hat{F}_{obj}}{\hat{v}_{obj}} = \frac{k_{11} V_3 + k_{12}(V_e - V_3)/Z_e}{k_{21} V_3 + k_{22}(V_e - V_3)/Z_e} \quad (8)$$

where k_{11}, k_{12}, k_{21}, and k_{22} are the components of the inverse matrix

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = (K_{tr} K_m K_w)^{-1}$$

in eqn. 6.

On the other hand, using the distributed-parameter system of the object layer, the input impedance $Z_{in-obj}(\omega)$ at the surface of the object layer is given by

$$Z_{in-obj}(\omega) = Z_{obj} \frac{1 + \Gamma_{o2} \exp(-2\gamma_{obj} l_{obj})}{1 - \Gamma_{o2} \exp(-2\gamma_{obj} l_{obj})} \quad (9)$$

where γ_{obj} and l_{obj} are the propagation constant and thickness of the object layer, respectively, and Γ_{o2} is the reflection coefficient from the object to medium 2 defined by $\Gamma_{o2} \triangleq (Z_{w'} - Z_{obj})/(Z_{w'} + Z_{obj})$. Thus, the reflection characteristic, $\Gamma(\omega)$, from medium 1 to the object is given by

$$\Gamma(\omega) = \frac{Z_{in-obj}(\omega) - Z_w}{Z_{in-obj}(\omega) + Z_w} = \frac{\Gamma_{1o} + \Gamma_{o2} \exp(-2\gamma_{obj} l_{obj})}{1 + \Gamma_{1o}\Gamma_{o2} \exp(-2\gamma_{obj} l_{obj})} \quad (10)$$

where $\Gamma_{1o} \triangleq (Z_{obj} - Z_w)/(Z_{obj} + Z_w)$. Moreover, $\Gamma(\omega)$ of eqn. 10 is described by

$$\Gamma(\omega) = \Gamma_{1o} + (1+\Gamma_{1o}) \frac{\Gamma_{o2} \exp(-2\gamma_{obj} l_{obj})}{1 + \Gamma_{1o}\Gamma_{o2} \exp(-2\gamma_{obj} l_{obj})} (1 - \Gamma_{1o}) \quad (11)$$

Since the second term of the right side of eqn. 11 is decomposed into an infinite series, $\Gamma(\omega)$ is described by

$$\Gamma(\omega) = \Gamma_{1o} + (1+\Gamma_{1o})\Gamma_{o2} e^{-2\gamma_{obj} l_{obj}} (1 - \Gamma_{1o}) \times \sum_{n=0}^{\infty} (-1)^n \{\Gamma_{1o}\Gamma_{o2} e^{-2\gamma_{obj} l_{obj}}\}^n \quad (12)$$

By describing γ_{obj} by $\alpha_{obj} + j\omega/v_{obj}$ and defining $\tau_{obj} = l_{obj}/v_{obj}$, the term $\gamma_{obj} l_{obj}$ is given by $\alpha_{obj} l_{obj} + j\omega\tau_{obj}$. Thus, $\Gamma(\omega)$ of eqn. 12 corresponds to the impulse train $r(t)$ in the time domain, given by

$$r(t) = \Gamma_{1o}\delta(t) + (1+\Gamma_{1o})\Gamma_{o2}(1 - \Gamma_{1o}) \times \sum_{n=1}^{\infty} (-1)^n \Gamma_{1o}^n \Gamma_{o2}^n e^{-n\alpha_{obj} l_{obj}} \delta(t - 2n\tau_{obj}) \quad (13)$$

where $\delta(t)$ is the Dirac delta function. Therefore, using $\hat{Z}_{in-obj}(\omega)$ of eqn. 9 estimated from V_3 and V_e, both of which are measured at the electrical port, the reflection characteristic $\hat{\Gamma}(\omega)$ in eqn. 10 can be determined. By applying the inverse Fourier transform to the resultant $\hat{\Gamma}(\omega)$, the impulse train $\hat{r}(t)$ can be obtained. The estimated characteristics $Z_{in-obj}(\omega)$ and $\Gamma(\omega)$ are independent of the input voltage. Since the characteristics of the transducer and the matching layer are completely removed in the resultant estimate $\hat{r}(t)$, the transit time of the ultrasound in the object layer is accurately determined from the interval between the impulses.

Simulation experiments: A PZT-5A piezoelectric disc having a centre frequency of 3MHz and a diameter of 10mm was tested to confirm the principle. The parameters assumed were as follows: $Z_w = Z_{w'} = 1.5 \times 10^6 \text{kg/m}^2\text{s}$, $Z_{obj} = 1.6 \times 10^6 \text{kg/m}^2\text{s}$, $l_w = 100\text{mm}$, $l_{obj} = 0.2\text{mm}$. Fig. 3 shows the impulse train $\hat{r}(t)$ of eqn. 13 as estimated by the inverse Fourier transform of $\Gamma(\omega)$, obtained from $Z_{in-obj}(\omega)$ in eqn. 9. The transient response of the transducer is completely removed and an impulse train with an interval corresponding to the thickness of the thin layer is obtained.

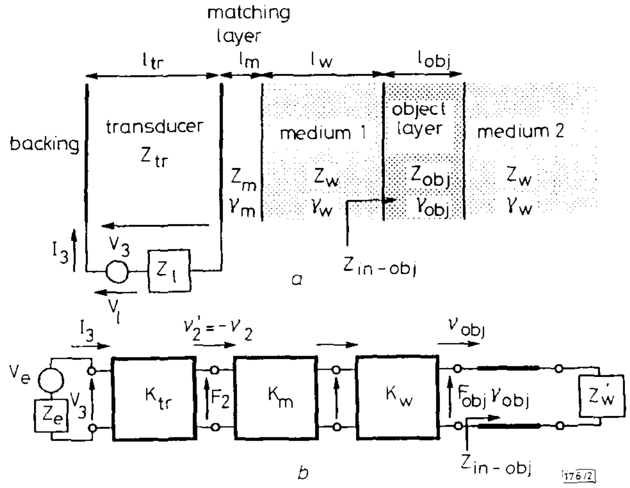

Fig. 2 *Model of system*

a Physical model of estimation of thickness and acoustical impedance of object from electrical port of transducer via matching layer and acoustical medium 1
b Exact equivalent circuit of model for compressional plane wave using backing material

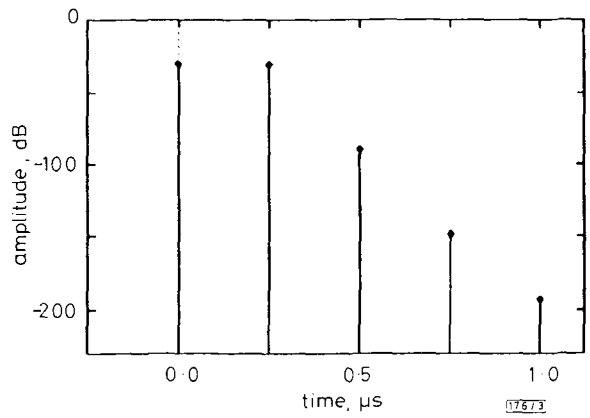

Fig. 3 *Impulse train 10 log|$\hat{r}(t)$|² of eqn. 13 estimated by inverse Fourier transform of $\hat{\Gamma}(\omega)$ obtained from $\hat{Z}_{in-obj}(\omega)$ in eqn. 9*

Conclusions: We have proposed a new method to increase the spatial resolution in the measurement of the thickness of an object using a piezoelectric transducer in the pulse-echo method. Since the reflection characteristics obtained from the acoustic input impedance have a wide range of frequencies, this procedure is optimum for determining the thickness of a thin layer.

© IEE 1995 *2 May 1995*

Electronics Letters Online No: 19950716

H. Kanai, T. Kimura and N. Chubachi (*Department of Electrical Engineering, Faculty of Engineering, Tohoku University, Sendai 980-77, Japan*)

References

1 HAYWARD, G., and JACKSON, M.N.: 'Discrete-time modeling of the thickness mode piezoelectric transducer', *IEEE Trans.*, 1984, **SU-31**, (3), pp. 137–150

2 REDWOOS, M.: 'Transient performance of a piezoelectric transducer', *J. Acoust. Soc. Am.*, 1961, **33**, (4), pp. 527–536